THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2016

MATH3303 Abstract Algebra and Number Theory

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - no materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - No calculators permitted

Materials To Be Supplied To Students:

none

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

For Examiner Use Only

Question	Mark
1	
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Total ________

Part A: General knowledge

1. State the second isomorphism theorem for groups. **(5 marks)**

2. Give the definition of a solvable group. **(5 marks)**

3. State Wedderburn's theorem, and give the definition of all mathematical structures involved. **(5 marks)**

4. Let R be a ring.

   (a) Under what conditions does R have a field of fractions, Frac(R)? **(1 mark)**

   (b) Describe the full construction of Frac(R). You do not need to prove any of the (implicit) claims that make this construction work. **(5 marks)**

   (c) Show that there is no smaller field in which R can be embedded. **(4 marks)**

5. Give the definition of a unique factorisation domain and explain the meaning of each of the notions used in the definition. **(5 marks)**

Part B: Open problems

6. Let $f : G \to H$ be a homomorphism between groups. Prove that $\ker f \lhd G$.
(Show both the subgroup and normality property.) **(5 marks)**

7. Let m, n, l be positive integers. For which values of m, n, l is it true that

$$(\mathbb{Z}/m\mathbb{Z})/(\mathbb{Z}/n\mathbb{Z}) \cong \mathbb{Z}/l\mathbb{Z}$$

as an isomorphism of groups? Fully justify your answer. **(10 marks)**

8. Show that all finite integral domains are fields. **(10 marks)**

9. Let $R = \mathbb{Q}[x]$ and $I = (x - m)\mathbb{Q}[x]$ for a fixed $m \in \mathbb{Z}$.
   Identify the quotient ring R/I. All your claims must be fully justified. **(10 marks)**

10. Show that an ideal I of a commutative ring R is prime if and only if R/I is an integral domain. **(10 marks)**

11. We say that a ring $R \neq 0$ is *local* if the set of nonunits, J, is an ideal of R.

(a) Let R be a local ring. Show that R/J is a division ring. **(5 marks)**

(b) Let R be local. Show that if $I \subseteq J$ is an ideal of R then R/I is local. **(10 marks)**

(c) Let I be an ideal of R such that all elements of I are nilpotent and such that R/I is a division ring. Show that R is local. **(10 marks)**

END OF EXAMINATION