The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2014

JUNIOR DIVISION

Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *The examiners will attach considerable weight to the method whereby a solution is presented. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The* ***six*** *questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *While you may need to do considerable rough work, you should write your final solution neatly, stating your arguments carefully.*

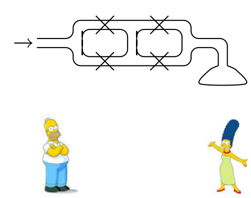
(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

Textbooks, electronic calculators and computers are ***NOT*** *allowed. Otherwise, normal examination conditions apply.*

1. Old boots and used socks. Former NSW Premier Barry O'Farrell finally has the time to open the $3000 bottle of 1959 Grange Hermitage for which he lost his job. After wine expert Huon Hooke described the 1959 Grange as resembling "an infusion of old boots, used socks and outback dust", Mr O'Farrell decides not to risk drinking it pure.
There is 750ml of Grange in the bottle. He pours 250ml of it into a large jug containing 1 litre of lemonade, and stirs thoroughly. He then pours 250ml of the mixture back into the bottle, so that it is full again.
How much wine does Mr O'Farrell's bottle now contain?

2. Springfield University – School Mathematics Competition. Due to the tremendous success of The University of Melbourne – School Mathematics competition, Springfield University instructs Homer Simpson to run a similar competition. Homer decides to award each of the top 30 students a cash prize, and sets aside the amount of $450 for this purpose. He wishes to give every winner a different whole dollar amount, with the student ranked first receiving the biggest prize and the student ranked 30th the smallest prize.
Show why the Springfield University – School Mathematics Competition is doomed to fail.

3. Wet and wild. Having just lost his job at Springfield University, Homer decides to try his hand at plumbing. To practise his skills he replaces the water pipe leading to his shower with the contraption shown on the right.
Each cross represents a location of a tap. If Marge leaves each of these opened or closed completely at random, what is the probability that she will actually get wet?



4. Thanassis. On a Saturday night around 300BC, Archimedes, Euclid and Pythagoras play a game of Thanassis, a popular game in ancient Greece. Much is at stake. Every time a player wins a game, the two losers must each give him one of their favourite mathematics books. At the end of the night, Archimedes has won 4 games, Euclid has won 4 more books than he has lost, and poor Pythagoras has lost 5 more books than he has won.
How many games of Thanassis did Archimedes, Euclid and Pythagoras play?

5. Circling the square. As part of his recent trip to South-East Asia, Prime Minister Tony Abbott hosted a dinner for Chinese President Xi Jinping, Japanese Prime Minister Shinzō Abe and (the now former) South Korean Prime Minister Jung Hong-won. They were seated at a 1×1 metre square table. Because these four powerful leaders are all notoriously messy eaters, Mr Abbott's Chief of Staff, Peta Credlin, decided to put a plastic table cloth on the table to collect the leftovers. Unfortunately, all she could find was a stack of round tablecloths, each of diameter 1 metre.
What is the minimum number of round tablecloths Ms Credlin needed to cover the entire table?

6. Friends. All students at your school with an even number of school friends send an email to those friends. All students at your school with an odd number of school friends send an email to those at your school who are not their friends.
Show that you will receive an even number of emails from fellow students at your school.
Hint: First show that there is an even number of students with an odd number of friends.