Kinetic Theory Model of Positron Thermalisation in Atomic Gases and Liquids

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Introduction

An accurate quantitative understanding of the behavior of positrons in gaseous [1] and soft-condensed matter [2] mediums is of interest for many technological applications as well as to fundamental physics research. The prime motivation behind the present work is improvements to the nuclear imaging technique of Positron Emission Tomography (PET) [3]. To optimize PET requires knowledge of how positrons behave prior to annihilation in soft-condensed matter, i.e. dense gases and liquids approximating biologic material.

Kinetic Theory

The fundamental kinetic equation describing a dilute swarm of light particles moving through a background gas or soft-condensed matter medium and subject to an external force, \( F \), is the Boltzmann kinetic equation for the phase-space distribution function, \( f(r, c, t) \), [4]:

\[
\frac{\partial f}{\partial t} + c \cdot \nabla f + \frac{F}{m} \cdot \frac{\partial f}{\partial c} = - \frac{\partial}{\partial t} f(s)
\]

where \( r, c \) and \( m \) are the position, velocity and mass of the swarm particle respectively. The \( f(s) \) represents the collision integral, which can be decomposed as:

\[
j = J_{1} + J_{2} + J_{3} + J_{4} + \cdots
\]

which then accounts for all of the relevant scattering processes. \( J_{1} \) must be modified to include interference effects from the coherent scattering through a modified differential cross-section,

\[
\tilde{\sigma}(c, \varphi) = S \left( \frac{2mc}{y} \sin \left( \frac{\varphi}{2} \right) \right) \sigma(c, \varphi)
\]

where \( S(K) \) is a static structure factor [5]. The multi-term [6] solution of the Boltzmann equation when there is a preferred direction is obtained by the expansion of \( f(r, c, t) \) in terms of Legendre polynomials, \( P_{l} \), i.e.

\[
f(r, c, t) \approx \sum_{l=0}^{\infty} f_{l}(r, c, t) P_{l}(\cos \varphi)
\]

The resulting hierarchy of coupled partial differential equations can be solved for the \( f_{l} \), from which experimental measurables can be calculated, e.g.,

\[
\langle \varepsilon \rangle = \frac{1}{\pi} \int \frac{1}{2} mc^{2} f dc,
\]

\[
\langle Z_{\text{eff}} \rangle = \frac{1}{\pi} \int z_{\text{eff}} f dc,
\]

where \( \varepsilon \) is the swarm mean energy and \( Z_{\text{eff}} \) is related to the positron annihilation rate.

Results: He

![Fig. 1. Momentum transfer cross-section and \( z_{\text{eff}} \) for helium. [7, J. Mitroy: Priv. Comm.]

![Fig. 2. Variation of \( Z_{\text{eff}} \) over time for positrons in dilute gaseous helium. [7,8]

![Fig. 3. Variation of \( Z_{\text{eff}} \) with reduced electric field for positrons in dilute gaseous helium. [9]

![Fig. 4. Variation of \( Z_{\text{eff}} \) with reduced electric field for positrons in liquid helium. [10]

![Fig. 5. Momentum transfer cross-section and \( z_{\text{eff}} \) for neon. \( \sigma_{e} - \sigma_{a} \) are obtained from cross-section calculations with different fitting parameters. [11, Mitroy: Priv. Comm.]

![Fig. 6. Variation of \( Z_{\text{eff}} \) over time for positrons in neon. [12]

Table 1. Average \( Z_{\text{eff}} \) at zero electric field for positrons in helium, neon and argon.

<table>
<thead>
<tr>
<th>( Z_{\text{eff}} )</th>
<th>Theoretical</th>
<th>Experimental</th>
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<tbody>
<tr>
<td>He</td>
<td>3.90 (3.88 [13])</td>
<td>3.94 ± 0.02 [8]</td>
</tr>
<tr>
<td>Ne</td>
<td>5.98 (6.98 [13])</td>
<td>5.99 ± 0.06 [8]</td>
</tr>
<tr>
<td>Ar</td>
<td>33.77 (30.5 [13])</td>
<td>33.8 [14]</td>
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References