Problems to hand in: 1,2,3

Problem 1: Peak in the Planck blackbody spectrum

In lectures we showed that the energy density of thermal radiation is given by \( U/V = \int_0^\infty u(\epsilon)d\epsilon \), where the Planck spectrum is given by
\[
u(\epsilon) = \frac{8\pi \epsilon^3}{(hc)^3 \exp(\epsilon/k_B T) - 1}
\]

(a) Show that the peak in the Planck spectrum occurs at \( h\nu = 2.82k_B T \).

(b) Change the variables in the expression for the energy density so that it is an integral over the wavelength, rather than the energy, and thus derive a formula for the photon spectrum as a function of wavelength. Find a formula for the wavelength where the spectrum peaks in terms of \( hc/k_B T \). Explain why this peak doesn’t occur at \( hc/2.82k_B T \).

Problem 2: Spectrum of the sun

Given that the temperature at the surface of the sun is approximately 5800K

(a) How much energy is contained in the electromagnetic radiation filling a cubic metre of space at the sun’s surface?

(b) Sketch the spectrum of this radiation as a function of photon energy, and mark the region that corresponds to visible wavelengths between 400 nm and 700 nm.

(c) What fraction of the energy is in the visible portion of the spectrum? (N.B. This has to be done numerically — a rough answer using your calculator is perfectly acceptable.)

Problem 3: Radiation from the body

(a) Roughly estimate the total power radiated by your body, neglecting any energy that is returned by your clothes and the environment (assume an emissivity of \( e=1 \)).

(b) Compare the total energy radiated by your body in one day to the energy in the food you eat. Why is there such a large discrepancy?

(c) The sun has a mass of \( 2 \times 10^{30} \) kg and radiates energy at a rate of \( 3.9 \times 10^{26} \) W. Which puts out more power per unit mass — the sun or your body?
Group problems: To be presented on Thursday 4th May 2006

**Problem 4: Surface temperature of the sun**

The value of the total radiant energy flux density at the earth from the sun normal to the incident rays is called the solar constant of the earth. The observed value integrated over all emission wavelengths and referred to the mean earth-sun distance is:

\[
\text{solar const.} = 1360 \text{ J s}^{-1} \text{m}^{-2}.
\]

Take the earth-sun distance as \(1.5 \times 10^{11} \text{ m}\) and the radius of the sun as \(R_C = 7 \times 10^{8} \text{ m}\).

(a) Show that the total rate of energy generation of the sun is \(4 \times 10^{26} \text{ J s}^{-1}\).

(b) Assume that the sun can be treated as a black body. From the result of (a) and the Stefan-Boltzmann constant \(\sigma_B = 5.67 \times 10^{-8} \text{ J s}^{-1} \text{m}^{-2} \text{K}^{-4}\), show that the effective temperature of the surface of the sun is \(T \sim 5800 \text{ K}\).

(c) At what wavelength does the emitted radiation have maximum intensity?

(d) Assume that the earth emits radiation as a perfect blackbody at all wavelengths (although it is not in equilibrium with the sun, but instead steady state.) What temperature would you estimate the surface to be?

**Problem 5: Schroeder problem 7.64**