The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2018
JUNIOR DIVISION
Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

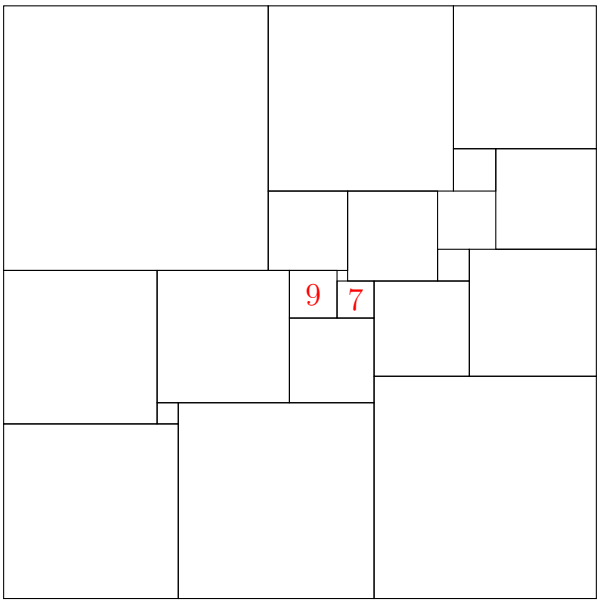
1. Girl power. At this year's Commonwealth Games, Shayna Jack, Emma McKeon, and sisters Bronte and Cate Campbell broke the world record for the 4×100 metres freestyle by more than half a second, swimming a new best time of 3:30.05 (three minutes, 30 seconds and 5 hundredths of a second). If Cate had swum 25% faster, the team would have finished in the even faster time of 3:20.05. What was Cate's time for her 100-metre leg of the relay?

2. Bad boy. Australian cricket vice-captain David W. is in the dressing room, using a piece of sandpaper to roughen a cricket ball. He hears captain Steve S. approaching and quickly hides the ball in a bag containing 99 untampered balls. Steve S. expects foul play and decides to check the balls. He can only compare two balls at a time, and he can only observe whether two balls are the same or different. In other words, if two balls are different he cannot tell which one has been tampered with. Assuming a worst-case scenario (i.e., Steve S. does not have any luck) but the best possible strategy (i.e., Steve S. is almost as smart as you are), what is the maximum number of pairs of balls he may need to compare before he can identify which is the tampered ball? Clearly justify your answer.

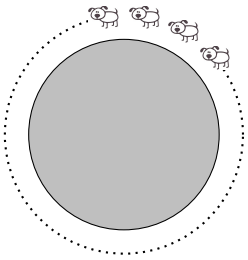
3. Dominoes. The popular game of dominoes originated during the reign of Chinese Emperor Xiaozong, the 11th emperor of the Song dynasty. In recent excavations around the city of Hangzhou—known as Wulin during Song's reign in the 12th century—archaeologists found an ancient domino instruction manual. One of the diagrams from the manual shows a 4×5 board covered with 10 dominoes, as seen in the picture on the right (translated into Arabic numerals for your convenience). Unfortunately, the outlines of the individual dominoes have completely faded. Assuming that the 10 dominoes are all different, determine the outlines of each of the 10 pieces. (Recall that a domino is a 1×2 tile containing two numbers, such as [1|3] or [2|2]. All dominoes being different means, for example, that only one of [0|2], $\left[\begin{smallmatrix}2\\0\end{smallmatrix}\right]$, [2|0] and $\left[\begin{smallmatrix}0\\2\end{smallmatrix}\right]$ may occur.)

2	0	3	1	4
4	4	1	0	2
5	5	1	4	2
2	2	1	2	0

4. Grace. From 30 March till 22 July, the National Gallery of Australia is presenting '*Cartier* The Exhibition', an exhibition of over 300 pieces of jewellery designed by famous Parisian jeweller Louis-François Cartier (1975–1942). The pièce de résistance of the exhibition is film star Grace Kelly's diamond engagement ring (she married Prince Rainier III of Monaco in 1956, becoming Princess Grace of Monaco). A photo (not to scale) of Grace Kelly's ring is shown on the right. The ring consists of 21 square diamonds, forming one larger square. The side-lengths of two of the individual stones are indicated in the photo. Find the total side-length of Grace's ring. (Partial marks are available for finding the side-lengths of some of the individual stones.)



5. Year of the dog. To celebrate the 'Year of the dog', 2018 dogs have gathered around a very large circular table to enjoy a nice meal of bones. The dogs are numbered from 1 to 2018, but sit in a random order around the table. For example, the poodle numbered 123 could have the shepherd numbered 18 to its left and the labradoodle numbered 1973 to its right. Show that there must be three consecutive dogs at the table the sum of whose numbers is at least 3029. (To earn a bonus mark, can you show the same for 3030?)



6. Blackbeard. Edward Teach, better known as Blackbeard, is one of the most notorious pirates to have ever lived. Known for his unusual fighting style, holding a cutlass in each hand, he robbed over 40 merchant ships in the Caribbean between 1695 and 1718. One day, on Treasure Island, he discovered a cave containing three locked treasure chests. A note pinned to one of the chests told him that two chests contained treasure and one contained rocks. His small barge could only carry one chest, so he had a difficult choice to make. His magic eye-patch allowed him to see inside one chest of his choice. The magic eye-patch, however, was not totally reliable, and in 25% of cases it would show a chest containing rocks as containing treasure, and in 25% of cases it would show a chest containing treasure as containing rocks. Blackbeard selected one of the chests, and his eye-patch showed that it contained rocks. Assuming the note told the truth, what is the probability that this chest actually contained treasure?