THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2013

MATH3303 Abstract Algebra and Number Theory

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 0 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - no materials permitted

No reading time

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - No calculators permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

Instructions To Students:

Each question is worth 10 points for a total of 80 points.

For Examiner Use Only

Question	Mark

Total ________

1. Write out all elements of $\mathrm{GL}_2(\mathbb{Z}/2\mathbb{Z})$ and compute the order of each element.

2. Let G be any group. Prove that the map from G to itself defined by $g \mapsto g^{-1}$ is a homomorphism if and only if G is abelian.

3. (a) State Lagrange's Theorem.

   (b) Use this theorem to show that if H and K are finite subgroups of G whose orders are relatively prime then $H \cap K = 1$.

4. Decide which of the following are subrings of $\mathbb{Q}$. Justify your answer.

   (a) The set of all rational numbers with odd denominators (when the fraction is completely reduced)

   (b) The set of all rational numbers with even denominators (again when the fraction is completely reduced)

   (c) The set of nonnegative rational numbers.

   (d) The set of squares of rational numbers

   (e) The set of all rational numbers with odd numerators (when the fraction is completely reduced).

5. Determine all maximal and prime ideals of the polynomials ring $\mathbb{C}[x]$. Justify your answer.

6. Let R be a commutative ring with identity, and let I be an ideal of R. Prove that I is maximal if and only if R/I is a field.

7. (a) Prove that every ideal in a Euclidean domain is principal.

   (b) Exhibit an ideal in $\mathbb{Z}[x]$ which is not principal.

8. Write the character table for S_3.

End of Examination