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# Journal of Combinatorial Theory, Series A

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## Fifty years of The Journal of Combinatorial Theory



The idea of a specialist journal focused on combinatorics was first conceived in the early 1960s by Frank Harary and Gian-Carlo Rota, and successfully brought to fruition a few years later with the publication of the first issue, in 1966, of the Journal of Combinatorial Theory. Many eminent mathematicians contributed to this issue, including Paul Erdős, Ron Graham, Daniel Kleitman, and Marcel-Paul Schützenberger. These formative years, as well as the split of the journal into a Series A and B, is described wonderfully in an article commemorating Rota, published in JCTA in 2000 and written by Edwin Beschler, an editor at Academic Press at the time the journal was founded [2]. We will continue the story by highlighting some of the developments of JCTA during the time we were involved in the journal, both as Managing Editors and Editors-in-Chief. This roughly covers the period 1969–2015.

When Bruce Rothschild was at MIT as a post-doc, Theodore Motzkin was passing through Cambridge, Massachusetts, on his way back to UCLA. He met with Gian-Carlo Rota who told him about the pending reorganization of JCT into JCTA and JCTB. As one of the original editorial board members of JCT, Rota asked Motzkin if he would consider becoming the Editor-in-Chief of Series A. Motzkin said he would think about it. As Rota told it, after a short pause, Motzkin said “yes”! Much later, Motzkin explained that he did a quick ‘calculation’, figuring that since Bruce Rothschild was soon to be joining UCLA, and Basil Gordon was already there, he could ask both to serve as Managing Editors. Gordon had previously worked with the Pacific Journal of Mathematics, which was edited at UCLA, and Motzkin knew he was an extraordinary scholar, very broad in his mathematical knowledge and interests, and extremely quick. He also knew he would be willing to take on this task, and so it transpired that in 1969 the new journal JCTA was effectively run from UCLA with Motzkin as Editor-in-Chief and Basil Gordon and Bruce Rothschild as Managing Editors.

The first thing that had to be done was to decide the fate of papers submitted to JCT, but which would only appear after the split into series A and B. Together with the editors for JCTB, Bill Tutte and Dan Younger, it had to be decided which papers should be routed to JCTB and which to JCTA. Typically papers on graph theory or matroids went

to JCTB and the rest to JCTA. This amounted to a rough split of about 40/60. Naturally, papers often did not fit obviously into one series or the other and judgements had to be made. This has remained a recurrent problem which we still face to this day. Over the years the distinction based on graphs and matroids has become much less pronounced as, with the emergence of new subfields such as additive combinatorics, combinatorial commutative algebra and physical combinatorics, the field of combinatorics has grown significantly.

In 1971 the first issues of JCTA and JCTB were published. There is a sad note at the start of the first issue of the JCTA; a single page in memory of Motzkin, who had suddenly died at the age of 62, a few months prior to publication. Besides a great loss at a personal level, the journal was also deprived of Motzkin's guidance. He remained listed as Editor-in-Chief until 1972, when Marshall Hall took on the role of Editor-in-Chief. Since he was at Caltech, it was not too difficult for the Managing Editors to consult with him. Before the ubiquity of e-mail, the Internet and free long-distance phone services, geographic proximity was very important. Hall continued to serve even after retirement, and was still JCTA's Editor-in-Chief at the age of 79. As another sad footnote to JCTA's history, he passed away on his way to a conference in the UK in honor of his 80th birthday. As with Motzkin, Hall remained listed as Editor-in-Chief till the end of the year, and after 1990 JCTA continued to operate with two Managing Editors without an Editor-in-Chief.

One of Hall's important impacts on the journal was the expansion of the editorial board. The division of JCT into JCTA and JCTB had been paralleled by the division of the editorial board, and only Dick de Bruijn, Paul Erdős and Bill Tutte, continued to serve on both. The board for JCTA was expanded first under Hall's editorship, and over the years continued to grow. During the time JCTA was edited at UCLA, only Herb Ryser succeeded in retiring from the board (for health reasons). All others were members for life. The editorial board reflected the development and expansion of the field of combinatorics over time. When JCT started, it was the only journal specifically devoted to combinatorics. A decade later there were several others, including *Discrete Mathematics* ('71), *Ars Combinatorica* ('76) and *Journal of Graph Theory* ('77), as well as a number of journals publishing extensively in combinatorics in the broadest sense. Despite the rapid rise in the number of journals, JCTA's submission rate kept increasing steadily, as did its publication rate.

In the early years the operation of JCTA was fairly primitive by modern standards. Papers would come in, and would either be sent to referees or to an editorial board member to have it refereed. All of this, as well as subsequent communications, were done by post. The only upside was that it was easy to track the journal backlog; it was an actual physical stack. Eventually computers held all the files and papers were transmitted by e-mail. But JCTA only adopted an online submission system in 2001, when H el ene Barcelo took over as Editor-in-Chief and the journal moved to ASU in Arizona. Not long thereafter, Elsevier, who had acquired Academic Press in 2000, took over the publishing of JCTA. Around the time of the changeover JCTA tried to clear up all pending papers, many still with delinquent referees, the bane of any editor. One particular paper was

accepted, modulo a few minor changes, more than 20 years earlier! At the editor’s final urging, the author, who shall remain nameless, did in fact get the paper ready. Because of the long time that had passed, the entire paper required retyping into a more modern format before it could be published.

Apart from going online, and developing an online submission system (maintained and run locally), during the years at ASU (2001–2009) H el ene Barcelo introduced an Advisory Board. The members of this board acted as a first port of call for the Editor-in-Chief, and would briefly comment on most new submissions, suggesting suitable editors, possible referees, or external experts who could be consulted for quick opinions. With the increased number of submissions it was no longer practical to have all papers refereed, and about 30% of submissions, would be returned to the authors based on brief expert evaluations.

The journal further modernized at the start of 2009 when, under new Editor-in-Chief, Ole Warnaar, it adopted Elsevier’s online submission system EES. This had the great advantage that papers were much easier to track. Also author and referee histories became more readily accessible. The adoption of EES also made it easier for authors to submit papers, perhaps explaining the significant increase in submissions, further boosted by the rapidly increasing mathematical output of several previously under-represented countries, including China, Iran, and Korea.

In its 50 year history, many influential papers in combinatorial theory have appeared in JCT(A). Although it is impossible to mention the majority of these explicitly, we have nonetheless decided to highlight a few exceptional contributions. We happily admit that our selection is highly subjective, reflecting our personal tastes and biases.

The first work we have chosen is Wilson’s celebrated triplet of papers, *An existence theory for pairwise balanced designs I–III*, published between 1972 and 1975, [6]. Let  $X$  be a set of  $v$  elements, thought of as points. A  $t$ – $(v, k, \lambda)$  design is a set  $B = \{b_1, b_2, \dots, b_{\lambda_0}\}$  of ‘blocks’, where each block  $b_i$  is a subset of  $X$  consisting of  $k$  points, such that (1) every point of  $X$  is contained in exactly  $\lambda_1$  blocks and (2) every  $t$ –subset of points of  $X$  appears in exactly  $\lambda$  blocks. It is easy to see that the numbers  $\lambda_0$  and  $\lambda_1$  are fixed by  $t, v, k, \lambda$  as  $\lambda_i = \lambda \binom{v-i}{t-i} / \binom{k-i}{t-i}$  for  $i = 0, 1$ . It is not much harder to show that a necessary condition for existence is that  $\lambda_i$ , as defined by the above ratio of binomial coefficients, is an integer for all  $0 \leq i \leq t$ . Much harder is showing that, for sufficiently large  $v$ , these conditions are also sufficient, and it was Wilson in his series of papers who made the first real breakthrough by settling the case  $t = 2$ . It took another 40 years before Keevash, in another major breakthrough in 2014, finally proved the existence conjecture for all  $t$  using his method of randomized algebraic constructions.

Our second highlight is Hindman’s 1974 paper, *Finite sums from sequences within cells of partition of  $\mathbb{N}$* , [3]. In this paper Hindman settled an open problem in Ramsey theory posed by Erdős, now widely known as ‘Hindman’s Theorem’. The original question asked when coloring the natural numbers  $\mathbb{N}$  (the  $N$  from the title) with two colors, is there always a sequence  $X = \{x_k\}_{k \geq 1} \subseteq \mathbb{N}$  and a color  $C$  so that all the finite sums of elements of  $X$  have color  $C$ ? Hindman answered this in the affirmative for the more

general case where there are any finite number of colors. Hindman's theorem implies Folkman's theorem, which makes the same claim with the restriction that  $X$  is finite. An alternative formulation of the theorem is the claim that if we color the set  $P$  of finite (non-empty) subsets of  $\mathbb{N}$  with  $k$  colors, then there exists an infinite family  $D$  of pairwise disjoint sets in  $P$  such that all of the finite unions of sets in  $D$  have the same color. Hindman's proof was a lengthy inductive argument. Shortly after the proof was given, shorter proofs appeared by Baumgartner, Galvin, and Glazer. Ultimately this became one of the points of departure for work in topological dynamics, vastly generalizing the original results.

Our next choice is the first of 27 joint papers written by Bender and Canfield: *The asymptotic number of labeled graphs with given degree sequences*, [1], published in 1978. This paper has proven to be extremely influential and the asymptotic method it presents has been applied in literally hundreds of subsequent papers on asymptotic graph enumeration. The main result of Bender and Canfield concerns the asymptotic enumeration of symmetric  $n \times n$  matrices. Fix positive integers  $z$  and  $d$ , and let  $M = (m_{ij})$  be an  $n \times n$  symmetric  $(0, 1)$ -matrix such that its row sums are at least  $n - z$ . Given such a matrix, let  $G(M, (r_1, \dots, r_n), t)$  count the number of  $n \times n$  symmetric matrices  $G = (g_{ij})$  with entries in  $\{0, 1, \dots, t\}$  such that the  $i$ th row sum of  $G$  is  $r_i$  and  $g_{ij} = 0$  if  $m_{ij} = 0$ . Bender and Canfield provide an explicit asymptotic expression for  $G(M, (r_1, \dots, r_n), t)$  in terms of the cardinality of a certain set of involutions. The connection with graphs arises by taking  $t = 1$ . Then  $G$  is a symmetric  $(0, 1)$ -matrix, which can be viewed as the incidence (or adjacency) matrix of a graph. In particular, if  $M$  is 0 on the main diagonal and 1 everywhere else, then  $G(M, (r_1, \dots, r_n), 1)$  counts the number of labeled simple graphs with degree sequence given by  $(r_1, \dots, r_n)$ . If  $M$  is the all-ones matrix, then the count includes graphs in which loops are allowed.

Our next highlight is the 1983 Mills, Robbins and Rumsey paper *Alternating sign matrices and descending plane partitions*, [4], containing many remarkable conjectures concerning alternating sign matrices (ASMs), a natural generalization of permutation matrices. A square matrix is said to be an ASM if all its entries are in  $\{0, 1, -1\}$ , all row and column sums are 1, and non-zero entries in each row and column alternate in sign. The main conjecture that was put forward by Mills, Robbins and Rumsey is that  $n \times n$  ASMs are enumerated by  $\prod_{i=0}^{n-1} (3i + 1)! / (n + i)!$ , numbers which, according to work of Andrews, also count descending plane partitions and totally symmetric self-complementary plane partitions (TSSCPPs). The ASM conjecture was proved 13 years later, by Zeilberger and, shortly thereafter by Kuperberg, revealing a surprising connection with the six-vertex model in statistical mechanics and Yang–Baxter integrability. Many related conjectures, however, remain wide open, and it is one of the main open problems in enumerative combinatorics to construct a bijection between ASMs and TSSCPPs, a problem considered so hard that it is sometimes referred to as the Gog–Magog conjecture (and which can be rephrased in terms combinatorial objects called Gog and Magog trapezoids).

The final and most recent paper we have chosen is Reiner, Stanton and White's *The cyclic sieving phenomenon*, published in 2004 [5], and recently featured in the *What is ...* section of the Notices of the AMS. The cyclic sieving phenomenon (CSP) is a far-reaching generalization of Stembridge's  $q = -1$  phenomenon, and can roughly be described as follows. Let  $C$  be a cyclic group of order  $n$ , generated by  $c \in C$  and acting on a finite set  $X$ . Let  $X(q)$  be the generating function of  $X$  with respect to some statistic. This implies that  $X(q)$  is a polynomial with non-negative integer coefficients and  $X(1) = |X|$ , the cardinality of  $X$ . Then the triple  $(X, X(q), C)$  is said to exhibit CSP if, for all  $d = 0, 1, \dots, n-1$ ,  $X(e^{2\pi id/n})$  counts the number of elements of  $X$  fixed by  $c^d$ . When  $n = 2$  we are simply dealing with a set  $X$  equipped with an involution, in which case  $X(-1)$  counts the number of fixed points of the involution. This is the afore-mentioned  $q = -1$  phenomenon. Surprisingly, a very wide array of seemingly disparate combinatorial sets exhibit CSP, such as  $k$ -element subsets of the  $n$ -set, plane partitions, standard Young tableaux, clusters in finite type cluster algebras, and certain cosets in Coxeter groups. As a wonderful example of the interconnectedness of mathematics, also the alternating sign matrices of Mills, Robbins and Rumsey exhibit CSP, with  $C$  the cyclic group of order 4, acting by rotations over  $\pi/2$ , and  $X(q)$  a simple  $q$ -deformation of the  $\prod_{i=0}^{n-1} (3i+1)/(n+i)!$ .

We reiterate that the above five papers provide only a tiny glimpse of the many wonderful papers that have appeared in JCTA, and there is little doubt that in the next 50 years many more breakthrough papers will appear in the journal.

To conclude, we would like to express our sincere gratitude to the many people who have assisted us in running JCTA over the years: past and present Editorial Board members, Editorial Assistants, referees, and of course, all of our authors. It has been a wonderful experience being part of a vibrant combinatorics community.

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