Dynamic polarizabilities for the low-laying states of Ca$^+$

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Introduction

The dynamic polarizability of an atom or ion gives a measure of the energy shift of the atom or ion when immersed in an electromagnetic field[1]. The calculations of dynamic polarizabilities can be used to identify magic wavelength of atomic transition states and tune-out wavelength of atomic states [2, 3]. The advantage of magic and tune-out wavelength measurements are that they are effectively null experiments. They measure the frequencies at which polarizability related quantities are equal to zero. Therefore they do not rely on a precise determination of the strength of a static electric field or the intensity of a laser field. This makes it possible to determine the magic and tune-out wavelengths to a high degree of precision[4].

Dynamic polarizabilities

The dynamic dipolar polarizability of a state at photon energy $\omega$ is defined

$$\alpha_j(\omega) = \sum_i \frac{f_{ij}^{(1)}}{\omega^2 - \varepsilon_{ij}}$$

(1)

The dipolar polarizability has tensor component for states with angular momentum $J = \frac{1}{2}$. This can be written

$$\alpha_J(\omega) = \frac{1}{6} \sum_{J_i | J_i J} \langle J_i | \frac{1}{J_{ij} + 1} | J \rangle$$

(2)

The polarizability for a state with nonzero angular momentum $J$ depends on the magnetic projection $M$: \(\alpha_{J,M} = \alpha_J + \frac{3M^2}{\omega} - J^2(J + 1)\)

$$\alpha_{J,M} = \alpha_J + \frac{3M^2}{\omega} - J^2(J + 1)$$

(3)

In present calculations, the wave functions and energies are obtained using Dirac-Fock plus core potential (DFCP) method which is based on B-splines basis set[5].

Conclusion

We calculated dynamic polarizability of the 4s, 4p, 3d states of Ca$^+$. A number of magic wavelengths have been identified. There are two relatively clean measurement of atomic structure parameters that could be made. Measurement of the magic wavelength near 395 nm could be used to determine a value of the oscillator strength $f(4s\rightarrow4p_{\frac{1}{2}})$. $f(4s\rightarrow4p_{\frac{3}{2}})$ with the uncertainty about 0.1%. Further, measurement of the two longest magic wavelengths for the 3d$^2$ states could give a good estimate of $f(3d_{\frac{3}{2}}\rightarrow4p_{\frac{3}{2}})$; $f(3d_{\frac{5}{2}}\rightarrow4p_{\frac{3}{2}})$.

Acknowledgments

This work was supported by the National Basic Research Program of China under Grants No. 2010CB832803 and No.2012CB821305 and by NSFC under Grants No.11274348. This research was supported by Australian Research Council Discovery Project No. DP-1092620.

References


Result 1. Magic wavelengths

![Figure 1. Dynamic polarizabilities of 4s$^{1/2}$, 4p$^{1/2}$, 4p$^{3/2}$, 3d$^{2}$ and 3d$^{3/2}$ states of Ca$^+$. Magic wavelengths are identified by arrows.](Image)

Result 2. Experiment measurement

<table>
<thead>
<tr>
<th>l (nm)</th>
<th>$\alpha_{4s\rightarrow4p_{\frac{1}{2}}}$</th>
<th>$\alpha_{4s\rightarrow4p_{\frac{3}{2}}}$</th>
<th>$\alpha_{4s\rightarrow4p_{\frac{5}{2}}}$</th>
<th>$\alpha_{4s\rightarrow4p_{\frac{3}{2}}}$</th>
<th>$\alpha_{4s\rightarrow4p_{\frac{5}{2}}}$</th>
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<tbody>
<tr>
<td>4s</td>
<td>24.0704</td>
<td>26.3971</td>
<td>27.0902</td>
<td>26.8051</td>
<td>28.0750</td>
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<tr>
<td>3p</td>
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<td>0.0098</td>
<td>0.0099</td>
<td>0.0099</td>
<td>0.0099</td>
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<td>4p</td>
<td>67.5032</td>
<td>52.2508</td>
<td>55.1513</td>
<td>49.8775</td>
<td>50.8848</td>
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<tr>
<td>5p</td>
<td>0.0045</td>
<td>0.0047</td>
<td>0.0049</td>
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<tr>
<td>Remainder</td>
<td>0.0172</td>
<td>0.0195</td>
<td>0.0199</td>
<td>0.0200</td>
<td>0.0199</td>
</tr>
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<td>Com.</td>
<td>3.2000</td>
<td>3.1261</td>
<td>3.2802</td>
<td>3.1793</td>
<td>3.3782</td>
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<td>Total</td>
<td>72.7511</td>
<td>62.1697</td>
<td>66.8437</td>
<td>60.0853</td>
<td>64.2630</td>
</tr>
</tbody>
</table>

$$\begin{align*}
\alpha_{4s\rightarrow4p_{\frac{1}{2}}} & = f(4s_{\frac{1}{2}} - 4p_{\frac{1}{2}}) + A' \\
\alpha_{4s\rightarrow4p_{\frac{3}{2}}} & = f(4s_{\frac{3}{2}} - 4p_{\frac{3}{2}}) + A' \\
\alpha_{4s\rightarrow4p_{\frac{5}{2}}} & = f(4s_{\frac{5}{2}} - 4p_{\frac{5}{2}}) + A' \\
\end{align*}$$

When the magic wavelength near 395 nm, one can see that the dominating contribution of the dynamic polarizability of 4s $\rightarrow$ 3d$^{2}$ come from $4s \rightarrow 4p_{\frac{1}{2}}$ and $4s \rightarrow 4p_{\frac{3}{2}}$. Equation (4) gives

$$\begin{align*}
\alpha_{4s\rightarrow4p_{\frac{1}{2}}} & = f(4s_{\frac{1}{2}} - 4p_{\frac{1}{2}}) + A' \\
\alpha_{4s\rightarrow4p_{\frac{3}{2}}} & = f(4s_{\frac{3}{2}} - 4p_{\frac{3}{2}}) + A' \\
\end{align*}$$

Experiment measurement

where A’ is the remainder contribution, which is about 0.1% of the dominating contribution $4s \rightarrow 4p_{\frac{1}{2}}$ or $4s \rightarrow 4p_{\frac{3}{2}}$. So, experiment measurement of the magic wavelength near 395 nm could be used to determine a value of the oscillator strength $f(4s\rightarrow4p_{\frac{1}{2}})$ and $f(4s\rightarrow4p_{\frac{3}{2}})$ with the uncertainty about 0.1%.