1. A “classical” spin can be defined by a three-dimensional vector $\mathbf{S}$ which has fixed magnitude $S = |\mathbf{S}|$ and can point in any direction. This is in contrast to a quantum spin which has the property that the component of its spin in the $z$-direction can have only certain discrete values. The state of each classical spin in a non-interacting paramagnet is defined by the co-ordinates $\theta$ (polar angle) and $\phi$ (azimuthal angle). Hence, in the presence of a magnetic field in the $z$-direction the energy of a spin with magnetic moment $\mathbf{m} = \gamma \mathbf{S}$ is

$$E(\theta, \phi) = -\mathbf{m} \cdot \mathbf{B} = -\gamma BS \cos \theta$$

(a) Evaluate the partition function for a set of $N$ non-interacting spins in thermal equilibrium at temperature $T$.

(b) Evaluate the internal energy $U$ as function of $B$ and $T$.

(c) Show that the magnetisation is $M(B, T) = M(B, T) \hat{z}$ where

$$\frac{M(B, T)}{N} = \gamma S \coth \left( \frac{\gamma BS}{k_B T} \right) - \frac{k_B T}{B}$$

(d) Find expressions for $M(B, T)$ at high and low temperatures. Explain why your expressions make physical sense.

(e) Sketch the magnetisation as a function of temperature, for two different values of the magnetic field, $B_1$ and $B_2$, where $B_2 = 2B_1$. 

2. Schroeder: Problem 7.5