INTERNATIONAL CONFERENCE ON "GEOMETRIC AND NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS", MISSION BEACH RESORT, QUEENSLAND, 30 AUGUST–03 SEPTEMBER 2010

ORGANISED BY: THE AUSTRALIAN NATIONAL UNIVERSITY

About the conference

This conference will be run similarly to a successful meeting at the same location in 1998 and since then there has been a lot of interest in a sequel. The conference will host leading international mathematicians as well as provide the opportunity for young researchers and PhD students to get together to discuss new work in the field. This conference enhances the strong relationship between Australia and overseas in geometric and nonlinear analysis and builds on previous interactions in this major scientific research area.

Supported by The Australian National University and Australian Mathematical Sciences Institute

Scientific Committee

- Neil Trudinger, (Chair) - The Australian National University
- Ben Andrews - The Australian National University
- Alan Carey - The Australian National University
- Nicola Fusco - Naples, Italy
- Richard Schoen - Stanford, USA
- Gang Tian - Princeton, USA

Organising Committee

- Neil Trudinger, (Chair) - The Australian National University
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- Florica Cirstea - The University of Sydney
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- Alison Irvine, (Secretary) - The Australian National University
- Xujia Wang - The Australian National University

Programme

Monday, 30th August 2010 **Day 1**

All events take place in the Rainforest Room.

9.00 am - 9.15 am	**Opening Remarks**
9.15 am - 10.00 am	Expository Lecture 1 **Gang Tian** (Beijing/Princeton) *Scalar V-soliton equation*
10.15 am - 11.00 am	Expository Lecture 2 **Richard Melrose** (MIT) *Einstein's equations and scattering*
11.15 am - 12 noon	Expository Lecture 3 **Michael Struwe** (ETH, Zurich) *Global smooth solutions to a 'supercritical' two-dimensional wave equation*
12.30 pm - 3.00 pm	Lunch
3.00 pm - 3.45 pm	Expository Lecture 4 **Juncheng Wei** (Chinese Univ. Hong Kong) *On the role of minimal surfaces in the Serrin's overdetermined problem*
4.00 pm - 4.45 pm	Expository Lecture 5 **Nicolai Krylov** (University of Minnesota) *On fully nonlinear elliptic and parabolic equations with VMO coefficients*
5.00 pm - 5.45 pm	Expository Lecture 6 **Lucio Boccardo** (Roma 1) *Dirichlet problems with singular terms: old and new*
7.00 pm	Buffet Dinner

ABSTRACTS OF INVITED TALKS

Dirichlet problems with singular terms: old and new

Lucio Boccardo (Università di Roma 1)

Let Ω be a bounded, open subset of $\boldsymbol{R}^N$ $(N > 2)$ $a : \Omega \times \boldsymbol{R} \times \boldsymbol{R}^N \to \boldsymbol{R}^N$, such that

$$\text{(1)} \qquad {}^1 \quad \alpha|\xi|^2 \leq a(x, s, \xi)\xi, \quad |a(x, s, \xi)| \leq \beta.$$

In the past century, in some joint papers with Thierry Gallouet, we proved the existence of distributional solutions (infinite energy solutions) of the boundary value problem

$$\text{(2)} \qquad \begin{cases} -\mathrm{div}(a(x, u, \nabla u)) = f(x), & \text{in } \Omega; \\ u = 0, & \text{on } \partial\Omega. \end{cases}$$

Here $f \in L^m(\Omega)$, $1 \leq m < \frac{2N}{N+2}$, (or $f \in M^m(\Omega)$, during this century and alone, unfortunately!) implies the existence of $u \in W_0^{1,q}(\Omega)$, with $q = m^*$, if $1 < m < \frac{2N}{N+2}$ (nonlinear Calderon-Zygmund [2]) and $q < N/(N-1)$, if $m = 1$. Furthermore, properties of u aw ay from the set where the datum f is singular are studied in a recent paper (with T. Leonori) dedicated to Neil. Moreover, with T. Gallouet and J.L. Vazquez, for the perturbed problem

$$\begin{cases} -\mathrm{div}(a(x, u, \nabla u)) + |u|^{p-1}u = f, & \text{in } \Omega; \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

we proved that (regularizing effect of the lower order term)

- $p \leq \frac{N}{N-2m} \Rightarrow u \in W_0^{1,m^*}(\Omega)$,
- $\frac{N}{N-2m} < p < \frac{1}{m-1} \Rightarrow u \in W_0^{1,\frac{2pm}{p+1}}(\Omega)$,
- $\frac{1}{m-1} \leq p \Rightarrow u \in W_0^{1,2}(\Omega) \cap L^p(\Omega)$.

Some techniques of the above papers are useful in the study of the Dirichlet problem

$$-\,\mathrm{div}(M(x)\nabla u - u\,E(x)) + \mu u = f(x) \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega.$$

Under the assumptions $E \in (L^N(\Omega))^N$ and $\mu > 0$ large enough, Guido Stampacchia proved that the problem has a unique weak solution u with s ome summability properties. If $\mu = 0$, it is possible to prove the coercivity of the differential operator only if $\|E\|_{L^N(\Omega)}$ is small enough; nevertheless, if $f \in L^m(\Omega)$, $m \geq 1$, I recently proved existence and summability properties (depending on m, but independent of the size of $\|E\|$) of solutions. In the talk, I will present some existence results if E belongs only to $(L^2(\Omega))^N$. The starting point here is the definition of solution since the distributional definition does not work. It is possible to give meaning to the solution thanks to the concept of

[1]In this talk only $p = 2$! and only elliptic problems!

[2]see also: R. Mingione "Opera omnia"

entropy solutions, introduced in a joint paper with Benilan-Gallouet-Vazquez. Then I will discuss some existence results about the system ($\theta \in (0,1)$)

$$\begin{cases} -\operatorname{div}(A(x)\nabla u) + u = -\operatorname{div}(u\, M(x)\nabla z) + f(x), & \text{in } \Omega; \\ -\operatorname{div}(M(x)\nabla z) = u^{\theta}, & \text{in } \Omega; \\ u = z = 0, & \text{on } \partial\Omega. \end{cases}$$

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Compactness of a class of constant scalar metrics

Alice Chang (Princeton University)

In this talk, I will report a joint work of Sophie Chen, Paul Yang and myself on the compactness of a class of constant scalar curvature metrics in the setting of conformal compact Einstein 4-manifolds with boundary. We impose some compactness condition of the Yamabe metrics on the boundary and study the effect of the compactness of the Yamabe metrics in the interior of the manifold. One of the main tool is to apply elliptic PDE technique to study the interplay between the order of decay of the Weyl curvature and the Ricci curvature at the ends of the manifold.

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Spreading speed revisited–A free boundary approach

Yihong Du (University of New England)

Much previous mathematical investigation on the spreading of population was based on the traveling wave solutions, which were first introduced in the pioneering works of Fisher (1937) and Kolmogorov et al (1937), and further developed later along several directions by many first rate mathematicians. In particular, making use of the traveling wave solutions, Aronson and Weinberger (1978) established the existence of a unique asymptotic spreading speed for the propagation front of a new or invasive species. This classical work has seen much ground breaking further developments in recent years.

In this talk, I will report some recent joint works with various collaborators (Z. Lin, Z. Guo, R. Peng, H. Matano) on a new approach to this problem, where a free boundary is used to describe the spreading front, instead of the level set of traveling wave solutions. Unlike the traditional traveling wave approach, which predicts persistent propagation of the new species regardless of its initial population size, our free boundary model exhibits a spreading-vanishing dichotomy for this model, namely, depending on the initial size of the population, the species either successfully spreads to all the new environment and stabilizes at a positive equilibrium state, or it fails to establish and dies out in the long run. Such a phenomenon appears to better reflect the natural spreading process. Moreover, we show that when spreading occurs, for large time, the expanding front (i.e., the free boundary) moves asymptotically at a constant speed, uniquely determined by an elliptic problem induced from the original model. Furthermore, the classical result of Aronson and Weinberger can be recovered by our research, as it can be shown that the Cauchy problem used by Aronson and Weinberger represents a limiting case of our free boundary problem.

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The Pohozaev and Kazdan-Warner identities

Rod Gover (The University of Auckland)

In 1965 Pohozaev proved the non-existence of solutions of certain non-linear equations, in a star shaped domain, by using an identity that now bears his name. Numerous variants of this identity have been developed for similar applications. Around a decade later Kazdan and Warner gave a (at the time) new condition for a function on the 2-sphere to possibly be the curvature of a metric. This result was generalised to the setting of conformal scalar curvature prescription in higher dimensions, and on closed manifolds, by Bourguignon and Ezin. Subsequently there have been many further generalisations of this result. A purely geometric interpretation of the original Pohozaev identity by Schoen demonstrated the close link between the Pohozaev and Kazdan-Warner identities; this was applied to the problem of conformally prescribing constant scalar curvature on a manifold with boundary. This talk describes recent joint work, with Bent Orsted, in which we develop an approach to these identities based around certain geometric principles. This leads to generalisations.

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Singular integral equations and convergence of their solutions to p-Laplace equations

Hitoshi Ishii (Waseda University)

Let $\Omega \subset \boldsymbol{R}^n$ be a bounded domain, and let $1 < p < \infty$ and $\sigma < p$. I will discuss the nonlinear singular integral equation

$$M[u](x) = f(x) \quad \text{in } \Omega$$

with the boundary condition $u = g$ on $\partial\Omega$, where $f \in C(\overline{\Omega})$ and $g \in C(\partial\Omega)$ are given functions and M is the singular integral operator given by

$$M[u](x) = \text{p.v.} \int_{B(0,\rho(x))} \frac{p-\sigma}{|z|^{n+\sigma}} |u(x+z) - u(x)|^{p-2}(u(x+z) - u(x))\, dz,$$

with some choice of $\rho \in C(\Omega)$ having the property, $0 < \rho(x) \leq \operatorname{dist}(x, \partial\Omega)$. I will explain the solvability (well-posedness) of this Dirichlet problem and the convergence uniform on $\overline{\Omega}$ as $\sigma \to p$, of the solution u_σ of the Dirichlet problem to the solution u of the Dirichlet problem for the p-Laplace equation $\nu\Delta_p u = f$ in Ω with the Dirichlet condition $u = g$ on $\partial\Omega$, where the factor ν is a positive constant. The talk will be based on a joint work with Gou Nakamura.

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On fully nonlinear elliptic and parabolic equations with WMO coefficients

Nicolai Krylov (University of Minnesota)

We prove the solvability in Sobolev spaces $W_p^{1,2}$, $p > d + 1$, of the terminal-boundary value problem for a class of fully nonlinear parabolic equations, including parabolic Bellman's equations, in bounded cylindrical domains with VMO "coefficients". The solvability in W_p^2, $p > d$, of the corresponding elliptic boundary-value problem is also obtained.

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Hölder continuity of maps optimizing non-negatively cross-curved costs

Robert McCann (University of Toronto)

I will sketch recent progress on the regularity theory of optimal mappings obtained jointly with Alessio Figalli and Young-Heon Kim.

Consider transportation of one distribution of mass onto another, chosen to optimize the total expected cost, where cost per unit mass transported from X to Y is given by a smooth function $c(X, Y)$. If the source density $f^+(X)$ is bounded away from zero and infinity in an open region $U \subset R^n$, and the target density $f^-(Y)$ is bounded away from zero and infinity on its support $V \subset R^n$, which is strongly c-convex with respect to U, and the transportation cost c is non-negatively cross-curved, we deduce continuity and injectivity of the optimal map inside U. This result provides a crucial missing step in the low/interior regularity setting: in subsequent work, we use it to establish regularity of optimal maps with respect to the Riemannian distance squared on arbitrary products of spheres. The result also includes an argument required by Figalli and Loeper to conclude in two dimensions continuity of optimal maps under the weaker (in fact, necessary) hypothesis (A3w). In higher dimensions, if the densities $f^\pm$ are Hölder continuous, our result permits continuous differentiability of the map inside U to be deduced from recent work of Liu, Trudinger and Wang without assuming their strict inequality (A3).

Connections to geometry and economics may be mentioned, time permitting.

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Einstein's equations and scattering

Richard Melrose (MIT)

For near-flat initial data to Einstein's equations global existence was shown by Christodoulou and Klainerman. Following work of Lindblad and Rodnianski, Fang Wang has recently given a detailed description of the long-time asymptotics in dimension greater than 4 including the existence of a radiation field in the sense of Friedlander. I will relate this to regularity for the Radon transform and scattering theory.

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Nonlinear aspects of Calderon-Zygmund theory

Rosario Mingione (University of Parma)

The classical Calderon-Zygmund theory allows to get, in a sharp way, information on integrability and differentiability of solutions to linear elliptic and parabolic equations in terms of the given data. I will give a survey of analogous results valid in the nonlinear context.

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Li-Yau-Hamilton type estimates on Kähler manifolds

Lei Ni (University of California)

I shall present some new results on Li-Yau-Hamilton type estimates for deforming (p, p) forms and explain the connections of Li-Yau-Hamilton type estimates with monotonicity etc.

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On manifolds of positive curvature and sphere theorems

Richard Schoen (Stanford University)

This talk is a survey on manifolds of positive curvature and sphere theorems, which includes some discussion of Ricci flow methods.

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Global smooth solutions to a 'supercritical' two-dimensional wave equation

Michael Struwe (ETHZ)

Extending the work of Ibrahim et al. on the Cauchy problem for wave equations with exponential nonlinearities in 2 space dimensions, we establish global well-posedness also in the supercritical regime of large energies for smooth, radially symmetric data.

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Scalar V-soliton equation

Gang Tian (Beijing/Princeton University)

In this talk, I will discuss a fully nonlinear equation which arises from studying the singularity formation of Kähler-Ricci flow at finite time.

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On Multi-Dimensional Compressible Navier-Stokes Systems

Zhouping Xin (The Chinese University of Hong Kong)

In this talk, I would like to survey the major progress on the global (in time) well-posedness theory of multi-dimensional Navier-Stokes Systems (CNS) and discuss some recent results on global classical solutions with small energy but possible large oscillations and vacuum for the isentropic CNS system.

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A fourth order invariant in CR geometry in 3-D

Paul Yang (Princeton University)

In CR geometry in 3-D, there is strong analogy with conformal geometry of dimension four. In particular, there are two conformally covariant operators which play critical role in the analysis: the analogue of the conformal Laplacian as well as the analogue of the fourth order operator of Paneitz. In recent work we found the positivity of both these operators have strong consequences: the positivity of mass as well as the imbeddability of CR structures.

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On the role of minimal surfaces in the Serrin's overdetermined problem

Juncheng Wei (The Chinese University of Hong Kong)

We consider the following so-called Serrin's overdetermined problem

$$\begin{cases} -\Delta u = f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ \dfrac{\partial u}{\partial \nu} = C & \text{on } \partial\Omega. \end{cases}$$

Serrin (1981) proved that if the domain is bounded, then it must be a ball. In the unbounded graph case, Berestycki-Caffarelli-Nirenberg (1987) showed that the graph must be a hyperplane if the slope is bounded. In this talk, I will show that in the higher-dimensions ($N \geq 9$), Serrin’s problem has a solution on the Bomberie-De Giorgi-Giusti graph. Furthermore, I will show how minimal surfaces and CMC are naturally embedded into Serrin’s problem. (Joint work with M. del Pino and F. Pacard.)

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Abstracts of Short Communications

Constant mean curvature surfaces and Darboux transforms

Emma Carberry (The University of Sydney)

I will discuss constant mean curvature surfaces from several viewpoints, and explain how these different points of view are related. The integrable systems approach stems from considering the classical associated family of constant mean curvature surfaces and leads to a complete description of constant mean curvature tori in terms of a linear flow in an abelian variety. Using a generalisation of the classical Darboux transform, I will give a geometric interpretation of this and explain geometric properties of the transformed surfaces. This is joint work with Katrin Leschke and Franz Pedit.

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Spectral gap on convex domains

Julie Clutterbuck (ANU)

The Laplacian on a convex domain has a discrete set of Dirichlet eigenvalues; the difference between the first two is called the spectral gap. Van den Berg and Yau conjectured that for domains of a given diameter, this gap is minimised on a vanishingly thin domain. I will report on recent work with Ben Andrews, where we use parabolic methods to show that the conjecture holds.

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High homogeneity curvature flow

James McCoy (University of Wollongong)

We discuss a new curvature pinching result for smooth strictly convex hypersurfaces which facilitates convergence of sufficiently pinched smooth convex hypersurfaces to round points under a class of high homogeneity fully nonlinear curvature flow, without the need for regularity results for porous medium type equations. This is joint work with Ben Andrews.

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Ricci flow and the determinant of the Laplacian on non-compact surfaces

Frederic Rochon (The Australian National University)

After defining the determinant of the Laplacian on non-compact surfaces with ends asymptotic to cusps or funnels, we briefly describe how the Ricci flow can be used to show that in a given conformal class of such surfaces, the determinant is maximal for the metric of constant scalar curvature. This is a joint work with P. Albin and C. Aldana.

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Scalar Curvature Behavior for Kähler-Ricci Flows over Closed Manifolds

Zhou Zhang (The University of Sydney)

The existence of Kähler-Ricci flow is determined by the cohomology data in an optimal way. The cases of finite and infinite time singularities both come up very naturally. The behavior of scalar curvature can be very different. We always have blow-up of scalar curvature towards finite time singularity, and there are examples with uniformly bounded scalar curvature for infinite time singularity. The corresponding (more classic) question for Ricci flow has been a very hot topic.

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