



The University of Melbourne–Department of Mathematics and
Statistics

School Mathematics Competition, 2017

JUNIOR DIVISION

Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

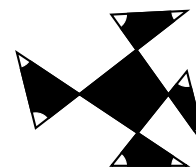
- (1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*
- (2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*
- (3) *It may be necessary to spend considerable time on a problem before any real progress is made.*
- (4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*
- (5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

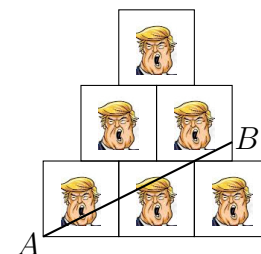
1. Darcy Vescio. Carlton's Darcy Vescio took out the inaugural award for the AFLW's Leading Goalkicker. Darcy kicked three more goals than runner up Sarah Perkins (Crows), but 6 fewer goals than Sarah and Alyssa Mifsud (Demons) combined. In total, Darcy, Sarah and Alyssa kicked 34 goals for the season. How many goals did Darcy kick?

2. Year of the (fire) rooster. In Chinese astrology, each year is associated with one of 12 animals as well as one of 5 elements: earth, fire, gold, water and wood. Currently, we are in the year of the fire rooster. According to Chinese astrology, a character trait of fire roosters is trustworthiness: they always tell the truth, unlike snakes, who always lie. A total of 2017 snakes and fire roosters form a long queue. Each animal, with the exception of the one at the front of the queue, claims that they are "standing" directly behind a snake. The animal at the front of the queue claims that all other animals are snakes. What is the total number of fire roosters in the queue?

3. Pablo Picasso. Pablo Picasso (1881–1973) is widely regarded as one of the most important painters of the last century. He was also a successful sculptor. His largest sculpture, the "Chicago Picasso", is over 15 metres high and weighs close to 150 tons. An artist's impression of the Chicago Picasso, which comprises four triangles and one quadrilateral, is shown on the right. What is the sum of the eight angles shown in white in Picasso's sculpture?



4. (S)he doesn't cut it. For his 70th birthday (held on June 14, 2016), Melania baked Donald a cake consisting of six squares, representing (some would say overestimating) his mental age. A photo of the cake, leaked by the fake media, is shown on the right. With one single cut Melania wishes to cut the cake into two equal-sized pieces, as is shown in the (failed) example on the right. If all squares have side length 10cm and Melania cuts from A on the bottom left corner to B on the right of the cake, determine the exact height of point B so that the cake is cut into two equal-sized pieces.



5. Malcolm in the middle. It is widely known that our Prime Minister, Malcolm Turnbull, loves to travel by public transport. However, as a Sydneysider, his knowledge of Melbourne's bus fares is limited. During a recent visit to Melbourne he took a bus trip from Carlton to Collingwood. He knew that the fare was at least \$3.00, but less than \$5.00. He also knew that the exact fare was required. What is the minimum number of coins (remember we have 5c, 10c, 20c, 50c, \$1 and \$2 coins) that he should have taken with him to be able to pay the exact fare?

6. Lucky numbers. You and your family attend a fair at the local primary school. Each family member buys exactly n raffle tickets, where n is some whole number greater than two. On each ticket a random positive whole number is printed. Your Mum, Dad, brother and sister are all amazed to discover that they can select three of their n tickets such that the sum of the numbers printed on these three is a multiple of 3. You explain to your family members that this is not amazing at all but a hard mathematical fact. What is the smallest value of n for which this is true, and why?