INTERNAL STUDENTS ONLY

THE UNIVERSITY OF QUEENSLAND

CANDIDATES MUST NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

First Semester Examination, June 2000 SAMPLE

MT140 SAMPLE

Basic Mathematics SAMPLE

(Unit Courses) SAMPLE

Time: TWO Hours for working

Ten minutes for perusal before examination begins

Check that this examination paper has 20 printed pages!

CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION PAPER!
There are 120 marks on the paper. All questions carry the indicated number of marks. Students should attempt all questions. Page 20 contains formulae. Pocket calculators allowed.

FAMILY NAME (PRINT): ______________________

GIVEN NAMES (PRINT): ______________________

STUDENT NUMBER: | | | | | | | – | | | |

SIGNATURE: ______________________

EXAMINER'S USE ONLY	
TOTAL	

1. Find R if

$$R = \sum_{i=1}^{3} \frac{1}{i}.$$

(3 marks)

2. Simplify $\dfrac{3^{n+1} - 3^{n-1}}{3^n - 3^{n-2}}$. (4 marks)

3. **(a)** Find all x such that $|2x - 3| < 5$. (4 marks)

(b) Write your answer to part (a) in interval format. (1 mark)

(c) Mark your answer to part (a) on the real line. (1 mark)

4. Write in summation (sigma) notation
$x - 2x + 3x - 4x + 5x$. (2 marks)

5. **(a)** Find the equation of the line passing through $(1, 2)$ and $(1, -2)$. (2 marks)

(b) Find the straight line distance between $(1, 2)$ and $(1, -2)$. (2 marks)

6. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$, $B = \{-2, 4\}$ and $C = \{-1, 5\}$.

(a) Mark the three sets A, B and C on a Venn diagram, with the elements of each set written in appropriate places on the diagram. (3 marks)

(b) Write down the elements of the set $B \cap C$. (1 mark)

(c) Write down the elements of the set $(A \cap B) \cup C$. (1 mark)

(d) Write down the elements of the set $A \setminus B$. (1 mark)

(e) Write down the elements of the set $(C \cap \emptyset) \cup (B \cap \emptyset) \cup (A \cap \emptyset)$. (1 mark)

7. Solve the following system of two simultaneous equations.

$$\begin{aligned} 3x + y &= 2 \\ x - 2y &= -4 \end{aligned}$$

(3 marks)

8. Find the equation of the line which is tangential to the curve $y = x^2 - 2x + 3$ at the point $(2, 3)$. (3 marks)

9. Find the point(s) at which the parabola $y = x^2 + x - 1$ intersects the line $y = x + 3$. Find both the x and y values at the intersection point(s).

(4 marks)

10. Find from first principles, using limits, the slope of the following function at any point x:

$$y = -2x + 3.$$

(5 marks)

11. I roll *two* fair, six-sided dice. What is the probability that:

(a) I roll a pair of odd numbers? Answer: ________ (2 marks)

(b) I roll exactly one odd number and one even number? Answer: ________ (2 marks)

(c) I roll a pair of even numbers? Answer: ________ (2 marks)

12. If $y = (x+1)^9$, find $\dfrac{dy}{dx}$. (3 marks)

13. If $y = (x^2+2)\sin x$, find $\dfrac{dy}{dx}$. (4 marks)

14. If $y = \dfrac{(x-1)(x+1)}{\ln x}$, find $\dfrac{dy}{dx}$. (5 marks)

15. Let $f(x) = x^3 - 3x$.

(a) Find all critical points of $f(x)$. (5 marks)

(b) Classify each critical point as a local maximum or minimum. (3 marks)

(c) Find the absolute maximum and minimum values attained by $f(x)$ on the interval $x \in [-1, 3]$. (3 marks)

16. Let

$$\begin{aligned} A &= \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \\ C &= \begin{pmatrix} 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} -1 \\ 1 \end{pmatrix}. \end{aligned}$$

Where possible, write down the answers to the following expressions. Where not possible, indicate why.

(i) $C + B$

(ii) $A - B$

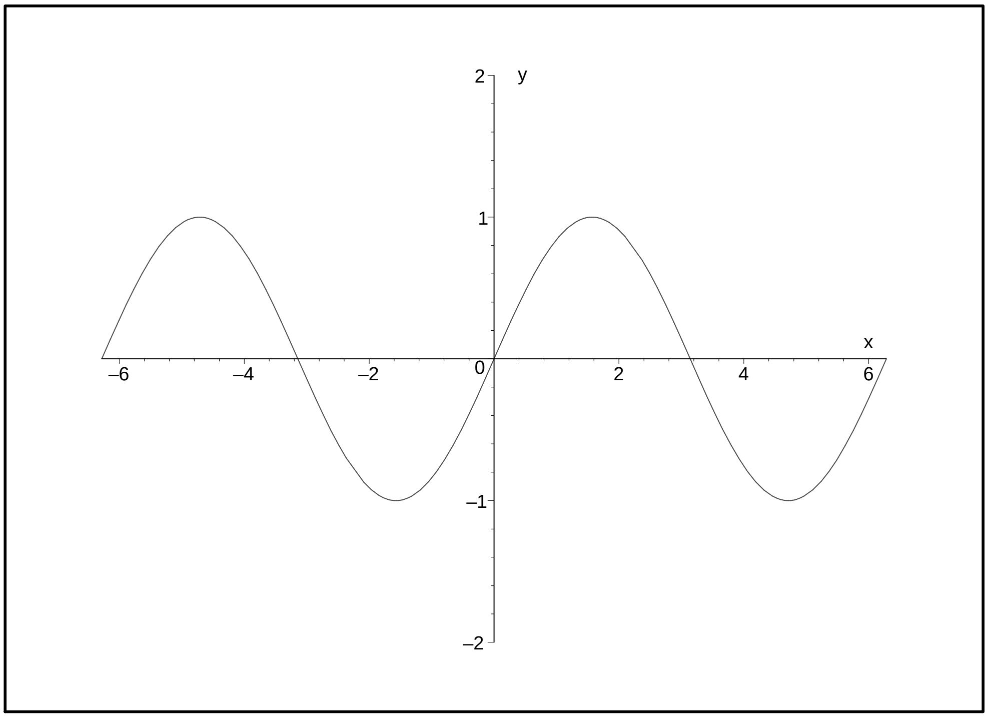
(iii) $B \times A$

(iv) $A + B - C$

(v) $B \times D$

(7 marks)

17. The following two sets of axes each contain a graph of $y = \sin x$, for $x \in [-2\pi, 2\pi]$. On the first set of axes, sketch a graph of $y = \sin 2x$. (2 marks)



On the second set of axes, sketch a graph of $y = 2\sin x$. (2 marks)

(Comment: an identical set of axes to the one above is included here; sketch your answer on this set of axes).

18. The following six equations have their graphs drawn below, in random order, with six extra graphs included. Match each equation with its graph, by writing the letter of the corresponding graph next to each equation. Note that the graphs are labelled A to L.

(6 marks)

Equation 1 is: .... Matching graph is labelled: ________

Equation 2 is: .... Matching graph is labelled: ________

Equation 3 is: .... Matching graph is labelled: ________

Equation 4 is: .... Matching graph is labelled: ________

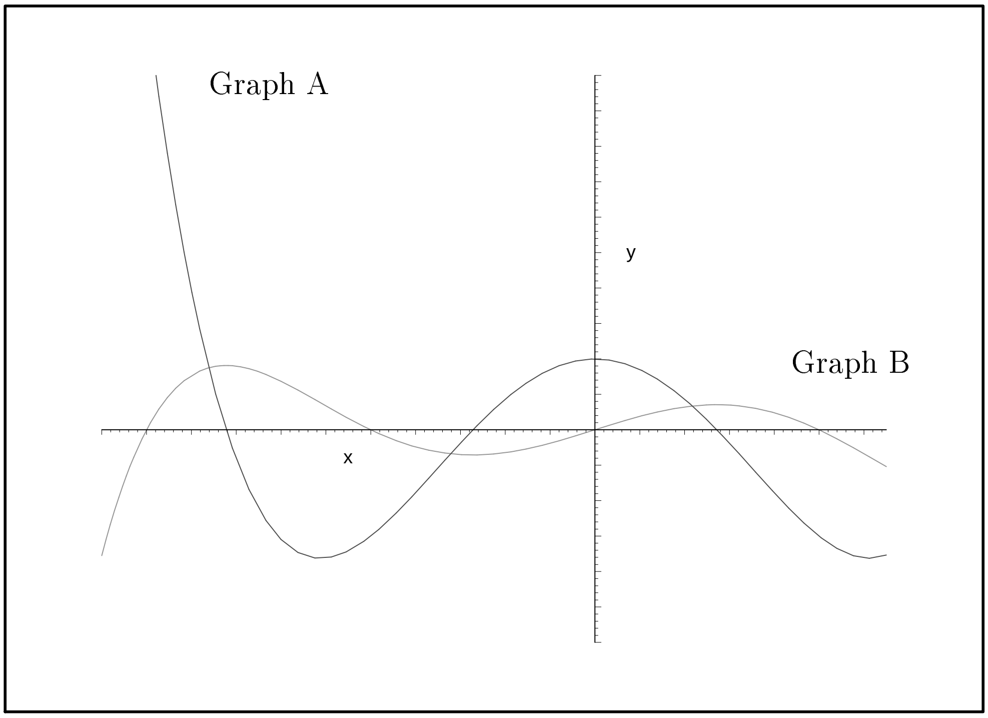
Equation 5 is: .... Matching graph is labelled: ________

Equation 6 is: .... Matching graph is labelled: ________

(Comment: see the midsemester exam for examples of graphs, plus graphs from the second part of the course).

19. The following set of axes contains the graph of a function and the graph of the derivative of the function. The two graphs have been labelled Graph A and Graph B. Clearly identify which graph represents the function, and which represents the derivative. Explain your answer. (Most marks will be assigned to your explanation.)

(3 marks)



The function is Graph ________.

The derivative is Graph ________.

Explanation:

20. A rocket takes off vertically at time $t = 0$, with velocity

$$v(t) = t + 1 \text{ metres/second.}$$

(i) Find an expression for the displacement of the rocket at any time t. Assume the rocket launching pad represents a displacement of 0 metres, so the displacement at $t = 0$ is 0.

(4 marks)

(ii) How far does the rocket travel between $t = 6$ and $t = 10$? (2 marks)

21. Find $\displaystyle\int \left(x - \frac{1}{x}\right) dx$. (3 marks)

22. Evaluate $\displaystyle\int_1^3 (e^x - 2x)\, dx$. (3 marks)

23. A mystery question. Let $y = \ldots$ (some function).

(a) Using the product rule (or otherwise), show that $\dfrac{dy}{dx} = \mathit{something}$.

(4 marks)

(b) Find all values of x for which $\dfrac{dy}{dx} = 0$. (2 marks)

(c) Show that $\dfrac{d^2y}{dx^2} = \mathit{something}$. (4 marks)

(d) Hence, or otherwise, show that the function y has a local maximum at $x =$ *some point*.

(2 marks)

24. A mystery question involving a number of 2 by 2 matrices. Be able to multiply such matrices, take scalar multiples of matrices, know what I is (the two by two identity matrix), and so on.

(6 marks)

TURN OVER FOR FORMULAE SHEET

Formulae Sheet:

Roots of $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Limits and derivatives:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Product rule:

$$(uv)' = u' \cdot v + u \cdot v'$$

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$$

Quotient rule:

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$