43 Statistical Formulas

This section gives a little background on some of the functions used in statistics, as well as "computing formulas" for some of the statistics we have used. Computing formulas were once an important part of doing statistical calculations, giving equivalent expressions to the original definitions which are easier to do by hand with a simple calculator. However these are all now routinely calculated by software and most are available on cheap scientific calculators. We include these formulas mainly for historical interest and just in case you find yourself stuck on a desert island with only a primitive calculator and this book. In fact, we start with logarithms so that you could probably survive with a calculator too.

43.1 Logarithms

The *logarithm* of x to the base b is the number y such that $b^y = x$, and is denoted by $\log_b(x)$. For example, $\log_{10}(100) = 2$ and $\log_2(0.25) = -2$.

Logarithms obey some simple rules, two of which are used several times in this book. The first rule,

$$\log_b(xy) = \log_b(x) + \log_b(y),$$

says that the logarithm of a product is the *sum* of the logarithms. Similarly,

$$\log_b(x/y) = \log_b(x) - \log_b(y),$$

so the logarithm of a ratio is the difference of the logarithms. From these rules it is not hard to see that logarithms turn powers into multiplication, so that

$$\log_b(x^y) = y \log_b(x).$$

Bases

Two popular bases for logarithms are 10 and e. Base 10 is useful since it is directly related to our decimal number system. We'll see how to exploit that below when using logarithm tables.

Base e logarithms, known as *natural* logarithms, are very important in mathematics because of a fundamental role they play in calculus.

For most of the transformations used in this book, it does not matter which base you use...

Log Tables

In the days before calculators, logarithms and their properties were an important part of life through logarithm tables and slide rules. The reason for this is that it is fairly easy to add two numbers together but it is usually harder to multiply two numbers, particularly when they are large. Since logarithms turn multiplication into addition they can be used to convert the harder problem into the easier problem.

Table 43.1 gives a small table of logarithms to the base 10. Only logs for two digits and points half-way in between are given, whereas a standard log table would give logs for four or more digits. Table 43.2 gives a corresponding table of "antilogarithms", the process of raising the base to a number.

Suppose we want to multiply 69 times 240. Table 43.1 only gives logs for numbers between 1 and 10, but using our rules we can find

$$\log_{10}(69) = \log_{10}(10 \times 6.9) = \log_{10}(10) + \log_{10}(6.9) = 1 + .839 = 1.839.$$

In general, you simply move the decimal point to the left and add 1 to your log for each step (since these are base 10). Similarly, $\log_{10}(240) = 2.380$ since the table gives $\log_{10}(2.4) = .380$. Together we find that

$$\log_{10}(69 \times 240) = \log_{10}(69) + \log_{10}(240) = 1.839 + 2.380 = 4.219.$$

Looking at Table 43.2, we find that $10^{.220} = 1.66$. Thus

$$10^{4.219} = 10^4 \times 10^{.219} \simeq 10000 \times 1.66 = 16600.$$

Hence our value for 69 times 240 is 16600. The real value is 16560, so this is not too bad, limited in accuracy by having tables with only 2 digits.

To calculate 240 divided by 69, we use

$$\log_{10}(69/240) = \log_{10}(69) - \log_{10}(240) = 1.839 - 2.380 = -.541.$$

Now $-.541 = -1 + .459$. Table 43.2 gives $10^{.460} = 2.88$, so

$$\frac{69}{240} = 10^{-1} \times 10^{.460} = 0.1 \times 2.88 = .288.$$

The correct answer is .2875.

As a final example, suppose you wanted to find $.0365^5$. Table 43.1 gives $\log_{10}(3.65) = .562$ so

$$\log_{10}(.0365^5) = 5\log_{10}(.0365) = 5(-2 + .562) = 5 \times -1.438 = -7.190.$$

This is $-8 + .810$ and Table 43.2 gives $10^{.810} = 6.46$. Thus

$$.0365^5 = 6.46 \times 10^{-8} = .0000000646.$$

Table 43.1: Base 10 Logarithms

	First place of x									
x	0	1	2	3	4	5	6	7	8	9
1	.000	.041	.079	.114	.146	.176	.204	.230	.255	.279
	.021	.061	.097	.130	.161	.190	.217	.243	.267	.290
2	.301	.322	.342	.362	.380	.398	.415	.431	.447	.462
	.312	.332	.352	.371	.389	.407	.423	.439	.455	.470
3	.477	.491	.505	.519	.531	.544	.556	.568	.580	.591
	.484	.498	.512	.525	.538	.550	.562	.574	.585	.597
4	.602	.613	.623	.633	.643	.653	.663	.672	.681	.690
	.607	.618	.628	.638	.648	.658	.667	.677	.686	.695
5	.699	.708	.716	.724	.732	.740	.748	.756	.763	.771
	.703	.712	.720	.728	.736	.744	.752	.760	.767	.775
6	.778	.785	.792	.799	.806	.813	.820	.826	.833	.839
	.782	.789	.796	.803	.810	.816	.823	.829	.836	.842
7	.845	.851	.857	.863	.869	.875	.881	.886	.892	.898
	.848	.854	.860	.866	.872	.878	.884	.889	.895	.900
8	.903	.908	.914	.919	.924	.929	.934	.940	.944	.949
	.906	.911	.916	.922	.927	.932	.937	.942	.947	.952
9	.954	.959	.964	.968	.973	.978	.982	.987	.991	.996
	.957	.961	.966	.971	.975	.980	.985	.989	.993	.998

The first line of each block gives $\log_{10}(x)$ for $1 \leqslant x < 10$. The second line of each block gives $\log_{10}(x+.05)$. For example, $\log_{10}(4.2) = .623$ and $\log_{10}(6.95) = .842$.

Table 43.2: Base 10 Antilogarithms

	Second place of y									
y	0	1	2	3	4	5	6	7	8	9
0	1.00	1.02	1.05	1.07	1.10	1.12	1.15	1.17	1.20	1.23
	1.01	1.04	1.06	1.08	1.11	1.14	1.16	1.19	1.22	1.24
1	1.26	1.29	1.32	1.35	1.38	1.41	1.45	1.48	1.51	1.55
	1.27	1.30	1.33	1.36	1.40	1.43	1.46	1.50	1.53	1.57
2	1.58	1.62	1.66	1.70	1.74	1.78	1.82	1.86	1.91	1.95
	1.60	1.64	1.68	1.72	1.76	1.80	1.84	1.88	1.93	1.97
3	2.00	2.04	2.09	2.14	2.19	2.24	2.29	2.34	2.40	2.45
	2.02	2.07	2.11	2.16	2.21	2.26	2.32	2.37	2.43	2.48
4	2.51	2.57	2.63	2.69	2.75	2.82	2.88	2.95	3.02	3.09
	2.54	2.60	2.66	2.72	2.79	2.85	2.92	2.99	3.05	3.13
5	3.16	3.24	3.31	3.39	3.47	3.55	3.63	3.72	3.80	3.89
	3.20	3.27	3.35	3.43	3.51	3.59	3.67	3.76	3.85	3.94
6	3.98	4.07	4.17	4.27	4.37	4.47	4.57	4.68	4.79	4.90
	4.03	4.12	4.22	4.32	4.42	4.52	4.62	4.73	4.84	4.95
7	5.01	5.13	5.25	5.37	5.50	5.62	5.75	5.89	6.03	6.17
	5.07	5.19	5.31	5.43	5.56	5.69	5.82	5.96	6.10	6.24
8	6.31	6.46	6.61	6.76	6.92	7.08	7.24	7.41	7.59	7.76
	6.38	6.53	6.68	6.84	7.00	7.16	7.33	7.50	7.67	7.85
9	7.94	8.13	8.32	8.51	8.71	8.91	9.12	9.33	9.55	9.77
	8.04	8.22	8.41	8.61	8.81	9.02	9.23	9.44	9.66	9.89

The first line of each block gives 10^y for $0 \leqslant y < 1$. The second line of each block gives $10^{x+.005}$. For example, $10^{.84} = 6.92$ and $10^{.625} = 4.22$.