Spontaneous vortices in the formation of Bose-Einstein condensates

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Phase transitions are ubiquitous in nature, ranging from protein folding and denaturisation, to the superconductor-insulator quantum phase transition, to the decoupling of forces in the early universe. Remarkably, phase transitions can be arranged into universality classes, where systems having unrelated microscopic physics exhibit identical scaling behaviour near the critical point. Here we study the dynamics of Bose-Einstein condensation, reporting the first experimental observation and characterisation of spontaneous vortex formation. Using a quantum-mechanical theory of the microscopic dynamics of finite-temperature Bose gases, we observe spontaneous vortex formation in simulations of condensation, finding close quantitative agreement with our experimental results. Vortex formation in the birth of superfluids is a consequence of spontaneous symmetry breaking, a phenomenon common to many phase transitions. Our studies of this process in condensates provide further understanding of the development of coherence in superfluids, and allow for the direct investigation of universal phase-transition models.
The spontaneous formation of topological defects, such as vortices, is common in continuous phase transitions\textsuperscript{1}. Defect formation has been experimentally observed in a broad variety of systems including superconductors\textsuperscript{2,3}, liquid crystals\textsuperscript{4,5}, nonlinear optics\textsuperscript{6}, hydrodynamic convection\textsuperscript{7,8}, and quenched ferromagnetic spinor Bose-Einstein condensates\textsuperscript{9}. Spontaneous defect formation has also been proposed as a mechanism for the creation of cosmic strings in the evolution of the universe\textsuperscript{10}. However, the microscopic dynamics of defect formation are often difficult to investigate, particularly for thermal-to-superfluid phase transitions\textsuperscript{11–13}; in superfluid $^4$He for example, vortex formation may be attributable to convective stirring mechanisms\textsuperscript{14}, significantly complicating studies of phase transition dynamics. Because of their amenability to manipulation and probing, Bose-Einstein condensates (BECs) offer unique experimental opportunities for studying microscopic details of phase transitions and topological defect formation. Furthermore, these superfluid systems can be described by microscopic theories\textsuperscript{15–23} that incorporate atomic interactions as well as quantum and thermal fluctuations. Although some theoretical estimates have predicted that vortices can be spontaneously formed in the BEC transition\textsuperscript{24,25}, they are energetically unfavorable and it has been suggested that this cost is sufficiently large that they are unlikely to be observed\textsuperscript{16,26}. Here we present an experimental and theoretical study of BEC growth in an evaporatively cooled atomic gas confined by harmonic and toroidal potentials, providing the first observations and statistical characterisation of spontaneous vortex formation in the BEC transition. Of particular interest and importance is the remarkable quantitative agreement between our experimental and theoretical results.

The physical process behind spontaneous vortex formation in superfluids is intimately con-
nected to the dynamics of superfluid growth. A simple picture, illustrated in Fig. 1, suggests that near the critical point of the phase transition proto-condensates of characteristic size $\xi$ form with individual macroscopic wave functions and random relative phases. These causally isolated regions then merge together as the transition proceeds with the requirement that the resulting macroscopic wave function is continuous. The initially discrete phase differences lead to a phase gradient in the merged state, with the lowest energy configuration having the smallest possible gradient; for example, regions with phases of $\pi/4$ and $7\pi/4$ will connect through a continuous phase change of $\pi/2$, rather than $3\pi/2$ in the opposite direction. Occasionally three or more merging regions can trap a $2\pi$ phase loop within the condensate, as illustrated in Fig. 1. Due to the requirement of continuity of the macroscopic wave function, the condensate density at these locations is topologically constrained to be zero, resulting in the formation of a quantised vortex. In the context of superfluid growth, this vortex formation process is a fundamental component of the Kibble-Zurek (KZ) mechanism\textsuperscript{1,10,27,28}. Based on universality classes for second-order phase transitions, the KZ mechanism provides a prescription for estimating a correlation length $\xi$ and hence the density of defects that may spontaneously form. In the KZ scenario, a system falls out of equilibrium when the relaxation rate drops below a quench rate $1/\tau_Q$, often characterised by the rate of cooling; $\xi$ then essentially remains static for a period of time on either side of the critical point. A principle result is that $\xi$ scales with $\tau_Q^{1/4}$; faster quenches (smaller quench timescales $\tau_Q$) produce smaller and more numerous proto-condensates, and hence higher densities of spontaneously formed defects.

Svistunov and co-workers have proposed a qualitative yet more complex three-stage scenario of superfluidity and condensate formation in a homogeneous Bose gas\textsuperscript{15,29–31} (for a summary see
Ref. [32]). In the kinetic (or weak-turbulent) stage, a wave in momentum space propagates towards zero momentum as energy is removed from the system, resulting in the macroscopic occupation of a number of low-energy atomic field modes. Interference between these modes with random relative phases leads to nodes in the total field, which appear as lines of zero atomic density. In the subsequent coherent (or strong-turbulent) stage, a quasi-condensate having local coherence but no long-range coherence grows around the lines of zero density, which simultaneously evolve into well-structured vortex cores. The final stage involves long-range ordering, where the superfluid relaxes into equilibrium and a true condensate with global phase coherence is achieved. Berloff and Svistunov numerically demonstrated this scenario for the homogeneous Bose gas in simulations of the Gross-Pitaevskii equation. These three stages, however, do not necessarily occur for the harmonically trapped Bose gas. If the mean time between collisions of trapped particles is greater than the time it takes for them to traverse the trap, a pure condensate whose phase coherence spans the entire system may form from the beginning. This assumption has always been made in previous models of condensate formation in harmonic traps, and with the exception of condensate growth in the hydrodynamic regime, has been broadly successful in describing experimental observations. In contrast, here we experimentally study condensate growth in a harmonic trap and observe vortices in the resulting condensates. We implement a new theoretical method incorporating interactions and fluctuations to model our experiments, revealing the role of excited states in condensate growth and in the dynamics of vortex formation.
Vortices in harmonic traps

In previous work we demonstrated that vortices can form during the controlled merging of three independent BECs with uncorrelated phases\textsuperscript{45}. In contrast, here we form a single BEC by evaporatively cooling an atomic gas through the BEC phase transition in a weak, non-rotating oblate harmonic trap with $\frac{\omega_z}{\omega_r} \approx 2$, where $\omega_z$ ($\omega_r$) is the axial (radial) oscillator frequency (see Methods for further details). In this section we describe data obtained with two different temperature quenches: Quench A uses a 6-s radio-frequency (rf) evaporative cooling ramp, and Quench B uses a sudden jump of the rf frequency to a final value. We use these two substantially different rf trajectories in order to show that vortex formation rates are not overly sensitive to the details of evaporation, as will be described in more detail below. Plots of condensate number versus time for both quenches are shown in Fig. 2a, with the measured temperature trajectories shown in the inset.

To first determine an estimate for the correlation length $\xi$ near the critical point, we follow the prescription outlined by Anglin and Zurek\textsuperscript{24}, finding $\xi \approx 0.7 \mu$m for both Quenches A and B. This is about a factor of 5 smaller than the radial harmonic oscillator length $a_r \equiv (\hbar/m\omega_r)^{1/2} \sim 3.8 \mu$m (where $m$ is the mass of a $^{87}$Rb atom) that characterises the condensate radius for small atom numbers. This estimate suggests that vortices can be created during the birth of the condensate in our experiments.

To look for the presence of vortex cores in BECs created in the laboratory, we suddenly remove the trapping potential at the end of the 6-s evaporative cooling ramp of Quench A (the BEC starts to form $\sim 2.5$ s into this quench), or $\sim 1.5$ s after the rf jump for Quench B (the BEC
starts to form ∼0.5 s into this quench). We allow each BEC to ballistically expand before imaging the atom cloud along the vertical direction (the z axis), which coincides with the symmetry axis of our trap (see Methods). Vortex cores well-aligned with the z axis appear as holes in the column-density distribution, as shown in Fig. 3a. We emphasise that our experimental procedure does not impart net angular momentum to the atomic cloud, such as through stirring or phase imprinting; our observations thus represent a new regime for the study of quantised vortex nucleation in BECs (see Supplementary Information for further discussion).

We have simulated condensate formation for these parameter regimes using the Stochastic Gross-Pitaevskii equation (SGPE) formalism\textsuperscript{19, 20} that represents the highly-occupied, low-energy modes of a Bose gas as a classical field. These evolve according to a generalised Gross-Pitaevskii equation that includes dissipation and thermal noise describing collisions between the partially condensed matter waves and the high-energy atoms in the thermal cloud. The details of rf evaporative cooling leading to condensation are difficult to simulate realistically\textsuperscript{39} and are often qualitatively unimportant. We instead implement an idealised cooling model with a sudden jump in chemical potential and temperature of the thermal cloud through the critical point for Bose-Einstein condensation. This leaves the SGPE classical field out of equilibrium with the thermal cloud; the subsequent dynamics and relaxation results in condensate formation (see Methods for details). The dashed blue (solid red) curves in Fig. 2a show the growth in condensate number for the simulations of Quench A (B), and the good agreement with experiment allows a meaningful comparison of other observables.
We find that vortices are spontaneously formed in our simulations, as shown in the column-density images of Fig. 3b and the phase-profile images of Fig. 3c. Within the Wigner formalism underpinning the SGPE we can interpret each trajectory as corresponding to a single experimental realisation. We therefore study vortex dynamics as each condensate grows and compare the vortex observation statistics with our experimental results. In both our laboratory and numerical procedures, we repeat the BEC creation procedure for each quench, and analyze statistics of vortex observations. For each data set, we count the number \( n_j \) and extract the fraction \( P_j \) of images that show \( j = 0, 1, \) or 2 vortex cores within a radius of vortex core displacements \( d_c < 0.8 \), where \( d_c \equiv |\mathbf{R}|/R_{TF} \), \( \mathbf{R} \) is the radial position vector of the vortex core relative to the BEC centre, and \( R_{TF} \) is the Thomas-Fermi radius of the BEC in the \( z = 0 \) plane. We use \( P_j \) as our estimate of the probability of observing spontaneously formed vortices in a single run.

For Quench A, 23% to 28% of our experimentally obtained images contain at least one visible vortex core. Our quoted error ranges are defined by our uncertainty in determining whether or not an image shows a vortex. For example, a vortex core tilted or bent with respect to the imaging axis will decrease the visibility of the core; localised decreases in the density profile of a given image thus may or may not clearly indicate the presence of a core, and we use these uncertain cases to define our error. For Quench B, 15% to 20% of our images show at least one vortex core. While the two quenches utilise quite different rf evaporation trajectories, they exhibit similar cooling and BEC growth rates. For this reason we can expect the clear statistical similarities observed between the two data sets. Further details of our observations are summarised in Table 1.
From our simulations we can additionally determine vortex observation probabilities as a function of time for each quench. To determine the presence of a vortex we consider an instantaneous slice of the classical field in the $z = 0$ plane of the trap, and detect all phase-loops of $\pm 2\pi$, with $d_c < 0.8$, where the Thomas-Fermi radius is based on the (time-dependent) condensate number. We find that the majority of vortices (although not all) are aligned with the $z$ axis of the trap. The vortex observation probabilities obtained from our simulations are plotted against time in Fig. 2b for Quenches A and B, where the experimentally measured probabilities are plotted as horizontal grey bars with widths corresponding to our measurement error ranges.

According to our SGPE simulations the number of vortices decreases as a function of time; this is consistent with the model as the thermal bath is independent of time and has no angular momentum, so the thermodynamic final state should be a condensate without any vortices. In this respect the simulations diverge from our experimental observations, where we have found no significant variation of the vortex observation probabilities with time. For example, with the conditions of Quench B, experimental vortex observation statistics are approximately constant between observation times of 1.5 s and 6 s after the initiation of the quench, indicating negligible damping of vortices on this timescale. This low damping rate is consistent with the comparatively small thermal atomic component observed, indicating that a kinetic theory of thermal cloud dynamics may be needed to fully account for the long-time behaviour of the experiment. We therefore quote our results (summarised in Table 1) at $t = 3.5$ s for Quench A, and $t = 1.5$ s for Quench B, based upon the experimental observations that vortex damping is negligible, and because the BEC is nearly fully formed at these times.
Vortices in toroidal potentials

Adding a 2D potential barrier to the centre of a 3D harmonic trap forms a multiply-connected, toroidal potential in which a BEC may display both a persistent superfluid current\(^{46}\) as well as free vortices circulating around the barrier. The pinning of superfluid flow, which can be viewed as trapping a vortex core at the central barrier, may influence both vortex dynamics during BEC growth and observations of vortices after the BEC is formed: if a vortex becomes pinned by the barrier the likelihood of complete self-annihilation between pairs of spontaneously formed vortices of opposite charge are reduced, and the probability of finding vorticity in the fully formed BEC is increased.

By focusing a blue-detuned laser beam propagating along the \(z\) axis into the centre of the trap (see Methods for beam details), we experimentally studied BEC growth in a toroidal potential with a 6-s final evaporative cooling ramp identical to Quench A; we identify this data set as Quench C. An *in-situ* phase-contrast image of a BEC in the toroidal trap is given in the leftmost image of Fig. 3d. Note that the dark region in the BEC centre is due to atomic fluid displacement by the laser beam, and does not indicate the presence of a vortex core. After creating the BEC, we remove the optical potential by ramping down the beam's power over 100 ms and immediately allow the BEC to expand from the trapping potential. For these conditions, we found 56\% to 62\% of our images contained at least one visible vortex core, more than a factor of two increase over the Quench A statistics found with our harmonic trap. Example expanded BEC images are shown as the centre and rightmost images in Fig. 3d.
Condensate formation rates were not experimentally measured for Quench C; for the simulations we use the parameters of Quench A but with an additional repulsive Gaussian barrier with a height of $33 \hbar \omega_r$. A plot of simulated condensate growth versus time is shown as the green dot-dashed curve in Fig. 2a, resulting in smaller condensates compared to Quench A, as also observed experimentally. Three examples of the numerically obtained column density and phase are shown in Figs. 3e–f. The vortex observation statistics are plotted as a green dot-dashed line in Fig. 2b; we find that the vortex observation probability is approximately twice that of the harmonic case (as with the experimental data) but somewhat lower in overall magnitude than the experimental observations. We note that the curve does not exhibit decay below 40% in contrast to the harmonic case — this corresponds to vortices that are pinned by the central barrier. Additional statistics for the experimental and simulated data are provided in Table 1. (In the experimental data, 1–2 images, or 2%–4%, showed 3 cores. This is not represented in Table 1.)

We observe in the simulated BEC growth dynamics that the central potential can pull in and pin a single spontaneously formed vortex in some cases. By examining the distribution of vortex core positions for both the experimental and simulated data we see that the toroidal trap induces a central clustering of vortex cores, whereas in the harmonic trap the core positions are more evenly spread throughout the BECs. A visual comparison of these cases is given in Figs. 4a–d. Figures 4e–f show a histogram of vortex core displacements away from the mean core position for the two different trap geometries for theory and experiment. Due to the observation that the cores in the experimental toroidal trap data are likely to be found within a small region within the BEC, we interpret our results as indicating that the observed cores are likely to have been pinned
to the central barrier prior to expansion. This suggests possibilities for future controlled studies of spontaneous vortex formation, perhaps with multiple sites at which vortices may be pinned, in order to better understand the density and numbers of vortices created at early times in BEC growth.

**Dynamics of spontaneous vortex formation**

In the Supplementary Information, we provide movies of simulated condensate formation for Quenches A and C. Here we describe one run in which a single vortex persists to the final time step for a Quench A simulation in the purely harmonic trap. After the system temperature is initially lowered but before a bulk BEC has clearly formed, the density profile of the atomic field clearly fluctuates temporally and spatially. A snapshot illustrating this state is given in Fig. 5a, with iso-density surfaces shown in a three-dimensional rendering. As time progresses, a bulk BEC begins to grow, and a tangle of vortices is trapped within the BEC as shown in Fig. 5b, in qualitative agreement with both the predictions of Svistunov et al. and the KZ mechanism. At later stages, a nearly uniform condensate exists with clear vortex cores, as shown in Fig. 5c. This state eventually damps to a single core, seen in Fig. 5d.

In order to examine the relationship between our observed vortex statistics and the predictions of the KZ mechanism, one would ideally perform controlled ramps of either $\mu$ or $T$ through the critical point in order to control BEC growth rates. In our experiments, we have been able to increase our BEC growth rate by a factor of two to three over that of Quench A by optimizing rf
evaporation trajectories. In a set of 60 images obtained under these faster conditions, 47% to 53% had a vortex core visible, a factor of $\sim 2$ increase over the data of Quenches A and B. In simulations, we increase the condensate growth rates by increasing the coupling to the thermal cloud. We find that while this results in more vortices up to a point, eventually the damping rate is sufficient to ensure the condensate is phase coherent throughout its growth (see Supplementary Information).

Our work places the concept of spontaneous topological defect formation on a theoretical foundation that has not been available in analogous studies in other systems, with the quantitative agreement between our experimental and theoretical results of primary importance for their mutual interpretation. We have shown that defect formation is a natural consequence of the inherent fluctuations described by quantum-mechanical theories of trapped interacting gases. We conclude that even in the lowest-temperature phase transition that can be studied, thermal fluctuations can play an important role, and spontaneous topological defect formation is virtually unavoidable at the critical point of the transition. In the future we will study in greater detail exactly how a condensate forms in this regime: is the simple picture of proto-condensates merging as suggested by the Kibble-Zurek mechanism a sufficient description, or is the more complex scenario described by Svistunov et al. necessary? We anticipate that our results will motivate further investigations of the KZ mechanism and the universal dynamics of second-order phase transitions using ultracold gases, and these will provide new details of generic phase transition phenomena. Finally, with further simulations and comparisons with experiments, one may expect to discover new understanding of the development of coherence in the birth of a superfluid, a tantalizing prospect addressing the interface between the classical and quantum worlds.
METHODS SUMMARY

In our experiments, $^{87}$Rb atoms in the $|F = 1, m_F = -1\rangle$ hyperfine state are loaded into a time-averaged orbiting potential (TOP) trap\textsuperscript{47}. Forced evaporative cooling of the atoms proceeds until the atomic cloud temperature is just above the condensation critical temperature. The trap frequencies are then adiabatically relaxed, and a final stage of cooling as described in the text lowers the temperature below $T_c$. Additional details about our trap parameters and our data collection procedure are provided in the Methods section (available in the online version of the paper at www.nature.com/nature).

Our numerical approach describes both the condensate and the low-energy, highly occupied modes of the gas with a single classical field. This field is coupled to a bath of thermal atoms, parameterised with a chemical potential $\mu$ and temperature $T$ above a cut-off energy $E_{\text{cut}}$. Further details about our implementation of the SGPE approach are provided in the Methods section.

METHODS

Evaporative cooling. During the main evaporative cooling stages of our experimental procedure, our TOP trap has an initial instantaneous vertical magnetic field gradient of $B'_z = 300$ G/cm, and a $B_0 = 41$-G magnetic bias field that rotates in a horizontal plane at a frequency of $\omega_{\text{rot}} = (2\pi) \cdot 4$ kHz or $\omega_{\text{rot}} = (2\pi) \cdot 2$ kHz. Evaporative cooling then proceeds over 72 seconds as $B_0$ decreases to $\sim 5.2$ G, leaving a trapped cloud of atoms at a temperature just above the condensation critical temperature $T_c$. $B'_z$ is then adiabatically reduced to $\sim 54$ G/cm, weakening the harmonic oscillator
trapping frequencies to a measured radial (horizontal) trapping frequency of $\omega_r = 2\pi \cdot 7.81(9)$ Hz and an axial (vertical) trapping frequency of $\omega_z = 2\pi \cdot 15.3(2)$ Hz. In the final stage of our cooling cycle for Quench A, we use a continuous 6-s ramp of the rf field, which induces the evaporative cooling of the atom cloud from 70 nK to 20 nK, with $T_c \sim 42$ nK, to create condensates of $N_c \sim 5 \times 10^5$ atoms. For Quench B, the continuous rf evaporative cooling ramp is replaced with a sudden rf jump to a final rf value, followed by a hold of the atomic sample in the trap before release and imaging. In this situation we find $T_c \sim 35$ nK and the final condensate number is $N_c \sim 3 \times 10^5$ atoms.

**TOP trap.** To ensure that the rotating bias field of the TOP trap plays no significant role in the spontaneous formation of vortices, we measured the $z$-component of the net orbital angular momentum $L_z$ of our condensates using surface wave spectroscopy. We excite a quadrupolar oscillation of the BEC in the horizontal plane, and stroboscopically probe the BEC with a set of non-destructive in-trap phase-contrast images, obtained by probing along the $z$ axis$^{48,49}$. The quadrupolar oscillations will then precess with a rate and direction proportional to $L_z$. In our measurements, there was no significant biasing of surface mode precession in a direction corresponding to the TOP trap rotation direction, an indication that TOP trap temporal dynamics have little to no influence on spontaneous vortex formation. This is discussed further in Supplementary Information.

**Toroidal trap.** A potential barrier was added to the centre of the magnetic trap using a focussed blue-detuned laser beam with a wavelength of 660 nm, $\sim 18$ $\mu$W of power, and a 6-$\mu$m Gaussian
radius. The beam intensity corresponds to a potential barrier of approximately $k_B \cdot 20 \text{nK}$, where $k_B$ is Boltzmann’s constant. This can be compared with a $\sim k_B \cdot 10 \text{nK}$ chemical potential of a fully formed BEC in the purely harmonic trap. The beam was adiabatically ramped on prior to the final 6-s evaporation ramp, only slightly perturbing the thermal cloud but providing enough additional potential energy along the trap axis to exclude BEC atoms from the $z$ axis of the trap.

**Imaging.** Our main imaging procedure involves the sudden removal of the magnetic trap, and the subsequent ballistic expansion of the trapped cloud. After 59 ms of expansion in the presence of an additional magnetic field to support the atoms against gravity, the atomic cloud is illuminated with near-resonant laser light, and the absorption profile of the atomic density distribution is imaged onto a camera. In our grey scale images, lighter shades represent higher optical depth, proportional to integrated column density along the $z$-direction line-of-sight. A clear vortex core aligned along the $z$ axis appears as a dark hole in the density distribution.

**Stochastic Gross-Pitaevskii theory.** We denote the condensate and low-energy portion of the trapped gas with the field $\alpha(\mathbf{x}, t)$, and define the Gross-Pitaevskii operator

$$L_{GP} = -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{x}) + g|\alpha(\mathbf{x}, t)|^2,$$

where $m$ is the mass of an atom, $V(\mathbf{x})$ is the trapping potential, $g = 4\pi\hbar^2a/m$ characterises the strength of atomic interactions, and $a$ is the s-wave scattering length. The equation of motion for the field is

$$d\alpha(\mathbf{x}, t) = \mathcal{P} \left\{ -\frac{i}{\hbar} L_{GP} \alpha(\mathbf{x}, t) dt + \frac{G(\mathbf{x})}{k_BT} (\mu - L_{GP}) \alpha(\mathbf{x}, t) dt + dW_G(\mathbf{x}, t) \right\},$$
which has been derived from first principles using the Wigner phase-space representation\textsuperscript{20}. The first term on the right describes unitary evolution of the classical field according to the Gross-Pitaevskii equation. The second term represents growth processes, i.e. collisions that transfer atoms from the thermal bath to the classical field and vice-versa, and the form of $G(x)$ may be determined from kinetic theory\textsuperscript{23}. The third term is the complex noise associated with growth satisfying $\langle dW_G^*(x, t) dW_G(x', t') \rangle = 2G(x) dt \delta(x - x') \delta(t - t')$. This form of the noise correlation is consistent with the fluctuation-dissipation theorem. The projection operator $P$ restricts the dynamics to the low-energy region\textsuperscript{18,21} defined by all harmonic oscillator modes with energy $\epsilon < E_{\text{cut}} = 40\hbar\omega_r$ for these calculations, which for our parameters gives about three particles per mode at the cutoff. For typical experimental parameters this method is accurate from slightly above the critical temperature to colder temperatures where there is still a significant thermal fraction\textsuperscript{22}.

The initial states used in our simulations are independent field configurations generated by ergodic evolution of the SGPE at equilibrium with the thermal cloud with $\mu_i = \hbar \omega_r$ and $T_i = 45 (35)$ nK for Quench A (B), representing the thermalised Bose gas above the transition temperature\textsuperscript{22}. These parameters are suddenly changed to the final values $\mu_f = 25 (22) \hbar \omega_r$ and $T_f = 34 (25)$ nK for Quench A (B), and we perform simulations for 300 (298) sets of initial conditions. By averaging over the different realisations we can calculate any quantum-mechanical observable as a function of time, and in particular we diagonalise the single-particle density matrix to find the number of atoms in the condensate\textsuperscript{21}.

Because vortex formation is expected to depend upon the BEC growth rate, we adjust the
coupling rate describing Bose-enhanced collisions between the classical field and thermal cloud to obtain a close match of the experimental BEC growth curves. We choose a spatially constant dimensionless rate for the coupling $\gamma = \frac{hG(x)}{k_B T}$ to the high energy component; in principle this is specified by a quantum Boltzmann integral, but here we treat it as an experimental fitting parameter for the condensate growth rate; it is never more than a factor of two different from the result of Eq. (A11) in Bradley et al.\textsuperscript{23} The effect of these parameter choices is discussed further in Supplementary Information.


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**Supplementary Information** is linked to the online version of the paper at www.nature.com/nature.
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Author Information  The authors declare that they have no competing financial interests. Correspondence and requests for materials should be addressed to B.P.A. (bpa@optics.arizona.edu).
Table 1 | Harmonic and toroidal trap vortex observation statistics.

<table>
<thead>
<tr>
<th>Quench</th>
<th>runs</th>
<th>0 cores</th>
<th>1 core</th>
<th>2 cores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(n_0)</td>
<td>(P_0)</td>
<td>(n_1)</td>
</tr>
<tr>
<td>A, expt.</td>
<td>90</td>
<td>65 – 69</td>
<td>0.72 – 0.77</td>
<td>18 – 23</td>
</tr>
<tr>
<td>A, sim.</td>
<td>300</td>
<td>229</td>
<td>0.76</td>
<td>68</td>
</tr>
<tr>
<td>B, expt.</td>
<td>98</td>
<td>78 – 83</td>
<td>0.80 – 0.85</td>
<td>13 – 18</td>
</tr>
<tr>
<td>B, sim.</td>
<td>298</td>
<td>234</td>
<td>0.79</td>
<td>61</td>
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<tr>
<td>Toroidal, expt.</td>
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<td>20 – 23</td>
<td>0.38 – 0.44</td>
<td>15 – 25</td>
</tr>
<tr>
<td>Toroidal, sim.</td>
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<td>147</td>
<td>0.49</td>
<td>137</td>
</tr>
</tbody>
</table>

For each quench condition with total number of runs listed, \(n_j\) shows the number of times \(j\) vortices were observed, and \(P_j\) shows the corresponding measured probability of observing \(j\) vortices in a single run.
Figure 1  | Schematic of spontaneous vortex formation in a trapped BEC. (Left) As a thermal gas (mottled grey shade) is cooled through the BEC transition, isolated proto-condensates of approximate size $\xi$ and unpredictable phase may form. Quantum phase ranges from 0 to $2\pi$, and is represented here by shades of grey, as indicated by the gradient bar at the right. (Right) Proto-condensates eventually merge to form a single BEC (continuous greyscale region), potentially forming quantised vortices. Here, a positive (negative) vortex is labeled with a cross (circle), with the phase winding direction corresponding to the direction of superfluid flow and phase gradient around the vortex core.

Figure 2  | Condensate formation and vorticity.  

Figure 3  | Vortices in the harmonic and toroidal traps. a, Example 200-µm-square expansion images of BECs created in a harmonic trap, showing single vortices (left, centre) and two vortices (right). b, c, Sample simulation results from Quench B, showing
integrated column densities along $z$ (in b) and associated phase profiles in the $z = 0$ plane (in c), with vortices indicated by crosses and circles at $\pm 2\pi$ phase windings. d, Left image: 70-$\mu$m-square phase-contrast experimental image of a BEC in the toroidal trap. Remaining images: vortices in 200-$\mu$m-square expansion images of BECs created in the toroidal trap. e, f, Simulations of BEC growth in the toroidal trap show vortices (as in b,c) and persistent currents.

Figure 4 | Vortex core pinning. a, c, Representation of the experimentally measured positions of the vortex cores relative to the Thomas-Fermi radius (outer circles) for a, the harmonic trap and c, the toroidal trap. b, d, Corresponding theoretical results, crosses and circles indicate oppositely charged vortices. e, f, Comparison of the statistics of the vortex locations, binned in steps of $0.1 R_{TF}$, for the experimental data (left bar in each pair) and theoretical simulations (right bars). Harmonic trap results are shown in e, toroidal trap results are shown in f. For the experimental data, only images clearly showing a single core are considered.

Figure 5 | BEC growth dynamics. a–d, Four snapshots during the simulated growth of a BEC showing isodensity surfaces (in light red) as described in the text. Vortex cores of opposite charges about the $z$ axis are indicated as magenta and cyan lines. The corresponding times are a, 0.13 s; b, 0.45 s; c, 0.57 s; d, 1.57 s, where $t = 0$ s is the time when the quench is initiated in the simulation. The full movie from which these images were taken is provided as Supplementary Video 3.
Figure 1
Figure 2

(a) Graph showing the number of atoms $N_o$ (in $10^5$ atoms) as a function of time. The graph includes data points and lines indicating different stages of the process.

(b) Graph showing the vortex probability as a function of time. The graph includes data points and lines indicating different stages of the process.
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Figure 3
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Figure 5
Supplementary Information:  
Spontaneous vortices in the formation of Bose-Einstein condensates  
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Supplementary Discussion 1  
Experimental vortex observations. In our experiments on condensate formation we have not performed any uncommon experimental techniques in order to observe spontaneously formed vortices. The Bose gas of 87Rb atoms was evaporatively cooled in a similar manner to those of most other dilute-gas BEC experiments. One might assume that rapid cooling is necessary in order for several proto-condensates and hence spontaneous vortices to form, in contradiction with the seemingly long cooling time scales for Quenches A and C. It is then natural to wonder why our experiments have yielded observations of spontaneous vortex formation during the BEC phase transition given that this has not been reported previously. We believe that the major factors contributing to our observations are as follows:  
1. We evaporatively cool our gas to near-degeneracy in a tight trap, and then relax the trap to the final trapping frequencies before our final cooling ramp. This relaxation reduces the atomic collision rate at the point at which condensation occurs, and potentially decreases the ratio of the correlation length to the size of the ground state of the trap (the harmonic oscillator length) and thus increases the number of vortices expected to form. In the KZ mechanism, the correlation length $\xi$ depends on the quench time $\tau_Q$, so specific details of the ratio of correlation length to the harmonic oscillator length will depend upon specific cooling trajectories, as well as trap frequencies. Regardless of the initial number of vortices formed, we expect that vortices may survive for longer times in weaker traps, where the lower atomic density and longer trap oscillation time scale generally decrease the rates of dynamical processes.  
2. Our experiment is performed in an oblate trap with an approximately 2:1 aspect ratio, in which it is favourable for the vortices to align along the vertical symmetry axis of the trap. All condensate formation dynamics experiments reported to-date in the literature have been performed in cigar-shaped traps.  
3. We introduce a linear magnetic field gradient during expansion that supports the condensate against gravity and allows for long expansion times before imaging. During ballistic expansion of the condensate, the vortex cores expand relative to the condensate size such that our imaging system can optically resolve the cores. Unless one is actively looking for vortex cores, such long expansion times (59 ms in our case) are not necessarily needed or desired in other experiments.  
4. We have the ability to acquire images of trapped atoms along both the vertical axis (the direction of strongest confinement) as well as the horizontal axis, the more standard imaging axis for TOP trap experiments. The vertical imaging axis allows us to detect vortices that are well-aligned with the vertical trap axis.  
It seems entirely possible that spontaneous vortices have been present in other experiments, but simply could not be resolved, damped quickly, or were not sought.  
Influence of the TOP field. Using the method of quadrupolar surface mode excitation and precession detection, early experimental results from this work appeared to show a significant bias of the direction of fluid circulation for the observed spontaneously formed vortices. Our initial measurements seemed to indicate that the direction of vortex fluid flow for the spontaneously formed vortices was most often found to be circulating in the same azimuthal direction as the rotating magnetic field that forms our TOP trap. This suggested that some form of stirring or other influence from the TOP trap was responsible for the apparent directional bias. However, our later and more detailed measurements revealed that the apparent bias was a result of a scissors mode oscillation that was unintentionally excited. Coupled with the times that we initially chose to stroboscopically probe the surface mode precession, the scissors mode oscillation appeared as a bias to the precession direction. Such directional biasing is not detected in our more recent measurements, and we do not believe that the rotating field of the TOP trap plays a role in vortex formation.
Supplementary Video Legends

Here we describe the main features of a selection of movies of our condensate formation simulations. Ten example movies are given: five corresponding to condensate formation in the purely harmonic trap (Quench A), and five in the toroidal trap (Quench C). As specified in the title frames, all simulations in this section have initial chemical potential \( \mu_i = \hbar \omega_r \) and initial temperature \( T_i = 45 \) nK, and corresponding final values \( \mu_f = 25\hbar \omega_r \) and \( T_f = 34 \) nK. Quench C has an additional central repulsive gaussian barrier of height \( 33\hbar \omega_r \).

All movies begin at time \( t = 0 \) corresponding to the instant that the chemical potential and temperature are suddenly changed in order to induce condensation. The left panel of each movie shows a rendering of a three-dimensional isosurface of the density of the classical field, with magenta (cyan) dots indicating the presence of positive (negative) phase windings about the \( z \) direction. We indicate only the phase windings located in a spatial region where there is a significant condensate density, as determined by the Penrose-Onsager criterion. At earlier times, these points of fluid circulation seem to be distributed almost at random and at high spatial density, however at longer times these points can often be seen to form vortex lines that extend through the condensate. The top-right panel of each frame plots the column density along the \( z \)-axis of the classical field. The greyscale colourmap is fixed in time as indicated by the gradient bar at the far right (arbitrary units), and the numerical data appear similar to the type of images obtained in the experiment, although without the time-of-flight expansion. Vortices can often be seen as holes or low density regions in the column density image depending on their orientation with respect to the \( z \)-axis. The lower right panel shows the phase of the field in the \( z = 0 \) plane. Positive (negative) phase windings are indicated as magenta crosses (cyan circles) in the regions of significant condensate density. It should be noted that many additional \( 2\pi \) windings in this panel occur where there is no condensate density, and are hence not to be interpreted as vortices.

Condensate formation movies in a harmonic trap: Quench A. From 300 simulations of Quench A, 229 contained no vortex cores after 1.5 s. The following five movies show a selection of outcomes, including cases with 0, 1, and 2 long-lived vortices.

Supplementary Video 1: The first movie is an example of BEC formation in which there are no vortices trapped in the final condensate, and is provided for comparison purposes. At early times there is some indication of vortices in the condensate; however, these do not survive at longer times. In this situation the condensate appears to grow adiabatically in its ground state, as was assumed in the condensate formation calculations of Refs. [34–39,42,43]. The reader should note that cases such as this are typical for the data set corresponding to Quench A.

Supplementary Video 2: This movie provides an example of BEC formation with a single vortex line that survives at long times and remains close to the centre of the condensate. The vortex line remains approximately vertical, and is usually easily visible in the column density plot. The reader should note that there are many examples similar to this where the vortex line is not so close to the centre, and instead slowly spirals towards the boundary of the condensate over a time scale of several seconds.

Supplementary Video 3: In this example there are three clear vortices at early times. Two of these arise near the edge of the condensate and these damp out relatively quickly, leaving a single vortex line of opposite charge near the centre that survives to the end of the simulation. This is an example of a vortex that does not directly align with the \( z \) axis, and the column density often shows an elongated density dip. This movie uses the same simulation data as in Figure 5 of the main text.

Supplementary Video 4: This simulation results in two oppositely charged vortices that remain well separated, precessing about the centre in opposite directions. It should be noted that only 3 out of 300 simulations in this data set clearly exhibited more than one core at long times.

Supplementary Video 5: The final movie for the data set of Quench A shows a number of vortex cores that undergo some rather complicated dynamics as they move about within the condensate, including vortices that cross each other and reconnect in a different configuration, and an example where a vortex line flips (and hence changes colour). This example shows the most complicated vortex dynamics observed in this data set.

Condensate formation movies in a toroidal trap: Quench C. From the 300 simulations of Quench C, 147 contained no vortices after 1.5 s. The following movies show five examples where vortices arose.

Supplementary Video 6: The first movie from this data set shows the formation of a single vortex. This is not readily seen in the isodensity surface, as the \( 2\pi \) phase winding is located in the centre of the trap where there is no condensate density.

\[ T = \pi \]
However, the phase winding about the entire condensate is clear in the bottom-right plot of the phase in the \( z = 0 \) plane. Whereas in the harmonic trap all vortices will eventually make their way to the edge of the condensate and disappear, here the gaussian barrier pins the vortex to the centre — this is an example of a persistent current.

**Supplementary Video 7:** In this example it is possible to see a pair of oppositely charged vortices at \( t = 0.8 \) s, one of which becomes pinned to the central barrier while the other ends up precessing about the centre in the region of maximum condensate density.

**Supplementary Video 8:** Here there are some rather complicated early dynamics that result in a single vortex precessing about the outside of the condensate, but with no persistent current.

**Supplementary Video 9:** There are a number of vortices early on, and near 0.6 s it seems that there is a doubly charged persistent current. However, only one vortex ends up being trapped on the central gaussian barrier and the other vortex of the same charge remains within the bulk of the condensate, precessing rapidly about the centre. The rate of precession should be compared with the example from Supplementary Video 7, where the vortex near the edge is of the opposite charge to that pinned by the barrier.

**Supplementary Video 10:** The final example shows a stable doubly-charged persistent current. This is particularly interesting given the energy difference between this and the thermodynamic ground state of the condensate with no current. This is the only example from the data set where a stable doubly-charged persistent current was seen.

**Supplementary Discussion 2**

**Dependence of results on simulation fitting parameters.** In the main text we have compared our experimental results to simulations in which the chemical potential \( \mu \) and temperature \( T \) underwent an instantaneous quench to their final values. In this section we discuss the dependence of the simulations modelling Quench A on the parameters that characterise our idealisation of the cooling process.

The final temperature chosen for the simulations was based on the approximate value measured experimentally once the slope of the condensate growth curve began to flatten out. We have also performed a set of 100 simulations where the final temperature was \( T_f = 23 \) nK instead of 34 nK, and found no difference in the vortex probabilities within statistical noise. This implies that for our simulations the time-scale of the temperature decrease and the exact final value has little effect.

We have also performed the same simulations for 100 trajectories using a linear ramp of the temperature and chemical potential from their initial to final values over time scales of 1, 2, 4, 8, 16, and 24 radial trap periods, and the results are shown in Supplementary Figure 1a–b. We find that the first four of these give condensate growth curves that cannot be distinguished, apart from an increasing delay in the initiation of growth. The vortex observation probability appears to drop slightly with increasing ramp time, but this is difficult to distinguish within the statistical error. As we are not actually modeling the evaporative cooling trajectory of the experiment, and the time axis of our simulations is shifted to best fit the experimental data, it seems that the time scale of the quench is of little consequence for our simulations.

This brings into question the relation between the KZ scenario of formation of topological defects and the simulations. In the KZ scenario, the quench time \( \tau_Q \) is a crucial parameter for estimating the initial density of spontaneously formed vortices, and it describes the rate at which the temperature \( T \) is lowered relative to the critical temperature \( T_c \). However, given that the condensate forms at the same rate even for different \( \mu \) and \( T \) ramp times in our simulations (and hence the condensate “quench” is occurring on a similar time scale), similar vortex observation probabilities among these simulations seems reasonable.

For the longer time scale ramps of \( \mu \) and \( T \) over 16 and 24 radial trap periods, the initial sections of the condensate growth curves are much more rounded, and the maximum growth rate is reduced. Fewer vortices are observed after the condensate reached its final population, and it thus seems that over these time scales the “effective quench time” of the system is increased.

The other fitting parameter for the simulations was the coupling of the classical field to the thermal cloud, \( \gamma = 0.005 \), which is essentially a measure of the collision rate within the thermal cloud. As a check of our methods, we have generated data sets of 200 trajectories for \( \gamma = 0.0025, 0.01, \) and 0.02 with all other parameters unchanged. Doubling \( \gamma \) essentially halves the time scale for condensate formation as shown in Supplementary Figure 1c, and so might lead one to predict that larger \( \gamma \) would lead to a higher probability of vortex formation and observation. However, Supplementary Figure 1d shows that the results are more complicated than this. This can be partly understood from the fact that a larger \( \gamma \) means that there is stronger damping rate and the vortices disappear at a faster rate.

To make a definitive statement about the validity of the Kibble-Zurek scenario for trapped Bose gases, it will be necessary to better control the quenching of the thermal cloud relative to the condensate. We are currently working towards techniques to achieve this.
Supplementary Figure 1: Effect of altering simulation parameters. a,b, Results for condensate growth and vortex probability for a linear ramp of the chemical potential over time scales of 0 (thick solid blue line), 1 (thick red dashed line), 2 (thick green dot-dashed line), 4 (thick black dotted line), 8 (thin magenta solid line), 16 (thin blue dashed line), and 24 (thin red dot-dashed line) radial trap periods. The ramp begins at $t = 0$, and ends at the times indicated by the vertical black dotted lines. c, d, Condensate growth and vortex probability for different scattering rates: $\gamma = 0.0025$ (red dashed line), $\gamma = 0.005$ (blue solid line), $\gamma = 0.01$ (green dot-dash line), and $\gamma = 0.02$ (black dotted line).