THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2015

MATH3303 Abstract Algebra and Number Theory

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 0 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - no materials permitted

No reading time

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - No calculators permitted

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

Instructions To Students:

Each question is worth 10 points for a total of 80 points.

For Examiner Use Only

Question	Mark

1. Show that $\mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$ is cyclic if and only if $\gcd(m, n) = 1$.
2. Let G be a group and H be a subgroup of G. Suppose the order of $|G/H| = 2$. Show that H is normal in G.
3. Classify all abelian groups of order 135.
4. Let R be a commutative ring with identity and let $I \subset R$ be an ideal. Show that I is a prime ideal if and only if R/I is an integral domain.
5. Let x be a nilpotent element of a ring with identity 1. Show that $1 + x$ is a unit.
6. Determine all maximal and prime ideals of $\mathbb{C}[x]$. Justify your answer.
7. Show that the number of conjugacy classes of nilpotent $n \times n$ matrices equals the number of partitions of n (Hint: use the Jordan Form).
8. Recall that $G = \mathrm{Gl}_n(\mathbb{R})$ acts on $\mathbb{R}^n$ as follows: a matrix $g \in G$ sends a vector $v \in V$ to the vector gv.
   (i) Show that this action is transitive.
   (ii) Determine $\mathrm{Stab}_G((1, 0, \cdots, 0))$.
   (iii) Now let
   $$S = \{v \in \mathbb{R}^n \mid v^t v = 1\}.$$
   Let $v = (1, 0, \cdots, 0)$. Give an example of $g \in G$ such that $gv \notin S$.
   (iv) Show that $O(n)$ acts on S; that is, show that if $g \in O(n)$ and $s \in S$, then $gs \in S$.
   (v) Show that the action of $O(n)$ on S is transitive.
   (vi) Show that $\mathrm{Stab}_{O(n)}((1, 0, 0, \cdots, 0))$ is isomorphic to $O(n-1)$.

End of Examination