

Some edge-colored cube decompositions

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This research report contains a number of edge-colored cube decompositions, used in the constructions in [1].

Within this report, any vertex labelled ∞ remains fixed when cycling the given graphs. For any vertices labelled i_j , the subscript j remains fixed and the base i is cycled when cycling the given graphs. For any bipartite graph, use the obvious vertex partition.

References

- [1] P. Adams, D.E. Bryant and H. Gavlas, Edge-coloured cube decompositions, (submitted).

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G_1 designs

$6K_8$ Let the vertex set be $\mathbb{Z}_7 \cup \{\infty\}$. Cycle the following mod 7:

$$(((0, 1)(2, 4))_1, ((0, 2)(1, 4))_2, ((0, 3)(1, 6))_3, ((2, 5)(4, \infty))_4, ((3, 6)(5, \infty))_5, ((3, 5)(6, \infty))_6, \\ (((0, \infty)(2, 6))_1, ((2, \infty)(0, 6))_2, ((3, \infty)(0, 1))_3, ((2, 4)(5, 6))_4, ((1, 3)(4, 5))_5, ((3, 4)(1, 5))_6).$$

$6K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$$(((0, 1)(2, 5))_1, ((0, 2)(1, 5))_2, ((0, 3)(1, 6))_3, ((2, 4)(5, 8))_4, ((3, 6)(4, 8))_5, ((3, 4)(6, 8))_6, \\ (((0, 4)(3, 5))_1, ((0, 3)(4, 5))_2, ((0, 8)(4, 6))_3, ((2, 3)(1, 5))_4, ((6, 8)(1, 2))_5, ((2, 8)(1, 6))_6).$$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$$(((0, 1)(2, 4))_1, ((0, 2)(1, 4))_2, ((0, 3)(1, 5))_3, ((2, 6)(4, \infty))_4, ((3, 5)(6, \infty))_5, ((3, 6)(5, \infty))_6, \\ (((0, \infty)(1, 5))_1, ((1, \infty)(0, 5))_2, ((2, \infty)(0, 6))_3, ((1, 7)(5, 8))_4, ((2, 6)(7, 8))_5, ((2, 7)(6, 8))_6, \\ (((0, 3)(1, 7))_1, ((0, 1)(3, 7))_2, ((0, 9)(3, 4))_3, ((1, 2)(7, 5))_4, ((9, 4)(2, 5))_5, ((9, 2)(4, 5))_6).$$

$6K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$$(((0, 1)(2, 4))_1, ((0, 2)(1, 4))_2, ((0, 3)(1, 5))_3, ((2, 6)(4, 7))_4, ((3, 5)(6, 7))_5, ((3, 6)(5, 7))_6, \\ (((0, 3)(1, 7))_1, ((0, 1)(3, 7))_2, ((0, 5)(3, 10))_3, ((1, 12)(2, 7))_4, ((5, 10)(2, 12))_5, ((5, 12)(2, 10))_6, \\ (((0, 5)(6, 10))_1, ((0, 6)(5, 10))_2, ((0, 11)(4, 5))_3, ((6, 7)(3, 10))_4, ((4, 11)(3, 7))_5, ((7, 11)(3, 4))_6).$$

$6K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$$(((0, 4)(1, 5))_1, ((0, 5)(1, 4))_2, ((0, 6)(2, 4))_3, ((3, 5)(1, 7))_4, ((2, 6)(3, 7))_5, ((3, 6)(2, 7))_6, \\ (((0, 5)(1, 4))_1, ((0, 4)(1, 5))_2, ((0, 7)(2, 5))_3, ((3, 4)(1, 6))_4, ((2, 7)(3, 6))_5, ((3, 7)(2, 6))_6, \\ (((0, 6)(1, 7))_1, ((0, 7)(1, 6))_2, ((0, 4)(2, 6))_3, ((3, 7)(1, 5))_4, ((2, 4)(3, 5))_5, ((3, 4)(2, 5))_6, \\ (((0, 7)(1, 6))_1, ((0, 6)(1, 7))_2, ((0, 5)(2, 7))_3, ((3, 6)(1, 4))_4, ((2, 5)(3, 4))_5, ((3, 5)(2, 4))_6, \\ (((3, 4)(2, 5))_1, ((3, 5)(2, 4))_2, ((3, 6)(1, 4))_3, ((0, 5)(2, 7))_4, ((1, 6)(0, 7))_5, ((0, 6)(1, 7))_6, \\ (((3, 5)(2, 4))_1, ((3, 4)(2, 5))_2, ((3, 7)(1, 5))_3, ((0, 4)(2, 6))_4, ((1, 7)(0, 6))_5, ((0, 7)(1, 6))_6, \\ (((3, 6)(2, 7))_1, ((3, 7)(2, 6))_2, ((3, 4)(1, 6))_3, ((0, 7)(2, 5))_4, ((1, 4)(0, 5))_5, ((0, 4)(1, 5))_6, \\ (((3, 7)(2, 6))_1, ((3, 6)(2, 7))_2, ((3, 5)(1, 7))_3, ((0, 6)(2, 4))_4, ((1, 5)(0, 4))_5, ((0, 5)(1, 4))_6).$$

G_2 designs

$6K_8$ Let the vertex set be $\mathbb{Z}_7 \cup \{\infty\}$. Cycle the following mod 7:

$$(((0, 1)(0, 2))_1, ((1, \infty)(1, 4))_2, ((2, \infty)(2, 6))_3, ((0, 3)(3, 4))_4, ((3, 6)(6, 5))_5, ((5, \infty)(4, 5))_6, \\ (((1, 4)(1, \infty))_1, ((0, 1)(0, 2))_2, ((2, 4)(2, 3))_3, ((0, 5)(5, \infty))_4, ((4, 6)(6, \infty))_5, ((3, 5)(3, 6))_6).$$

$6K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$$(((0, 1)(0, 2))_1, ((1, 3)(1, 5))_2, ((2, 3)(2, 7))_3, ((0, 5)(4, 5))_4, ((5, 7)(6, 7))_5, ((3, 6)(4, 6))_6, \\ (((0, 3)(0, 4))_1, ((2, 3)(3, 6))_2, ((2, 4)(1, 4))_3, ((0, 6)(6, 8))_4, ((1, 6)(1, 7))_5, ((2, 7)(7, 8))_6).$$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$$(((0, 1)(0, 2))_1, ((1, \infty)(1, 4))_2, ((2, \infty)(2, 5))_3, ((3, 0)(3, 4))_4, ((5, 3)(5, 6))_5, ((6, \infty)(6, 4))_6, \\ (((1, 4)(1, \infty))_1, ((0, 1)(0, 2))_2, ((2, 4)(2, 6))_3, ((7, 0)(7, \infty))_4, ((4, 9)(9, \infty))_5, ((6, 7)(6, 9))_6, \\ (((0, 4)(0, 5))_1, ((4, 10)(4, 8))_2, ((5, 10)(5, 6))_3, ((2, 0)(2, 8))_4, ((6, 2)(6, 3))_5, ((3, 10)(3, 8))_6).$$

$6K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(0, 2))_1, ((1, 3)(1, 5))_2, ((2, 3)(2, 6))_3, ((0, 4)(4, 5))_4, ((4, 6)(6, 7))_5, ((3, 7)(5, 7))_6,$
 $((0, 3)(0, 4))_1, ((2, 3)(3, 8))_2, ((2, 4)(1, 4))_3, ((0, 5)(5, 8))_4, ((1, 5)(1, 7))_5, ((2, 7)(7, 8))_6,$
 $((0, 6)(0, 5))_1, ((6, 12)(6, 9))_2, ((5, 12)(5, 10))_3, ((0, 7)(7, 9))_4, ((7, 10)(2, 10))_5, ((2, 12)(2, 9))_6.$

$6K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The decomposition is given by:

$((0, 4)(0, 5))_1, ((1, 4)(2, 4))_2, ((1, 5)(3, 5))_3, ((0, 6)(2, 6))_4, ((3, 6)(3, 7))_5, ((1, 7)(2, 7))_6,$
 $((0, 6)(0, 7))_1, ((1, 6)(2, 6))_2, ((1, 7)(3, 7))_3, ((0, 4)(2, 4))_4, ((3, 4)(3, 5))_5, ((1, 5)(2, 5))_6,$
 $((1, 4)(1, 5))_1, ((0, 4)(3, 4))_2, ((0, 5)(2, 5))_3, ((1, 6)(3, 6))_4, ((2, 6)(2, 7))_5, ((0, 7)(3, 7))_6,$
 $((1, 6)(1, 7))_1, ((0, 6)(3, 6))_2, ((0, 7)(2, 7))_3, ((1, 4)(3, 4))_4, ((2, 4)(2, 5))_5, ((0, 5)(3, 5))_6,$
 $((2, 5)(2, 4))_1, ((3, 5)(0, 5))_2, ((3, 4)(1, 4))_3, ((2, 7)(0, 7))_4, ((1, 7)(1, 6))_5, ((3, 6)(0, 6))_6,$
 $((2, 7)(2, 6))_1, ((3, 7)(0, 7))_2, ((3, 6)(1, 6))_3, ((2, 5)(0, 5))_4, ((1, 5)(1, 4))_5, ((3, 4)(0, 4))_6,$
 $((3, 5)(3, 4))_1, ((2, 5)(1, 5))_2, ((2, 4)(0, 4))_3, ((3, 7)(1, 7))_4, ((0, 7)(0, 6))_5, ((2, 6)(1, 6))_6,$
 $((3, 7)(3, 6))_1, ((2, 7)(1, 7))_2, ((2, 6)(0, 6))_3, ((3, 5)(1, 5))_4, ((0, 5)(0, 4))_5, ((2, 4)(1, 4))_6.$

G_3 designs

$4K_{10}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2\}$. Cycle the following mod 5:

$((0_1, 1_1)(2_1, 0_2)(3_1, 4_2))_1, ((0_1, 2_1)(1_1, 0_2)(2_2, 3_2))_2, ((0_1, 3_1)(2_1, 2_2)(3_2, 4_2))_3,$
 $((1_1, 4_2)(0_2, 3_2)(3_1, 2_2))_4,$
 $((0_1, 3_1)(2_2, 4_2)(4_1, 3_2))_1, ((0_1, 2_2)(3_1, 4_2)(2_1, 0_2))_2, ((0_1, 4_1)(0_2, 2_2)(2_1, 3_2))_3,$
 $((3_1, 3_2)(2_1, 4_2)(4_1, 0_2))_4,$
 $((0_2, 1_2)(1_1, 3_2)(2_1, 2_2))_1, ((0_2, 3_2)(1_1, 1_2)(0_1, 4_1))_2, ((2_1, 0_2)(4_1, 3_2)(0_1, 2_2))_3,$
 $((1_2, 2_2)(0_1, 1_1)(2_1, 4_1))_4.$

$4K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(2, 4)(3, 6))_1, ((0, 2)(1, 4)(5, 9))_2, ((0, 3)(2, 9)(5, 6))_3, ((1, 6)(4, 5)(3, 9))_4,$
 $((0, 4)(1, 9)(2, 8))_1, ((0, 1)(4, 9)(6, 12))_2, ((0, 8)(1, 12)(2, 6))_3, ((2, 4)(6, 9)(8, 12))_4.$

$4K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$((0_1, 1_1)(2_1, 0_2)(3_1, 2_2))_1, ((0_1, 2_1)(1_1, 0_2)(1_2, \infty))_2, ((0_1, 3_1)(2_1, 1_2)(2_2, \infty))_3,$
 $((1_1, 2_2)(0_2, \infty)(3_1, 1_2))_4,$
 $((1_1, \infty)(2_1, 2_2)(3_1, 0_2))_1, ((2_1, \infty)(1_1, 2_2)(1_2, 3_2))_2, ((3_1, \infty)(2_1, 3_2)(0_2, 1_2))_3,$
 $((1_1, 0_2)(1_2, 2_2)(3_1, 3_2))_4,$
 $((0_2, \infty)(0_1, 0_3)(2_1, 4_1))_1, ((0_1, 0_2)(0_3, \infty)(1_1, 4_2))_2, ((2_1, 0_2)(0_1, 1_1)(4_1, 4_2))_3,$
 $((4_1, \infty)(4_2, 0_3)(1_1, 2_1))_4,$
 $((0_3, 1_3)(0_1, 1_2)(2_3, \infty))_1, ((0_1, 0_3)(1_2, 1_3)(4_2, 3_3))_2, ((0_3, \infty)(0_1, 3_3)(4_2, 2_3))_3,$
 $((1_3, 2_3)(1_2, 4_2)(3_3, \infty))_4,$
 $((0_2, 1_2)(4_1, 1_3)(2_2, 0_3))_1, ((0_2, 1_3)(4_1, 1_2)(0_1, 4_3))_2, ((0_2, 2_2)(0_1, 1_3)(0_3, 4_3))_3,$
 $((1_2, 0_3)(4_1, 4_3)(0_1, 2_2))_4,$
 $((0_2, 1_3)(1_2, 4_2)(2_1, 0_3))_1, ((0_2, 1_2)(4_2, 1_3)(2_3, 4_3))_2, ((0_2, 0_3)(1_2, 2_3)(2_1, 4_3))_3,$
 $((2_1, 1_3)(4_2, 4_3)(0_3, 2_3))_4,$
 $((0_2, 4_3)(2_1, 3_3)(2_2, 2_3))_1, ((0_2, 3_3)(2_1, 4_3)(0_1, 4_1))_2, ((0_2, 2_3)(4_1, 3_3)(0_1, 2_2))_3,$
 $((2_2, 4_3)(0_1, 2_1)(4_1, 2_3))_4,$
 $((0_3, 3_3)(2_1, 1_3)(0_2, 2_3))_1, ((0_3, 1_3)(2_1, 3_3)(1_1, 4_3))_2, ((0_3, 2_3)(1_1, 1_3)(0_2, 4_3))_3,$
 $((0_2, 3_3)(2_1, 4_3)(1_1, 2_3))_4.$

$4K_{19}$ Let the vertex set be \mathbb{Z}_{19} . Cycle the following mod 19:

$$\begin{aligned} &(((0, 1)(2, 4)(3, 6))_1, ((0, 2)(1, 4)(7, 12))_2, ((0, 3)(2, 7)(6, 12))_3, ((1, 6)(4, 12)(3, 7))_4), \\ &(((0, 4)(1, 8)(2, 7))_1, ((0, 1)(4, 8)(6, 12))_2, ((0, 2)(1, 12)(6, 7))_3, ((4, 7)(6, 8)(2, 12))_4), \\ &(((0, 8)(11, 17)(1, 10))_1, ((0, 11)(8, 17)(4, 16))_2, ((0, 10)(4, 11)(1, 16))_3, ((1, 8)(16, 17)(4, 10))_4). \end{aligned}$$

$4K_{22}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_{11}, j = 1, 2\}$. Cycle the following mod 11:

$$\begin{aligned} &(((3_1, 3_2)(2_2, 7_2)(2_1, 8_2))_1, ((3_1, 2_2)(3_2, 7_2)(1_1, 8_1))_2, ((3_1, 8_2)(1_1, 2_2)(2_1, 8_1))_3, \\ & \quad \quad \quad ((2_1, 3_2)(8_1, 7_2)(1_1, 8_2))_4), \\ &(((1_1, 5_1)(3_1, 8_2)(2_1, 8_1))_1, ((3_1, 5_1)(1_1, 8_2)(4_1, 6_2))_2, ((5_1, 8_1)(3_1, 4_1)(2_1, 6_2))_3, \\ & \quad \quad \quad ((1_1, 2_1)(6_2, 8_2)(4_1, 8_1))_4), \\ &(((3_1, 4_1)(7_2, 10_2)(8_1, 0_2))_1, ((4_1, 10_2)(3_1, 7_2)(10_1, 2_2))_2, ((4_1, 8_1)(10_1, 10_2)(0_2, 2_2))_3, \\ & \quad \quad \quad ((3_1, 0_2)(2_2, 7_2)(8_1, 10_1))_4), \\ &(((0_1, 2_1)(1_1, 0_2)(9_1, 2_2))_1, ((0_1, 1_1)(2_1, 0_2)(1_2, 3_2))_2, ((0_1, 9_1)(1_1, 3_2)(1_2, 2_2))_3, \\ & \quad \quad \quad ((2_1, 2_2)(0_2, 1_2)(9_1, 3_2))_4), \\ &(((0_1, 3_1)(9_1, 5_2)(8_1, 6_2))_1, ((0_1, 5_2)(3_1, 9_1)(4_2, 9_2))_2, ((0_1, 6_2)(5_2, 9_2)(8_1, 4_2))_3, \\ & \quad \quad \quad ((3_1, 8_1)(9_1, 4_2)(6_2, 9_2))_4), \\ &(((0_2, 1_2)(0_1, 8_2)(3_2, 5_2))_1, ((0_2, 8_2)(0_1, 1_2)(5_1, 8_1))_2, ((0_2, 3_2)(5_1, 8_2)(8_1, 5_2))_3, \\ & \quad \quad \quad ((1_2, 5_2)(0_1, 8_1)(5_1, 3_2))_4), \\ &(((0_2, 7_2)(10_1, 1_2)(4_1, 5_2))_1, ((0_2, 1_2)(10_1, 7_2)(3_1, 3_2))_2, ((0_2, 5_2)(3_1, 1_2)(4_1, 3_2))_3, \\ & \quad \quad \quad ((4_1, 7_2)(10_1, 3_2)(3_1, 5_2))_4). \end{aligned}$$

$4K_{6,6}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_6, j = 1, 2\}$. Cycle the following mod 6:

$$\begin{aligned} &(((0_1, 0_2)(2_1, 1_2)(5_1, 2_2))_1, ((0_1, 1_2)(2_1, 0_2)(3_1, 5_2))_2, ((0_1, 2_2)(3_1, 1_2)(5_1, 5_2))_3, \\ & \quad \quad \quad ((5_1, 0_2)(2_1, 5_2)(3_1, 2_2))_4), \\ &(((0_1, 1_2)(4_1, 0_2)(1_1, 5_2))_1, ((0_1, 0_2)(4_1, 1_2)(3_1, 2_2))_2, ((0_1, 5_2)(3_1, 0_2)(1_1, 2_2))_3, \\ & \quad \quad \quad ((1_1, 1_2)(4_1, 2_2)(3_1, 5_2))_4). \end{aligned}$$

$4K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$$\begin{aligned} &(((0_1, 0_3)(0_2, 1_3)(1_1, 2_3))_1, ((0_1, 1_3)(0_2, 0_3)(1_2, 0_4))_2, ((0_1, 2_3)(1_2, 1_3)(1_1, 0_4))_3, \\ & \quad \quad \quad ((1_1, 0_3)(0_2, 0_4)(1_2, 2_3))_4), \\ &(((0_1, 2_3)(0_2, 0_3)(1_1, 1_4))_1, ((0_1, 0_3)(0_2, 2_3)(2_1, 0_5))_2, ((0_1, 1_4)(2_1, 0_3)(1_1, 0_5))_3, \\ & \quad \quad \quad ((1_1, 2_3)(0_2, 0_5)(2_1, 1_4))_4), \\ &(((0_1, 1_4)(1_1, 2_5)(2_2, 0_4))_1, ((0_1, 2_5)(1_1, 1_4)(2_1, 1_3))_2, ((0_1, 0_4)(2_1, 2_5)(2_2, 1_3))_3, \\ & \quad \quad \quad ((2_2, 1_4)(1_1, 1_3)(2_1, 0_4))_4), \\ &(((0_2, 0_4)(2_1, 1_4)(2_2, 1_3))_1, ((0_2, 1_4)(2_1, 0_4)(1_2, 0_5))_2, ((0_2, 1_3)(1_2, 1_4)(2_2, 0_5))_3, \\ & \quad \quad \quad ((2_2, 0_4)(2_1, 0_5)(1_2, 1_3))_4), \\ &(((0_2, 1_5)(1_1, 0_5)(2_2, 2_5))_1, ((0_2, 0_5)(1_1, 1_5)(2_1, 1_4))_2, ((0_2, 2_5)(2_1, 0_5)(2_2, 1_4))_3, \\ & \quad \quad \quad ((2_2, 1_5)(1_1, 1_4)(2_1, 2_5))_4), \\ &(((0_2, 2_5)(1_2, 0_4)(0_1, 0_5))_1, ((0_2, 0_4)(1_2, 2_5)(2_2, 0_3))_2, ((0_2, 0_5)(2_2, 0_4)(0_1, 0_3))_3, \\ & \quad \quad \quad ((0_1, 2_5)(1_2, 0_3)(2_2, 0_5))_4). \end{aligned}$$

G_4 designs

$4K_{10}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2\}$. Cycle the following mod 5:

$$\begin{aligned} &(((0_1, 1_1)(0_1, 2_1)(3_1, 1_2))_1, ((1_1, 3_1)(1_1, 0_2)(0_1, 4_1))_2, ((4_1, 0_2)(0_2, 1_2)(2_1, 4_2))_3, \\ & \quad \quad \quad ((4_1, 4_2)(1_2, 4_2)(2_1, 3_1))_4), \\ &(((0_2, 3_2)(4_1, 0_2)(1_1, 1_2))_1, ((1_1, 3_2)(3_2, 4_2)(0_1, 0_2))_2, ((0_1, 4_2)(1_2, 4_2)(4_1, 2_2))_3, \\ & \quad \quad \quad ((0_1, 2_2)(1_2, 2_2)(1_1, 4_1))_4), \\ &(((0_2, 4_2)(3_1, 0_2)(2_1, 1_2))_1, ((1_2, 4_2)(1_1, 4_2)(4_1, 0_2))_2, ((1_1, 4_1)(1_1, 2_1)(3_1, 3_2))_3, \\ & \quad \quad \quad ((4_1, 3_2)(2_1, 3_2)(3_1, 1_2))_4). \end{aligned}$$

$4K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$$\begin{aligned} &(((0, 1)(0, 2)(3, 9))_1, ((1, 3)(1, 6)(0, 4))_2, ((4, 6)(6, 9)(2, 7))_3, ((4, 7)(7, 9)(2, 3))_4), \\ &(((0, 5)(0, 3)(8, 12))_1, ((5, 12)(4, 5)(0, 10))_2, ((4, 10)(4, 8)(2, 3))_3, ((2, 10)(2, 8)(3, 12))_4). \end{aligned}$$

$4K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$$\begin{aligned} &(((0_1, 1_1)(0_1, 2_1)(3_1, \infty))_1, ((1_1, 3_1)(1_1, 0_2)(0_1, 4_1))_2, ((4_1, 0_2)(0_2, \infty)(2_1, 1_2))_3, \\ & \quad \quad \quad ((4_1, 1_2)(1_2, \infty)(2_1, 3_1))_4), \\ &(((0_1, 0_2)(0_1, 1_2)(2_1, 0_3))_1, ((2_1, 0_2)(3_1, 0_2)(0_1, \infty))_2, ((3_1, \infty)(3_1, 0_3)(4_1, 1_2))_3, \\ & \quad \quad \quad ((4_1, \infty)(4_1, 0_3)(2_1, 1_2))_4), \\ &(((0_2, 1_2)(1_1, 0_2)(4_1, 2_2))_1, ((1_2, 2_2)(1_2, 0_3)(0_2, \infty))_2, ((0_3, \infty)(4_1, 0_3)(1_1, 1_3))_3, \\ & \quad \quad \quad ((1_3, \infty)(4_1, 1_3)(1_1, 2_2))_4), \\ &(((0_2, \infty)(0_3, \infty)(2_2, 4_2))_1, ((0_2, 2_2)(0_2, 2_3)(1_3, \infty))_2, ((1_3, 2_3)(4_2, 2_3)(1_1, 0_3))_3, \\ & \quad \quad \quad ((1_1, 1_3)(1_1, 4_2)(2_2, 0_3))_4), \\ &(((0_1, 2_2)(0_1, 4_3)(4_2, 0_3))_1, ((2_2, 0_3)(2_2, 3_3)(0_1, 1_2))_2, ((1_2, 3_3)(4_2, 3_3)(1_3, 4_3))_3, \\ & \quad \quad \quad ((1_2, 1_3)(4_2, 1_3)(0_3, 4_3))_4), \\ &(((0_1, 2_3)(0_1, 0_3)(1_2, 3_3))_1, ((2_3, 3_3)(2_2, 2_3)(0_1, 0_2))_2, ((0_2, 2_2)(1_2, 2_2)(4_2, 0_3))_3, \\ & \quad \quad \quad ((0_2, 4_2)(1_2, 4_2)(0_3, 3_3))_4), \\ &(((0_3, 3_3)(1_2, 0_3)(1_1, 2_3))_1, ((1_1, 3_3)(4_1, 3_3)(0_1, 0_3))_2, ((0_1, 4_1)(4_1, 2_3)(3_1, 1_2))_3, \\ & \quad \quad \quad ((0_1, 3_1)(3_1, 2_3)(1_1, 1_2))_4), \\ &(((0_3, 4_3)(2_2, 0_3)(1_2, 1_3))_1, ((1_3, 4_3)(1_1, 4_3)(4_1, 0_3))_2, ((1_1, 4_1)(1_1, 1_2)(2_2, 2_3))_3, \\ & \quad \quad \quad ((4_1, 2_3)(1_2, 2_3)(2_2, 1_3))_4). \end{aligned}$$

$4K_{19}$ Let the vertex set be \mathbb{Z}_{19} . Cycle the following mod 19:

$$\begin{aligned} &(((0, 1)(0, 2)(3, 9))_1, ((1, 3)(1, 6)(0, 4))_2, ((4, 6)(6, 9)(2, 7))_3, ((4, 7)(7, 9)(2, 3))_4), \\ &(((0, 3)(0, 4)(9, 14))_1, ((3, 9)(3, 10)(0, 1))_2, ((1, 10)(10, 14)(4, 5))_3, ((1, 5)(5, 14)(4, 9))_4), \\ &(((0, 8)(0, 7)(9, 18))_1, ((8, 18)(8, 16)(0, 3))_2, ((3, 16)(9, 16)(7, 15))_3, ((3, 15)(9, 15)(7, 18))_4). \end{aligned}$$

$4K_{22}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_{11}, j = 1, 2\}$. Cycle the following mod 11:

$$\begin{aligned} &(((5_2, 9_2)(8_1, 5_2)(9_1, 7_2))_1, ((9_1, 9_2)(5_1, 9_2)(10_1, 5_2))_2, ((5_1, 10_1)(5_1, 7_2)(6_1, 8_1))_3, \\ & \quad \quad \quad ((6_1, 10_1)(6_1, 7_2)(8_1, 9_1))_4), \\ &(((4_1, 7_2)(1_1, 7_2)(3_2, 5_2))_1, ((4_1, 3_2)(4_1, 9_2)(1_2, 7_2))_2, ((1_2, 9_2)(5_2, 9_2)(1_1, 2_2))_3, \\ & \quad \quad \quad ((1_2, 2_2)(2_2, 5_2)(1_1, 3_2))_4), \\ &(((3_1, 10_2)(8_1, 10_2)(2_1, 5_1))_1, ((2_1, 3_1)(3_1, 6_1)(9_2, 10_2))_2, ((6_1, 9_2)(5_1, 6_1)(8_1, 2_2))_3, \\ & \quad \quad \quad ((2_2, 9_2)(5_1, 2_2)(2_1, 8_1))_4), \\ &(((0_1, 1_1)(0_1, 2_1)(5_1, 9_2))_1, ((1_1, 5_1)(1_1, 3_1)(0_1, 6_1))_2, ((3_1, 6_1)(3_1, 9_2)(2_1, 0_2))_3, \\ & \quad \quad \quad ((6_1, 0_2)(0_2, 9_2)(2_1, 5_1))_4), \\ &(((0_1, 4_1)(0_1, 5_1)(0_2, 8_2))_1, ((4_1, 0_2)(4_1, 2_2)(0_1, 1_2))_2, ((1_2, 2_2)(2_2, 8_2)(1_1, 5_1))_3, \\ & \quad \quad \quad ((1_1, 1_2)(1_1, 8_2)(5_1, 0_2))_4), \\ &(((0_2, 1_2)(10_1, 0_2)(4_1, 9_2))_1, ((1_2, 9_2)(1_2, 8_2)(9_1, 0_2))_2, ((9_1, 8_2)(4_1, 8_2)(10_1, 7_2))_3, \\ & \quad \quad \quad ((9_1, 7_2)(4_1, 7_2)(10_1, 9_2))_4), \\ &(((0_2, 5_2)(0_1, 0_2)(2_1, 1_2))_1, ((2_1, 5_2)(3_2, 5_2)(3_1, 0_2))_2, ((3_1, 3_2)(1_2, 3_2)(0_1, 7_2))_3, \\ & \quad \quad \quad ((3_1, 7_2)(1_2, 7_2)(0_1, 2_1))_4). \end{aligned}$$

$4K_{6,6}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_6, j = 1, 2\}$. Cycle the following mod 6:

$$\begin{aligned} &(((0_1, 0_2)(0_1, 1_2)(1_1, 5_2))_1, ((1_1, 0_2)(2_1, 0_2)(0_1, 2_2))_2, ((2_1, 2_2)(2_1, 5_2)(3_1, 1_2))_3, \\ & \quad \quad \quad ((3_1, 2_2)(3_1, 5_2)(1_1, 1_2))_4), \\ &(((0_1, 2_2)(0_1, 5_2)(1_1, 4_2))_1, ((1_1, 2_2)(5_1, 2_2)(0_1, 0_2))_2, ((5_1, 0_2)(5_1, 4_2)(3_1, 5_2))_3, \\ & \quad \quad \quad ((3_1, 0_2)(3_1, 4_2)(1_1, 5_2))_4). \end{aligned}$$

$4K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$$\begin{aligned} &(((0_1, 0_3)(0_1, 1_3)(1_1, 1_4))_1, ((1_1, 0_3)(2_1, 0_3)(0_1, 0_4))_2, ((2_1, 0_4)(2_1, 1_4)(0_2, 1_3))_3, \\ & \quad \quad \quad ((0_2, 0_4)(0_2, 1_4)(1_1, 1_3))_4), \\ &(((0_1, 2_3)(0_1, 1_4)(2_1, 0_5))_1, ((2_1, 2_3)(0_2, 2_3)(0_1, 2_4))_2, ((0_2, 2_4)(0_2, 0_5)(1_1, 1_4))_3, \\ & \quad \quad \quad ((1_1, 2_4)(1_1, 0_5)(2_1, 1_4))_4), \\ &(((0_1, 2_4)(0_1, 0_5)(1_2, 1_4))_1, ((1_2, 2_4)(0_2, 2_4)(0_1, 2_5))_2, ((0_2, 2_5)(0_2, 1_4)(1_1, 0_5))_3, \\ & \quad \quad \quad ((1_1, 2_5)(1_1, 1_4)(1_2, 0_5))_4), \\ &(((0_2, 2_4)(0_2, 1_3)(2_2, 1_5))_1, ((2_2, 2_4)(1_1, 2_4)(0_2, 2_5))_2, ((1_1, 2_5)(1_1, 1_5)(1_2, 1_3))_3, \\ & \quad \quad \quad ((1_2, 2_5)(1_2, 1_5)(2_2, 1_3))_4), \\ &(((0_2, 0_5)(0_2, 1_4)(2_2, 2_3))_1, ((2_2, 0_5)(0_1, 0_5)(0_2, 1_3))_2, ((0_1, 1_3)(0_1, 2_3)(1_2, 1_4))_3, \\ & \quad \quad \quad ((1_2, 1_3)(1_2, 2_3)(2_2, 1_4))_4), \\ &(((0_2, 1_5)(0_2, 2_3)(0_1, 2_5))_1, ((0_1, 1_5)(1_2, 1_5)(0_2, 0_3))_2, ((1_2, 0_3)(1_2, 2_5)(2_1, 2_3))_3, \\ & \quad \quad \quad ((2_1, 0_3)(2_1, 2_5)(0_1, 2_3))_4). \end{aligned}$$

G_5 designs

$4K_{10}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2\}$. Cycle the following mod 5:

$$\begin{aligned} &(((0_1, 1_1)(0_1, 2_1)(0_1, 0_2))_1, ((1_1, 3_1)(2_1, 3_1)(3_1, 3_2))_2, ((1_1, 4_1)(4_1, 0_2)(4_1, 3_2))_3, \\ & \quad \quad \quad ((2_1, 1_2)(0_2, 1_2)(1_2, 3_2))_4), \\ &(((0_1, 1_2)(0_1, 2_2)(0_1, 3_2))_1, ((0_2, 1_2)(0_2, 2_2)(2_1, 0_2))_2, ((3_1, 1_2)(3_1, 3_2)(2_1, 3_1))_3, \\ & \quad \quad \quad ((1_1, 2_2)(1_1, 3_2)(1_1, 2_1))_4), \\ &(((0_2, 1_2)(0_2, 2_2)(1_1, 0_2))_1, ((0_1, 1_2)(0_1, 2_2)(0_1, 4_2))_2, ((1_2, 3_2)(1_1, 3_2)(3_2, 4_2))_3, \\ & \quad \quad \quad ((4_1, 2_2)(1_1, 4_1)(4_1, 4_2))_4). \end{aligned}$$

$4K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$$\begin{aligned} &(((0, 1)(0, 2)(0, 4))_1, ((1, 3)(2, 3)(3, 7))_2, ((1, 5)(4, 5)(5, 7))_3, ((2, 6)(4, 6)(6, 7))_4), \\ &(((0, 3)(0, 5)(0, 6))_1, ((3, 8)(5, 8)(1, 8))_2, ((3, 9)(6, 9)(1, 9))_3, ((5, 11)(6, 11)(1, 11))_4). \end{aligned}$$

$4K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$$\begin{aligned} &(((0_1, 1_1)(0_1, 2_1)(0_1, 0_2))_1, ((1_1, 3_1)(2_1, 3_1)(3_1, \infty))_2, ((1_1, 4_1)(4_1, 0_2)(4_1, \infty))_3, \\ & \quad \quad \quad ((2_1, 2_2)(0_2, 2_2)(2_2, \infty))_4), \\ &(((0_1, \infty)(0_1, 1_2)(0_1, 2_2))_1, ((0_2, \infty)(0_2, 1_2)(4_1, 0_2))_2, ((3_2, \infty)(2_2, 3_2)(4_1, 3_2))_3, \\ & \quad \quad \quad ((3_1, 1_2)(3_1, 2_2)(3_1, 4_1))_4), \\ &(((0_2, 1_2)(0_2, \infty)(1_1, 0_2))_1, ((1_2, 0_3)(0_3, \infty)(0_3, 1_3))_2, ((1_2, 3_2)(1_1, 3_2)(3_2, 1_3))_3, \\ & \quad \quad \quad ((3_1, \infty)(1_1, 3_1)(3_1, 1_3))_4), \\ &(((0_3, 1_3)(0_1, 0_3)(0_3, \infty))_1, ((0_2, 1_3)(0_1, 0_2)(2_1, 0_2))_2, ((1_3, 2_3)(2_3, \infty)(2_1, 2_3))_3, \\ & \quad \quad \quad ((0_1, 4_3)(4_3, \infty)(2_1, 4_3))_4), \\ &(((0_1, 1_3)(0_1, 2_3)(0_1, 3_3))_1, ((1_1, 1_3)(1_1, 2_3)(1_1, 0_3))_2, ((1_2, 1_3)(1_2, 3_3)(1_2, 0_3))_3, \\ & \quad \quad \quad ((2_2, 2_3)(2_2, 3_3)(2_2, 0_3))_4), \\ &(((0_2, 0_3)(0_2, 3_3)(0_2, 4_3))_1, ((3_2, 0_3)(3_2, 3_3)(1_1, 3_2))_2, ((2_1, 0_3)(2_1, 4_3)(1_1, 2_1))_3, \\ & \quad \quad \quad ((2_3, 3_3)(2_3, 4_3)(1_1, 2_3))_4), \\ &(((0_2, 2_3)(2_1, 0_2)(0_2, 3_2))_1, ((0_3, 2_3)(2_1, 0_3)(3_1, 0_3))_2, ((2_3, 4_3)(3_2, 4_3)(3_1, 4_3))_3, \\ & \quad \quad \quad ((2_1, 4_2)(3_2, 4_2)(3_1, 4_2))_4), \\ &(((0_3, 3_3)(1_1, 0_3)(4_2, 0_3))_1, ((0_2, 3_3)(1_1, 0_2)(0_2, 2_2))_2, ((4_1, 3_3)(4_1, 4_2)(4_1, 2_2))_3, \\ & \quad \quad \quad ((1_1, 1_3)(4_2, 1_3)(2_2, 1_3))_4). \end{aligned}$$

$4K_{19}$ Let the vertex set be \mathbb{Z}_{19} . Cycle the following mod 19:

$$\begin{aligned} &(((0, 1)(0, 2)(0, 4))_1, ((1, 3)(2, 3)(3, 7))_2, ((1, 5)(4, 5)(5, 7))_3, ((2, 6)(4, 6)(6, 7))_4), \\ &(((0, 3)(0, 5)(0, 6))_1, ((3, 8)(5, 8)(1, 8))_2, ((3, 9)(6, 9)(1, 9))_3, ((5, 11)(6, 11)(1, 11))_4), \\ &(((0, 9)(0, 7)(0, 11))_1, ((9, 15)(7, 15)(6, 15))_2, ((9, 16)(11, 16)(6, 16))_3, ((7, 14)(11, 14)(6, 14))_4). \end{aligned}$$

$4K_{22}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_{11}, j = 1, 2\}$. Cycle the following mod 11:

$$\begin{aligned} &(((4_1, 5_1)(5_1, 0_2)(5_1, 9_1))_1, ((4_1, 1_2)(0_2, 1_2)(8_1, 1_2))_2, ((4_1, 6_2)(9_1, 6_2)(8_1, 6_2))_3, \\ & \quad \quad \quad ((0_1, 0_2)(0_1, 9_1)(0_1, 8_1))_4), \\ &(((5_1, 6_2)(6_2, 10_2)(6_1, 6_2))_1, ((5_1, 5_2)(5_2, 10_2)(9_1, 5_2))_2, ((5_1, 1_2)(6_1, 1_2)(9_1, 1_2))_3, \\ & \quad \quad \quad ((8_2, 10_2)(6_1, 8_2)(9_1, 8_2))_4), \\ &(((6_1, 5_2)(4_1, 6_1)(6_1, 4_2))_1, ((5_2, 9_2)(4_1, 9_2)(3_1, 9_2))_2, ((5_1, 5_2)(5_1, 4_2)(3_1, 5_1))_3, \\ & \quad \quad \quad ((4_1, 10_1)(10_1, 4_2)(3_1, 10_1))_4), \\ &(((0_1, 3_1)(0_1, 5_1)(0_1, 2_2))_1, ((1_1, 3_1)(1_1, 5_1)(1_1, 4_1))_2, ((3_1, 8_1)(8_1, 2_2)(4_1, 8_1))_3, \\ & \quad \quad \quad ((5_1, 1_2)(1_2, 2_2)(4_1, 1_2))_4), \\ &(((0_1, 3_2)(0_1, 4_2)(0_1, 7_2))_1, ((1_1, 3_2)(1_1, 4_2)(1_1, 7_1))_2, ((0_2, 3_2)(0_2, 7_2)(7_1, 0_2))_3, \\ & \quad \quad \quad ((6_1, 4_2)(6_1, 7_2)(6_1, 7_1))_4), \\ &(((0_2, 3_2)(0_2, 2_2)(3_1, 0_2))_1, ((4_1, 3_2)(4_1, 2_2)(4_1, 5_1))_2, ((2_1, 3_2)(2_1, 3_1)(2_1, 5_1))_3, \\ & \quad \quad \quad ((2_2, 9_2)(3_1, 9_2)(5_1, 9_2))_4), \\ &(((0_2, 5_2)(6_1, 0_2)(0_2, 1_2))_1, ((5_2, 7_2)(6_1, 7_2)(4_2, 7_2))_2, ((5_2, 6_2)(1_2, 6_2)(4_2, 6_2))_3, \\ & \quad \quad \quad ((6_1, 9_2)(1_2, 9_2)(4_2, 9_2))_4). \end{aligned}$$

$4K_{6,6}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_6, j = 1, 2\}$. Cycle the following mod 6:

$$\begin{aligned} &(((0_1, 0_2)(0_1, 1_2)(0_1, 2_2))_1, ((1_1, 0_2)(1_1, 1_2)(1_1, 3_2))_2, ((2_1, 0_2)(2_1, 2_2)(2_1, 3_2))_3, \\ & \quad \quad \quad ((3_1, 1_2)(3_1, 2_2)(3_1, 3_2))_4), \\ &(((0_1, 3_2)(0_1, 5_2)(0_1, 4_2))_1, ((2_1, 3_2)(2_1, 5_2)(2_1, 0_2))_2, ((1_1, 3_2)(1_1, 4_2)(1_1, 0_2))_3, \\ & \quad \quad \quad ((3_1, 5_2)(3_1, 4_2)(3_1, 0_2))_4). \end{aligned}$$

$4K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$$\begin{aligned} &(((0_1, 0_3)(0_1, 1_3)(0_1, 2_3))_1, ((1_1, 0_3)(1_1, 1_3)(1_1, 0_4))_2, ((2_1, 0_3)(2_1, 2_3)(2_1, 0_4))_3, \\ & \quad \quad \quad ((0_2, 1_3)(0_2, 2_3)(0_2, 0_4))_4), \\ &(((0_1, 0_4)(0_1, 1_4)(0_1, 2_4))_1, ((0_2, 0_4)(0_2, 1_4)(0_2, 0_3))_2, ((1_2, 0_4)(1_2, 2_4)(1_2, 0_3))_3, \\ & \quad \quad \quad ((1_1, 1_4)(1_1, 2_4)(1_1, 0_3))_4), \\ &(((0_1, 0_5)(0_1, 1_5)(0_1, 2_5))_1, ((0_2, 0_5)(0_2, 1_5)(0_2, 1_3))_2, ((2_1, 0_5)(2_1, 2_5)(2_1, 1_3))_3, \\ & \quad \quad \quad ((1_2, 1_5)(1_2, 2_5)(1_2, 1_3))_4), \\ &(((0_2, 2_3)(0_2, 2_5)(0_2, 0_3))_1, ((1_1, 2_3)(1_1, 2_5)(1_1, 0_5))_2, ((2_2, 2_3)(2_2, 0_3)(2_2, 0_5))_3, \\ & \quad \quad \quad ((0_1, 2_5)(0_1, 0_3)(0_1, 0_5))_4), \\ &(((0_2, 1_4)(0_2, 1_3)(0_2, 2_4))_1, ((2_2, 1_4)(2_2, 1_3)(2_2, 1_5))_2, ((2_1, 1_4)(2_1, 2_4)(2_1, 1_5))_3, \\ & \quad \quad \quad ((0_1, 1_3)(0_1, 2_4)(0_1, 1_5))_4), \\ &(((0_2, 0_5)(0_2, 0_4)(0_2, 1_5))_1, ((0_1, 0_5)(0_1, 0_4)(0_1, 1_4))_2, ((1_2, 0_5)(1_2, 1_5)(1_2, 1_4))_3, \\ & \quad \quad \quad ((2_2, 0_4)(2_2, 1_5)(2_2, 1_4))_4). \end{aligned}$$

G_6 designs

$4K_{10}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2\}$. Cycle the following mod 5:

$$\begin{aligned}
&(((0_1, 1_1)(1_1, 0_2)(2_1, 0_2))_1, ((1_1, 3_1)(3_1, 2_2)(0_2, 2_2))_2, ((0_1, 4_1)(0_1, 2_1)(2_1, 1_2))_3, \\
&\quad ((3_1, 4_1)(4_1, 1_2)(1_2, 2_2))_4), \\
&(((0_1, 2_1)(2_1, 3_2)(1_2, 3_2))_1, ((1_1, 2_1)(1_1, 2_2)(2_2, 3_2))_2, ((0_1, 0_2)(0_1, 1_2)(4_1, 1_2))_3, \\
&\quad ((1_1, 0_2)(4_1, 0_2)(4_1, 2_2))_4), \\
&(((0_2, 1_2)(1_1, 1_2)(1_1, 3_2))_1, ((4_1, 1_2)(4_1, 4_2)(1_1, 4_2))_2, ((2_1, 0_2)(0_2, 3_2)(2_2, 3_2))_3, \\
&\quad ((2_1, 4_1)(2_1, 2_2)(2_2, 4_2))_4).
\end{aligned}$$

$4K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$$\begin{aligned}
&(((0, 1)(1, 4)(2, 4))_1, ((1, 5)(5, 10)(4, 10))_2, ((0, 3)(0, 2)(2, 6))_3, ((3, 5)(3, 6)(6, 10))_4), \\
&(((0, 4)(4, 11)(6, 11))_1, ((1, 4)(1, 12)(11, 12))_2, ((0, 8)(0, 6)(6, 7))_3, ((1, 8)(7, 8)(7, 12))_4).
\end{aligned}$$

$4K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$$\begin{aligned}
&(((0_1, 1_1)(1_1, 0_2)(2_1, 0_2))_1, ((1_1, 0_3)(0_3, \infty)(0_2, \infty))_2, ((0_1, 4_1)(0_1, 2_1)(2_1, 1_3))_3, \\
&\quad ((4_1, 0_3)(4_1, 1_3)(1_3, \infty))_4), \\
&(((1_1, \infty)(1_1, 2_2)(2_1, 2_2))_1, ((1_1, 3_1)(3_1, 3_2)(2_2, 3_2))_2, ((0_2, \infty)(2_1, \infty)(2_1, 1_2))_3, \\
&\quad ((3_1, 0_2)(0_2, 1_2)(1_2, 3_2))_4), \\
&(((0_2, \infty)(0_3, \infty)(0_1, 0_3))_1, ((1_1, \infty)(1_1, 1_3)(0_3, 1_3))_2, ((2_1, 0_2)(0_1, 0_2)(0_1, 2_2))_3, \\
&\quad ((1_1, 2_1)(2_1, 2_2)(2_2, 1_3))_4), \\
&(((0_3, 1_3)(2_1, 1_3)(0_1, 2_1))_1, ((3_1, 1_3)(3_1, 4_3)(2_1, 4_3))_2, ((0_3, \infty)(0_1, 0_3)(0_1, 1_2))_3, \\
&\quad ((3_1, \infty)(1_2, \infty)(1_2, 4_3))_4), \\
&(((0_2, 1_2)(1_2, 1_3)(2_2, 1_3))_1, ((1_2, 3_2)(3_2, 4_3)(1_3, 4_3))_2, ((0_2, 0_3)(0_2, 2_2)(2_2, 3_3))_3, \\
&\quad ((3_2, 0_3)(0_3, 3_3)(3_3, 4_3))_4), \\
&(((0_2, 3_2)(1_1, 3_2)(1_1, 4_3))_1, ((3_2, 1_3)(4_2, 1_3)(1_1, 4_2))_2, ((0_2, 1_2)(0_2, 4_3)(3_1, 4_3))_3, \\
&\quad ((1_2, 1_3)(3_1, 1_2)(3_1, 4_2))_4), \\
&(((0_2, 1_3)(0_1, 1_3)(0_1, 2_3))_1, ((2_2, 1_3)(1_1, 2_2)(0_1, 1_1))_2, ((0_2, 3_3)(0_2, 2_3)(4_1, 2_3))_3, \\
&\quad ((2_2, 3_3)(4_1, 3_3)(1_1, 4_1))_4), \\
&(((0_3, 2_3)(4_2, 2_3)(4_2, 1_3))_1, ((2_2, 2_3)(0_1, 2_2)(0_1, 4_2))_2, ((3_1, 0_3)(0_3, 1_3)(1_3, 3_3))_3, \\
&\quad ((3_1, 2_2)(3_1, 3_3)(0_1, 3_3))_4).
\end{aligned}$$

$4K_{19}$ Let the vertex set be \mathbb{Z}_{19} . Cycle the following mod 19:

$$\begin{aligned}
&(((0, 1)(1, 4)(2, 4))_1, ((1, 5)(5, 7)(4, 7))_2, ((0, 3)(0, 2)(2, 6))_3, ((3, 5)(3, 6)(6, 7))_4), \\
&(((0, 4)(4, 9)(1, 9))_1, ((4, 10)(2, 10)(2, 9))_2, ((0, 5)(0, 1)(1, 12))_3, ((5, 10)(5, 12)(2, 12))_4), \\
&(((0, 7)(1, 7)(1, 10))_1, ((7, 12)(2, 12)(1, 2))_2, ((0, 6)(0, 10)(10, 17))_3, ((6, 12)(6, 17)(2, 17))_4).
\end{aligned}$$

$4K_{22}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_{11}, j = 1, 2\}$. Cycle the following mod 11:

$$\begin{aligned}
&(((7_1, 1_2)(0_2, 1_2)(0_2, 8_2))_1, ((8_1, 1_2)(8_1, 5_2)(0_2, 5_2))_2, ((3_1, 7_1)(7_1, 8_2)(8_2, 10_2))_3, \\
&\quad ((3_1, 8_1)(3_1, 10_2)(5_2, 10_2))_4), \\
&(((2_1, 4_2)(2_1, 8_1)(8_1, 3_2))_1, ((2_1, 2_2)(2_2, 10_2)(8_1, 10_2))_2, ((5_1, 4_2)(3_2, 4_2)(7_1, 3_2))_3, \\
&\quad ((5_1, 2_2)(5_1, 7_1)(7_1, 10_2))_4), \\
&(((8_1, 5_2)(7_1, 8_1)(7_1, 8_2))_1, ((8_1, 9_1)(4_1, 9_1)(4_1, 7_1))_2, ((1_1, 5_2)(5_2, 8_2)(3_2, 8_2))_3, \\
&\quad ((1_1, 9_1)(1_1, 3_2)(4_1, 3_2))_4), \\
&(((0_1, 2_1)(2_1, 5_1)(1_1, 5_1))_1, ((2_1, 4_1)(4_1, 0_2)(5_1, 0_2))_2, ((0_1, 3_1)(0_1, 1_1)(1_1, 7_1))_3, \\
&\quad ((3_1, 4_1)(3_1, 7_1)(7_1, 0_2))_4), \\
&(((0_1, 0_2)(0_2, 5_2)(2_1, 5_2))_1, ((8_1, 0_2)(8_1, 6_2)(5_2, 6_2))_2, ((0_1, 2_2)(0_1, 2_1)(2_1, 10_2))_3, \\
&\quad ((8_1, 2_2)(2_2, 10_2)(6_2, 10_2))_4), \\
&(((0_1, 4_2)(5_1, 4_2)(5_1, 3_2))_1, ((0_2, 4_2)(1_1, 0_2)(1_1, 5_1))_2, ((0_1, 9_2)(0_1, 3_2)(3_2, 10_2))_3, \\
&\quad ((0_2, 9_2)(9_2, 10_2)(1_1, 10_2))_4), \\
&(((0_1, 7_2)(7_2, 9_2)(5_2, 9_2))_1, ((6_1, 7_2)(6_1, 0_2)(0_2, 9_2))_2, ((0_1, 6_2)(0_1, 5_2)(5_1, 5_2))_3, \\
&\quad ((6_1, 6_2)(5_1, 6_2)(5_1, 0_2))_4).
\end{aligned}$$

$4K_{6,6}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_6, j = 1, 2\}$. Cycle the following mod 6:

$$\begin{aligned} &(((0_1, 0_2)(2_1, 0_2)(2_1, 1_2))_1, ((3_1, 0_2)(3_1, 4_2)(2_1, 4_2))_2, ((0_1, 2_2)(0_1, 1_2)(1_1, 1_2))_3, \\ & \quad \quad \quad ((3_1, 2_2)(1_1, 2_2)(1_1, 4_2))_4), \\ &(((0_1, 2_2)(1_1, 2_2)(1_1, 4_2))_1, ((3_1, 2_2)(3_1, 1_2)(1_1, 1_2))_2, ((0_1, 3_2)(0_1, 4_2)(5_1, 4_2))_3, \\ & \quad \quad \quad ((3_1, 3_2)(5_1, 3_2)(5_1, 1_2))_4). \end{aligned}$$

$4K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$$\begin{aligned} &(((0_1, 0_3)(2_1, 0_3)(2_1, 1_3))_1, ((1_1, 0_3)(1_1, 0_4)(2_1, 0_4))_2, ((0_1, 2_3)(0_1, 1_3)(0_2, 1_3))_3, \\ & \quad \quad \quad ((1_1, 2_3)(0_2, 2_3)(0_2, 0_4))_4), \\ &(((0_1, 0_4)(0_2, 0_4)(0_2, 0_3))_1, ((2_2, 0_4)(2_2, 1_3)(0_2, 1_3))_2, ((0_1, 0_5)(0_1, 0_3)(1_2, 0_3))_3, \\ & \quad \quad \quad ((2_2, 0_5)(1_2, 0_5)(1_2, 1_3))_4), \\ &(((0_1, 1_4)(2_2, 1_4)(2_2, 0_4))_1, ((1_1, 1_4)(1_1, 1_5)(2_2, 1_5))_2, ((0_1, 2_4)(0_1, 0_4)(1_2, 0_4))_3, \\ & \quad \quad \quad ((1_1, 2_4)(1_2, 2_4)(1_2, 1_5))_4), \\ &(((0_2, 1_3)(2_2, 1_3)(2_2, 0_5))_1, ((1_2, 1_3)(1_2, 1_4)(2_2, 1_4))_2, ((0_2, 0_4)(0_2, 0_5)(1_1, 0_5))_3, \\ & \quad \quad \quad ((1_2, 0_4)(1_1, 0_4)(1_1, 1_4))_4), \\ &(((0_2, 0_5)(2_1, 0_5)(2_1, 1_4))_1, ((1_1, 0_5)(1_1, 2_3)(2_1, 2_3))_2, ((0_2, 2_5)(0_2, 1_4)(0_1, 1_4))_3, \\ & \quad \quad \quad ((1_1, 2_5)(0_1, 2_5)(0_1, 2_3))_4), \\ &(((0_2, 2_5)(2_1, 2_5)(2_1, 1_5))_1, ((2_2, 2_5)(2_2, 0_5)(2_1, 0_5))_2, ((0_2, 0_3)(0_2, 1_5)(0_1, 1_5))_3, \\ & \quad \quad \quad ((2_2, 0_3)(0_1, 0_3)(0_1, 0_5))_4). \end{aligned}$$

G_7 designs

$3K_8$ Let the vertex set be $\{0, 1, \dots, 7\}$. The design is given by:

$$\begin{aligned} &(((0, 1)(2, 3)(4, 6)(5, 7))_1, ((0, 2)(1, 3)(4, 5)(6, 7))_2, ((0, 4)(1, 5)(2, 6)(3, 7))_3), \\ &(((0, 2)(1, 3)(4, 5)(6, 7))_1, ((0, 1)(2, 3)(5, 7)(4, 6))_2, ((0, 5)(2, 7)(1, 4)(3, 6))_3), \\ &(((0, 3)(4, 7)(1, 2)(5, 6))_1, ((0, 4)(3, 7)(1, 5)(2, 6))_2, ((0, 1)(3, 5)(2, 4)(6, 7))_3), \\ &(((0, 4)(1, 5)(2, 6)(3, 7))_1, ((0, 5)(1, 4)(3, 6)(2, 7))_2, ((0, 6)(3, 4)(2, 5)(1, 7))_3), \\ &(((0, 5)(3, 6)(2, 7)(1, 4))_1, ((0, 3)(5, 6)(4, 7)(1, 2))_2, ((0, 7)(4, 5)(2, 3)(1, 6))_3), \\ &(((0, 6)(1, 7)(2, 5)(3, 4))_1, ((0, 7)(1, 6)(2, 4)(3, 5))_2, ((0, 2)(4, 6)(5, 7)(1, 3))_3), \\ &(((0, 7)(1, 6)(3, 5)(2, 4))_1, ((0, 6)(1, 7)(3, 4)(2, 5))_2, ((0, 3)(4, 7)(5, 6)(1, 2))_3). \end{aligned}$$

$3K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$$(((0, 1)(2, 5)(6, 8)(3, 7))_1, ((0, 2)(1, 5)(7, 8)(3, 6))_2, ((0, 8)(1, 7)(2, 6)(3, 5))_3).$$

$3K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$$\begin{aligned} &(((0, 4)(1, 5)(3, 6)(2, 7))_1, ((0, 5)(1, 4)(2, 6)(3, 7))_2, ((0, 6)(2, 4)(3, 5)(1, 7))_3), \\ &(((0, 5)(1, 4)(3, 7)(2, 6))_1, ((0, 4)(1, 5)(2, 7)(3, 6))_2, ((0, 7)(2, 5)(3, 4)(1, 6))_3), \\ &(((0, 6)(1, 7)(3, 4)(2, 5))_1, ((0, 7)(1, 6)(2, 4)(3, 5))_2, ((0, 4)(2, 6)(3, 7)(1, 5))_3), \\ &(((0, 7)(1, 6)(3, 5)(2, 4))_1, ((0, 6)(1, 7)(2, 5)(3, 4))_2, ((0, 5)(2, 7)(3, 6)(1, 4))_3). \end{aligned}$$

G_8 designs

$3K_8$ Let the vertex set be $\{0, 1, \dots, 7\}$. The design is given by:

$((0, 1)(0, 2)(3, 7)(4, 5))_1, ((1, 3)(2, 3)(4, 6)(5, 7))_2, ((2, 6)(6, 7)(0, 4)(1, 5))_3,$
 $((1, 2)(1, 3)(5, 7)(4, 6))_1, ((2, 5)(3, 5)(0, 6)(4, 7))_2, ((0, 3)(0, 7)(1, 6)(2, 4))_3,$
 $((0, 3)(2, 3)(1, 4)(5, 6))_1, ((0, 4)(2, 4)(6, 7)(1, 5))_2, ((2, 7)(1, 7)(3, 6)(0, 5))_3,$
 $((3, 4)(2, 4)(0, 7)(1, 6))_1, ((0, 3)(0, 2)(5, 6)(1, 7))_2, ((2, 5)(5, 7)(4, 6)(1, 3))_3,$
 $((3, 5)(2, 5)(0, 6)(4, 7))_1, ((3, 6)(2, 6)(1, 4)(0, 7))_2, ((1, 2)(0, 1)(4, 5)(3, 7))_3,$
 $((2, 6)(3, 6)(1, 7)(0, 5))_1, ((2, 7)(3, 7)(4, 5)(0, 1))_2, ((3, 4)(1, 4)(5, 6)(0, 2))_3,$
 $((6, 7)(2, 7)(1, 5)(0, 4))_1, ((1, 6)(1, 2)(3, 4)(0, 5))_2, ((2, 3)(3, 5)(4, 7)(0, 6))_3.$

$3K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$((0, 1)(0, 2)(4, 8)(3, 6))_1, ((1, 4)(2, 4)(6, 7)(3, 8))_2, ((2, 7)(7, 8)(0, 6)(1, 3))_3.$

$3K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$((0, 4)(0, 5)(1, 7)(2, 6))_1, ((1, 4)(1, 5)(3, 6)(2, 7))_2, ((3, 5)(3, 7)(0, 6)(2, 4))_3,$
 $((1, 4)(1, 5)(0, 7)(3, 6))_1, ((0, 4)(0, 5)(2, 6)(3, 7))_2, ((2, 5)(2, 7)(1, 6)(3, 4))_3,$
 $((2, 4)(2, 5)(3, 7)(0, 6))_1, ((3, 4)(3, 5)(1, 6)(0, 7))_2, ((1, 5)(1, 7)(2, 6)(0, 4))_3,$
 $((3, 4)(3, 5)(2, 7)(1, 6))_1, ((2, 4)(2, 5)(0, 6)(1, 7))_2, ((0, 5)(0, 7)(3, 6)(1, 4))_3.$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$((0, 1)(0, 2)(3, 6)(4, 5))_1, ((1, 3)(2, 3)(4, 7)(5, 6))_2, ((2, 7)(6, 7)(0, 4)(1, 5))_3,$
 $((0, 2)(0, 3)(5, 9)(4, \infty))_1, ((2, 5)(3, 5)(6, \infty)(4, 9))_2, ((3, 6)(6, 9)(0, \infty)(2, 4))_3,$
 $((0, 4)(0, 6)(5, 10)(9, \infty))_1, ((4, 10)(6, 10)(7, \infty)(5, 9))_2, ((6, 7)(5, 7)(0, \infty)(4, 9))_3.$

$6K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(0, 2)(3, 6)(4, 5))_1, ((1, 3)(2, 3)(4, 7)(5, 6))_2, ((2, 7)(6, 7)(0, 4)(1, 5))_3,$
 $((0, 2)(0, 3)(5, 11)(1, 7))_1, ((2, 5)(3, 5)(1, 9)(7, 11))_2, ((3, 9)(9, 11)(0, 1)(2, 7))_3,$
 $((0, 4)(0, 5)(1, 10)(2, 7))_1, ((4, 10)(5, 10)(2, 11)(1, 7))_2, ((5, 11)(1, 11)(0, 2)(4, 7))_3.$

G_9 designs

$3K_8$ Let the vertex set be $\{0, 1, \dots, 7\}$. The design is given by:

$((0, 1)(0, 2)(0, 4)(3, 7))_1, ((1, 5)(4, 5)(5, 7)(2, 3))_2, ((2, 6)(4, 6)(6, 7)(1, 3))_3,$
 $((0, 3)(0, 5)(0, 6)(2, 7))_1, ((3, 4)(4, 6)(4, 7)(2, 5))_2, ((1, 5)(1, 6)(1, 7)(2, 3))_3,$
 $((1, 5)(1, 2)(1, 7)(3, 4))_1, ((5, 6)(6, 7)(3, 6)(2, 4))_2, ((0, 2)(0, 7)(0, 3)(4, 5))_3,$
 $((2, 4)(2, 3)(2, 6)(0, 7))_1, ((1, 4)(1, 6)(1, 7)(0, 3))_2, ((3, 5)(5, 6)(5, 7)(0, 4))_3,$
 $((4, 7)(4, 5)(1, 4)(3, 6))_1, ((2, 7)(1, 2)(2, 6)(3, 5))_2, ((0, 5)(0, 1)(0, 6)(3, 7))_3,$
 $((5, 6)(5, 7)(2, 5)(1, 3))_1, ((0, 6)(0, 2)(0, 1)(3, 7))_2, ((4, 7)(2, 4)(1, 4)(3, 6))_3,$
 $((4, 6)(1, 6)(6, 7)(3, 5))_1, ((0, 4)(0, 7)(0, 5)(1, 3))_2, ((1, 2)(2, 7)(2, 5)(3, 4))_3.$

$3K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$((0, 1)(0, 3)(0, 5)(2, 4))_1, ((1, 8)(5, 8)(4, 8)(2, 3))_2, ((3, 7)(5, 7)(4, 7)(1, 2))_3.$

$3K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$((0, 4)(0, 5)(0, 6)(1, 7))_1, ((2, 4)(2, 6)(2, 7)(1, 5))_2, ((3, 5)(3, 6)(3, 7)(1, 4))_3,$
 $((1, 4)(1, 5)(1, 6)(0, 7))_1, ((3, 4)(3, 6)(3, 7)(0, 5))_2, ((2, 5)(2, 6)(2, 7)(0, 4))_3,$
 $((2, 4)(2, 5)(2, 6)(3, 7))_1, ((0, 4)(0, 6)(0, 7)(3, 5))_2, ((1, 5)(1, 6)(1, 7)(3, 4))_3,$
 $((3, 4)(3, 5)(3, 6)(2, 7))_1, ((1, 4)(1, 6)(1, 7)(2, 5))_2, ((0, 5)(0, 6)(0, 7)(2, 4))_3.$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$((0, 1)(0, 2)(0, 4)(3, 7))_1, ((1, 5)(4, 5)(5, 7)(2, 3))_2, ((2, 6)(4, 6)(6, 7)(1, 3))_3,$
 $((0, \infty)(0, 2)(0, 3)(4, 10))_1, ((6, \infty)(3, 6)(6, 10)(2, 4))_2, ((2, 7)(3, 7)(7, 10)(4, \infty))_3,$
 $((0, \infty)(0, 3)(0, 5)(7, 8))_1, ((10, \infty)(5, 10)(7, 10)(3, 8))_2, ((2, 3)(2, 5)(2, 7)(8, \infty))_3.$

$6K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(0, 2)(0, 4)(3, 7))_1, ((1, 5)(4, 5)(5, 7)(2, 3))_2, ((2, 6)(4, 6)(6, 7)(1, 3))_3,$
 $((0, 1)(0, 2)(0, 3)(4, 10))_1, ((1, 6)(3, 6)(6, 10)(2, 4))_2, ((2, 7)(3, 7)(7, 10)(1, 4))_3,$
 $((0, 5)(0, 8)(0, 3)(4, 11))_1, ((5, 10)(3, 10)(4, 10)(8, 11))_2, ((8, 9)(3, 9)(4, 9)(5, 11))_3.$

G_{10} designs

$3K_8$ Let the vertex set be $\{0, 1, \dots, 7\}$. The design is given by:

$((0, 1)(0, 2)(5, 7)(6, 7))_1, ((1, 3)(1, 5)(2, 6)(4, 6))_2, ((0, 4)(4, 5)(2, 3)(3, 7))_3,$
 $((0, 5)(0, 4)(1, 2)(1, 3))_1, ((5, 7)(2, 5)(3, 4)(3, 6))_2, ((0, 6)(2, 6)(4, 7)(1, 7))_3,$
 $((1, 4)(1, 7)(2, 3)(3, 6))_1, ((0, 4)(2, 4)(6, 7)(5, 6))_2, ((1, 5)(2, 5)(0, 7)(0, 3))_3,$
 $((3, 7)(0, 3)(2, 6)(5, 6))_1, ((1, 7)(2, 7)(0, 5)(4, 5))_2, ((3, 4)(2, 4)(0, 1)(1, 6))_3,$
 $((3, 4)(4, 6)(2, 7)(0, 7))_1, ((3, 5)(2, 3)(0, 6)(0, 1))_2, ((1, 4)(1, 2)(5, 6)(5, 7))_3,$
 $((4, 7)(2, 4)(3, 5)(1, 5))_1, ((0, 7)(3, 7)(1, 2)(1, 6))_2, ((4, 6)(3, 6)(0, 2)(0, 5))_3,$
 $((4, 5)(2, 5)(1, 6)(0, 6))_1, ((4, 7)(1, 4)(0, 2)(0, 3))_2, ((3, 5)(1, 3)(2, 7)(6, 7))_3.$

$3K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$((0, 1)(0, 2)(3, 6)(3, 8))_1, ((1, 4)(1, 3)(2, 7)(7, 8))_2, ((2, 4)(4, 8)(0, 6)(6, 7))_3.$

$3K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$((0, 4)(0, 5)(2, 6)(2, 7))_1, ((1, 4)(2, 4)(3, 5)(3, 7))_2, ((1, 5)(1, 7)(0, 6)(3, 6))_3,$
 $((0, 7)(0, 6)(2, 5)(2, 4))_1, ((1, 7)(2, 7)(3, 6)(3, 4))_2, ((1, 6)(1, 4)(0, 5)(3, 5))_3,$
 $((3, 5)(3, 4)(1, 7)(1, 6))_1, ((2, 5)(1, 5)(0, 4)(0, 6))_2, ((2, 4)(2, 6)(3, 7)(0, 7))_3,$
 $((3, 6)(3, 7)(1, 4)(1, 5))_1, ((2, 6)(1, 6)(0, 7)(0, 5))_2, ((2, 7)(2, 5)(3, 4)(0, 4))_3.$

G_{11} designs

$3K_8$ Let the vertex set be $\{0, 1, \dots, 7\}$. The design is given by:

$((0, 1)(1, 3)(2, 3)(4, 5))_1, ((0, 2)(2, 6)(4, 6)(3, 7))_2, ((1, 5)(5, 7)(6, 7)(0, 4))_3,$
 $((0, 2)(2, 6)(4, 6)(5, 7))_1, ((0, 4)(1, 4)(1, 5)(3, 6))_2, ((2, 7)(3, 7)(1, 3)(0, 5))_3,$
 $((0, 3)(3, 5)(5, 6)(1, 2))_1, ((0, 6)(6, 7)(1, 7)(4, 5))_2, ((2, 3)(2, 4)(4, 7)(0, 1))_3,$
 $((0, 4)(1, 4)(1, 5)(3, 7))_1, ((0, 5)(2, 5)(2, 7)(1, 6))_2, ((3, 4)(3, 6)(2, 6)(0, 7))_3,$
 $((0, 5)(2, 5)(2, 7)(3, 6))_1, ((0, 7)(4, 7)(3, 4)(1, 2))_2, ((5, 6)(1, 6)(1, 4)(0, 3))_3,$
 $((0, 6)(6, 7)(1, 7)(2, 4))_1, ((0, 1)(1, 3)(2, 3)(5, 7))_2, ((4, 6)(4, 5)(3, 5)(0, 2))_3,$
 $((0, 7)(4, 7)(3, 4)(1, 6))_1, ((0, 3)(3, 5)(5, 6)(2, 4))_2, ((1, 7)(1, 2)(2, 5)(0, 6))_3.$

$3K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$$(((0, 1)(1, 7)(2, 7)(3, 5))_1, ((0, 2)(2, 8)(3, 8)(6, 7))_2, ((1, 5)(5, 6)(6, 8)(0, 3))_3).$$

$3K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$$\begin{aligned} &(((0, 4)(1, 4)(1, 5)(2, 6))_1, ((0, 5)(3, 5)(3, 6)(1, 7))_2, ((2, 4)(2, 7)(3, 7)(0, 6))_3), \\ &(((2, 7)(3, 7)(3, 6)(0, 5))_1, ((2, 6)(1, 6)(1, 5)(3, 4))_2, ((0, 7)(0, 4)(1, 4)(2, 5))_3), \\ &(((3, 5)(3, 4)(2, 4)(0, 6))_1, ((2, 5)(2, 7)(0, 7)(1, 4))_2, ((3, 6)(1, 6)(1, 7)(0, 5))_3), \\ &(((1, 6)(1, 7)(0, 7)(2, 5))_1, ((0, 6)(0, 4)(2, 4)(3, 7))_2, ((1, 5)(3, 5)(3, 4)(2, 6))_3). \end{aligned}$$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$$\begin{aligned} &(((0, 1), (1, 3), (2, 3), (4, 6))_1, ((0, 2), (2, 5), (4, 5), (3, 7))_2, ((1, 6), (6, 7), (5, 7), (0, 4))_3), \\ &(((1, \infty), (1, 5), (2, 5), (3, 6))_1, ((2, \infty), (2, 7), (3, 7), (4, 5))_2, ((1, 6), (4, 6), (4, 7), (3, \infty))_3), \\ &(((1, \infty), (1, 6), (2, 6), (4, 9))_1, ((2, \infty), (2, 7), (7, 9), (3, 6))_2, ((1, 4), (3, 4), (3, 7), (9, \infty))_3). \end{aligned}$$

$6K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$$\begin{aligned} &(((0, 1), (1, 3), (2, 3), (4, 6))_1, ((0, 2), (2, 5), (4, 5), (3, 7))_2, ((1, 6), (6, 7), (5, 7), (0, 4))_3), \\ &(((0, 3), (3, 6), (1, 6), (2, 7))_1, ((0, 1), (1, 5), (2, 5), (4, 6))_2, ((3, 7), (4, 7), (4, 5), (0, 2))_3), \\ &(((0, 6), (6, 12), (8, 12), (7, 11))_1, ((0, 8), (1, 8), (1, 7), (4, 12))_2, ((6, 11), (4, 11), (1, 4), (0, 7))_3). \end{aligned}$$

G_{12} designs

$3K_8$ Let the vertex set be $\{0, 1, \dots, 7\}$. The design is given by:

$$\begin{aligned} &(((0, 1)(0, 2)(0, 4)(1, 3))_1, ((5, 7)(1, 5)(4, 5)(3, 7))_2, ((2, 6)(4, 6)(6, 7)(2, 3))_3), \\ &(((0, 3)(0, 5)(0, 7)(3, 4))_1, ((2, 6)(3, 6)(6, 7)(2, 4))_2, ((1, 5)(1, 7)(1, 2)(4, 5))_3), \\ &(((1, 5)(1, 7)(1, 4)(2, 5))_1, ((0, 6)(5, 6)(4, 6)(0, 2))_2, ((3, 7)(3, 4)(0, 3)(2, 7))_3), \\ &(((2, 7)(2, 6)(2, 4)(5, 7))_1, ((0, 3)(0, 7)(0, 4)(3, 5))_2, ((1, 6)(1, 4)(1, 3)(5, 6))_3), \\ &(((3, 6)(2, 3)(3, 7)(5, 6))_1, ((0, 1)(1, 6)(1, 7)(0, 5))_2, ((2, 4)(4, 7)(0, 4)(2, 5))_3), \\ &(((4, 5)(4, 6)(4, 7)(3, 5))_1, ((1, 2)(2, 5)(2, 7)(1, 3))_2, ((0, 6)(0, 7)(0, 1)(3, 6))_3), \\ &(((1, 6)(0, 6)(6, 7)(1, 2))_1, ((3, 4)(1, 4)(4, 7)(2, 3))_2, ((0, 5)(5, 7)(3, 5)(0, 2))_3). \end{aligned}$$

$3K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$$(((0, 1)(0, 2)(0, 5)(1, 4))_1, ((3, 7)(1, 7)(5, 7)(3, 4))_2, ((2, 6)(5, 6)(3, 6)(2, 4))_3).$$

$3K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$$\begin{aligned} &(((0, 4)(0, 5)(0, 6)(1, 4))_1, ((2, 7)(2, 4)(2, 6)(1, 7))_2, ((3, 5)(3, 6)(3, 7)(1, 5))_3), \\ &(((1, 7)(1, 6)(1, 5)(0, 7))_1, ((3, 4)(3, 7)(3, 5)(0, 4))_2, ((2, 6)(2, 5)(2, 4)(0, 6))_3), \\ &(((2, 5)(2, 4)(2, 7)(3, 5))_1, ((0, 6)(0, 5)(0, 7)(3, 6))_2, ((1, 4)(1, 7)(1, 6)(3, 4))_3), \\ &(((3, 6)(3, 7)(3, 4)(2, 6))_1, ((1, 5)(1, 6)(1, 4)(2, 5))_2, ((0, 7)(0, 4)(0, 5)(2, 7))_3). \end{aligned}$$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$$\begin{aligned} &(((0, 1), (0, 2), (0, 4), (1, 3))_1, ((5, 7), (1, 5), (4, 5), (3, 7))_2, ((2, 6), (4, 6), (6, 7), (2, 3))_3), \\ &(((0, 1), (0, 3), (0, 4), (1, \infty))_1, ((7, 9), (1, 7), (4, 7), (9, \infty))_2, ((3, 6), (4, 6), (6, 9), (3, \infty))_3), \\ &(((0, 3), (0, 6), (0, 5), (3, \infty))_1, ((7, 8), (3, 8), (5, 8), (7, \infty))_2, ((1, 6), (1, 5), (1, 7), (6, \infty))_3). \end{aligned}$$

$6K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1), (0, 2), (0, 4), (1, 3))_1, ((5, 7), (1, 5), (4, 5), (3, 7))_2, ((2, 6), (4, 6), (6, 7), (2, 3))_3,$
 $((0, 1), (0, 3), (0, 6), (1, 5))_1, ((4, 12), (1, 4), (4, 6), (5, 12))_2, ((3, 9), (6, 9), (9, 12), (3, 5))_3,$
 $((0, 3), (0, 5), (0, 8), (3, 10))_1, ((2, 7), (2, 3), (2, 8), (7, 10))_2, ((5, 12), (8, 12), (7, 12), (5, 10))_3.$

G_{13} designs

$2K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(1, 5)(2, 5)(0, 2)(0, 6)(1, 9))_1, ((3, 7)(3, 9)(6, 9)(6, 7)(2, 7)(3, 5))_2.$

$2K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$((0_1, 1_1)(1_1, 0_2)(2_1, 0_2)(0_1, 2_1)(0_1, 1_2)(1_1, 3_2))_1, ((3_1, 0_3)(3_2, 0_3)(1_2, 3_2)(3_1, 1_2)(2_1, 3_1)(0_2, 0_3))_2,$
 $((0_1, 0_2)(0_2, 1_2)(1_2, 0_3)(0_1, 0_3)(0_1, 1_3)(0_2, 3_2))_1, ((1_1, 4_1)(4_1, 3_2)(3_2, 1_3)(1_1, 1_3)(1_1, 0_3)(4_1, 1_2))_2,$
 $((0_2, 1_3)(2_1, 1_3)(2_1, 0_3)(0_2, 0_3)(0_2, 2_3)(3_2, 1_3))_1, ((3_3, \infty)(3_2, \infty)(3_2, 2_3)(2_3, 3_3)(0_3, 3_3)(2_1, \infty))_2,$
 $((0_3, \infty)(0_3, 4_3)(2_1, 4_3)(2_1, \infty)(1_2, \infty)(0_3, 2_3))_1, ((1_1, 2_2)(1_1, 2_3)(1_2, 2_3)(1_2, 2_2)(2_1, 2_2)(1_1, 4_3))_2.$

$2K_{6,6}$ Let the vertex set be $\{0, \dots, 5\} \cup \{6, \dots, 11\}$. The design is given by:

$((0, 6)(1, 6)(1, 7)(0, 7)(0, 8)(2, 6))_1, ((3, 9)(2, 9)(2, 8)(3, 8)(3, 7)(1, 9))_2,$
 $((0, 9)(1, 9)(1, 10)(0, 10)(0, 11)(2, 9))_1, ((3, 6)(2, 6)(2, 11)(3, 11)(3, 10)(1, 6))_2,$
 $((2, 7)(4, 7)(4, 8)(2, 8)(2, 10)(5, 7))_1, ((0, 6)(5, 6)(5, 10)(0, 10)(0, 8)(4, 6))_2,$
 $((3, 6)(4, 6)(4, 11)(3, 11)(3, 7)(5, 6))_1, ((0, 9)(5, 9)(5, 7)(0, 7)(0, 11)(4, 9))_2,$
 $((3, 9)(4, 9)(4, 10)(3, 10)(3, 8)(5, 9))_1, ((1, 11)(5, 11)(5, 8)(1, 8)(1, 10)(4, 11))_2,$
 $((5, 11)(1, 11)(1, 8)(5, 8)(5, 10)(2, 11))_1, ((4, 7)(2, 7)(2, 10)(4, 10)(4, 8)(1, 7))_2.$

$2K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$((0_1, 0_3)(0_2, 0_3)(0_2, 1_3)(0_1, 1_3)(0_1, 2_3)(1_2, 0_3))_1, ((1_1, 0_4)(1_2, 0_4)(1_2, 2_3)(1_1, 2_3)(1_1, 1_3)(0_2, 0_4))_2,$
 $((0_1, 0_4)(0_2, 0_4)(0_2, 1_4)(0_1, 1_4)(0_1, 2_4)(1_2, 0_4))_1, ((1_1, 0_5)(1_2, 0_5)(1_2, 2_4)(1_1, 2_4)(1_1, 1_4)(0_2, 0_5))_2,$
 $((0_1, 0_5)(0_2, 0_5)(0_2, 1_5)(0_1, 1_5)(0_1, 2_5)(1_2, 0_5))_1, ((1_1, 0_3)(1_2, 0_3)(1_2, 2_5)(1_1, 2_5)(1_1, 1_5)(0_2, 0_3))_2.$

G_{14} designs

$2K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(1, 5)(2, 5)(0, 2)(0, 6)(5, 10))_1, ((3, 6)(6, 11)(10, 11)(3, 10)(1, 3)(2, 11))_2.$

$2K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$((0_1, 1_1)(1_1, 0_2)(2_1, 0_2)(0_1, 2_1)(0_1, 1_2)(0_2, 4_2))_1, ((3_1, 1_2)(1_2, 2_2)(2_2, 4_2)(3_1, 4_2)(1_1, 3_1)(2_1, 2_2))_2,$
 $((0_1, 0_2)(0_2, 0_3)(2_2, 0_3)(0_1, 2_2)(0_1, 1_3)(2_1, 0_3))_1, ((1_1, 1_3)(1_3, 4_3)(2_1, 4_3)(1_1, 2_1)(1_1, 0_2)(2_2, 4_3))_2,$
 $((0_2, 2_3)(2_3, 3_3)(2_2, 3_3)(0_2, 2_2)(0_2, 4_3)(4_1, 3_3))_1, ((1_2, 4_3)(4_3, \infty)(4_1, \infty)(4_1, 1_2)(1_2, 2_3)(2_2, \infty))_2,$
 $((1_1, \infty)(1_1, 3_3)(0_3, 3_3)(0_3, \infty)(2_2, \infty)(3_1, 3_3))_1, ((2_2, 2_3)(2_2, 1_3)(3_1, 1_3)(3_1, 2_3)(1_1, 2_3)(0_3, 1_3))_2.$

$2K_{6,6}$ Let the vertex set be $\{0, \dots, 5\} \cup \{6, \dots, 11\}$. The design is given by:

$((0, 6)(1, 6)(1, 7)(0, 7)(0, 8)(1, 9))_1, ((2, 8)(3, 8)(3, 9)(2, 9)(2, 6)(3, 7))_2,$
 $((0, 9)(2, 9)(2, 10)(0, 10)(0, 11)(2, 6))_1, ((1, 11)(3, 11)(3, 6)(1, 6)(1, 9)(3, 10))_2,$
 $((1, 8)(3, 8)(3, 10)(1, 10)(1, 11)(3, 6))_1, ((4, 11)(5, 11)(5, 6)(4, 6)(4, 8)(5, 10))_2,$
 $((2, 7)(4, 7)(4, 11)(2, 11)(2, 8)(4, 9))_1, ((5, 8)(0, 8)(0, 9)(5, 9)(5, 7)(0, 11))_2,$
 $((3, 9)(5, 9)(5, 11)(3, 11)(3, 7)(5, 10))_1, ((4, 7)(2, 7)(2, 10)(4, 10)(4, 9)(2, 11))_2,$
 $((4, 6)(5, 6)(5, 8)(4, 8)(4, 10)(5, 7))_1, ((0, 10)(1, 10)(1, 7)(0, 7)(0, 6)(1, 8))_2.$

$2K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$((0_1, 0_3)(0_2, 0_3)(0_2, 1_3)(0_1, 1_3)(0_1, 2_3)(0_2, 0_4))_1, ((1_1, 2_3)(2_2, 2_3)(2_2, 0_4)(1_1, 0_4)(1_1, 0_3)(2_2, 1_3))_2,$
 $((0_1, 0_4)(1_1, 0_4)(1_1, 0_5)(0_1, 0_5)(0_1, 1_5)(1_1, 2_4))_1, ((2_1, 1_5)(0_2, 1_5)(0_2, 2_4)(2_1, 2_4)(2_1, 0_4)(0_2, 0_5))_2,$
 $((0_2, 1_4)(2_2, 1_4)(2_2, 2_5)(0_2, 2_5)(0_2, 2_3)(2_2, 0_5))_1, ((1_2, 2_3)(2_1, 2_3)(2_1, 0_5)(1_2, 0_5)(1_2, 1_4)(2_1, 2_5))_2.$

$6K_8$ Let the vertex set be $\mathbb{Z}_7 \cup \{\infty\}$. Cycle the following mod 7:

$((0, \infty)(2, \infty)(1, 2)(0, 1)(0, 3)(2, 6))_1, ((3, 4)(3, 5)(5, 6)(4, 6)(4, \infty)(1, 5))_2,$
 $((0, 2)(2, 4)(3, 4)(0, 3)(0, 5)(4, \infty))_1, ((5, 6)(1, 5)(1, \infty)(6, \infty)(2, 6)(1, 3))_2.$

$6K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$((0, 1)(1, 3)(2, 3)(0, 2)(0, 4)(3, 8))_1, ((4, 5)(4, 6)(6, 8)(5, 8)(1, 5)(2, 6))_2,$
 $((0, 1)(1, 7)(3, 7)(0, 3)(0, 6)(5, 7))_1, ((4, 6)(2, 6)(2, 5)(4, 5)(1, 4)(2, 3))_2.$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$((0, 1)(1, 3)(2, 3)(0, 2)(0, 4)(3, 7))_1, ((4, 5)(4, 6)(6, 7)(5, 7)(1, 5)(2, 6))_2,$
 $((1, \infty)(1, 3)(2, 3)(2, \infty)(4, \infty)(3, 8))_1, ((4, 5)(4, 10)(8, 10)(5, 8)(1, 5)(2, 10))_2,$
 $((0, 3)(3, 10)(5, 10)(0, 5)(0, 8)(7, 10))_1, ((8, \infty)(2, 8)(2, 7)(7, \infty)(3, \infty)(2, 5))_2.$

$6K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$((3, 7)(3, 6)(0, 6)(0, 7)(2, 7)(1, 6))_1, ((2, 5)(2, 4)(1, 4)(1, 5)(3, 5)(0, 4))_2,$
 $((0, 5)(1, 5)(1, 4)(0, 4)(0, 6)(1, 7))_1, ((2, 6)(3, 6)(3, 7)(2, 7)(2, 5)(3, 4))_2,$
 $((3, 6)(3, 5)(0, 5)(0, 6)(2, 6)(1, 5))_1, ((2, 7)(2, 4)(1, 4)(1, 7)(3, 7)(0, 4))_2,$
 $((0, 4)(2, 4)(2, 5)(0, 5)(0, 7)(2, 6))_1, ((3, 7)(1, 7)(1, 6)(3, 6)(3, 4)(1, 5))_2,$
 $((1, 4)(3, 4)(3, 5)(1, 5)(1, 6)(3, 7))_1, ((2, 6)(0, 6)(0, 7)(2, 7)(2, 4)(0, 5))_2,$
 $((1, 4)(2, 4)(2, 7)(1, 7)(1, 6)(2, 5))_1, ((3, 6)(0, 6)(0, 5)(3, 5)(3, 4)(0, 7))_2,$
 $((2, 5)(3, 5)(3, 6)(2, 6)(2, 7)(3, 4))_1, ((0, 7)(1, 7)(1, 4)(0, 4)(0, 5)(1, 6))_2,$
 $((0, 4)(0, 7)(3, 7)(3, 4)(2, 4)(1, 7))_1, ((2, 6)(2, 5)(1, 5)(1, 6)(0, 6)(3, 5))_2.$

G_{15} designs

$2K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(1, 4)(2, 4)(5, 10)(5, 12)(3, 12))_1, ((1, 10)(3, 10)(3, 4)(0, 5)(0, 2)(2, 12))_2.$

$2K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$((0_1, 1_1)(1_1, 0_2)(2_1, 0_2)(4_1, 1_2)(4_1, 4_2)(2_2, 4_2))_1, ((1_1, 1_2)(1_2, 2_2)(0_2, 2_2)(0_1, 4_1)(0_1, 2_1)(2_1, 4_2))_2,$
 $((0_1, 2_1)(2_1, 0_3)(1_2, 0_3)(3_2, 1_3)(3_2, 3_3)(3_1, 3_3))_1, ((2_1, 1_3)(3_1, 1_3)(3_1, 0_3)(0_1, 3_2)(0_1, 1_2)(1_2, 3_3))_2,$
 $((0_3, 1_3)(4_1, 1_3)(4_1, 0_2)(1_2, 3_3)(1_2, \infty)(4_3, \infty))_1, ((1_3, 3_3)(3_3, 4_3)(4_1, 4_3)(1_2, 0_3)(0_2, 0_3)(0_2, \infty))_2,$
 $((1_1, \infty)(1_1, 0_3)(4_1, 0_3)(2_3, 4_3)(3_2, 4_3)(3_2, 4_2))_1, ((1_1, 2_3)(4_2, 2_3)(4_2, 0_3)(4_3, \infty)(4_1, \infty)(4_1, 3_2))_2.$

$2K_{6,6}$ Let the vertex set be $\{0, \dots, 5\} \cup \{6, \dots, 11\}$. The design is given by:

$((0, 6)(1, 6)(1, 7)(2, 8)(3, 8)(3, 9))_1, ((2, 6)(2, 9)(1, 9)(0, 8)(0, 7)(3, 7))_2,$
 $((0, 7)(2, 7)(2, 6)(1, 9)(4, 9)(4, 8))_1, ((1, 7)(1, 8)(2, 8)(0, 9)(0, 6)(4, 6))_2,$
 $((1, 8)(5, 8)(5, 10)(3, 11)(0, 11)(0, 9))_1, ((3, 8)(3, 9)(5, 9)(1, 11)(1, 10)(0, 10))_2,$
 $((2, 9)(5, 9)(5, 11)(4, 10)(0, 10)(0, 8))_1, ((4, 9)(4, 8)(5, 8)(2, 10)(2, 11)(0, 11))_2,$
 $((3, 10)(2, 10)(2, 11)(5, 6)(4, 6)(4, 7))_1, ((5, 10)(5, 7)(2, 7)(3, 6)(3, 11)(4, 11))_2,$
 $((4, 11)(1, 11)(1, 10)(5, 7)(3, 7)(3, 6))_1, ((5, 11)(5, 6)(1, 6)(4, 7)(4, 10)(3, 10))_2.$

$2K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$((0_1, 0_3)(2_1, 0_3)(2_1, 1_3)(1_1, 0_4)(0_2, 0_4)(0_2, 0_5))_1, ((1_1, 0_3)(1_1, 0_5)(2_1, 0_5)(0_1, 0_4)(0_1, 1_3)(0_2, 1_3))_2,$
 $((0_1, 0_4)(1_2, 0_4)(1_2, 2_4)(2_2, 0_5)(1_1, 0_5)(1_1, 1_5))_1, ((2_2, 0_4)(2_2, 1_5)(1_2, 1_5)(0_1, 0_5)(0_1, 2_4)(1_1, 2_4))_2,$
 $((0_2, 1_3)(1_2, 1_3)(1_2, 0_3)(2_2, 1_5)(0_1, 1_5)(0_1, 1_4))_1, ((2_2, 1_3)(2_2, 1_4)(1_2, 1_4)(0_2, 1_5)(0_2, 0_3)(0_1, 0_3))_2.$

$6K_8$ Let the vertex set be $\mathbb{Z}_7 \cup \{\infty\}$. Cycle the following mod 7:

$((0, \infty)(2, \infty)(1, 2)(3, 4)(3, 5)(5, 6))_1, ((4, \infty)(4, 6)(2, 6)(0, 3)(0, 1)(1, 5))_2,$
 $((0, 2)(2, 5)(1, 5)(4, 6)(3, 6)(3, \infty))_1, ((2, 4)(4, \infty)(5, \infty)(0, 6)(0, 1)(1, 3))_2.$

$6K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$((0, 1)(1, 3)(2, 3)(4, 5)(4, 8)(6, 8))_1, ((1, 5)(5, 6)(3, 6)(0, 4)(0, 2)(2, 8))_2,$
 $((0, 2)(2, 7)(1, 7)(3, 6)(3, 8)(5, 8))_1, ((2, 6)(5, 6)(5, 7)(0, 3)(0, 1)(1, 8))_2.$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$((0, 1)(1, 3)(2, 3)(4, 5)(4, 6)(6, 8))_1, ((1, 5)(5, 8)(3, 8)(0, 4)(0, 2)(2, 6))_2,$
 $((1, \infty)(1, 5)(2, 5)(3, 6)(3, 7)(4, 7))_1, ((1, 6)(4, 6)(4, 5)(3, \infty)(2, \infty)(2, 7))_2,$
 $((0, 4)(4, 9)(3, 9)(1, 5)(1, 6)(6, \infty))_1, ((4, 5)(5, \infty)(9, \infty)(0, 1)(0, 3)(3, 6))_2.$

$6K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$((2, 6)(3, 6)(3, 4)(0, 7)(1, 7)(1, 5))_1, ((0, 6)(0, 5)(3, 5)(2, 7)(2, 4)(1, 4))_2,$
 $((0, 4)(3, 4)(3, 7)(1, 5)(2, 5)(2, 6))_1, ((1, 4)(1, 6)(3, 6)(0, 5)(0, 7)(2, 7))_2,$
 $((0, 6)(0, 4)(2, 4)(1, 7)(1, 5)(3, 5))_1, ((0, 7)(3, 7)(3, 4)(1, 6)(2, 6)(2, 5))_2,$
 $((0, 4)(2, 4)(2, 5)(1, 6)(3, 6)(3, 7))_1, ((1, 4)(1, 7)(2, 7)(0, 6)(0, 5)(3, 5))_2,$
 $((0, 7)(3, 7)(3, 4)(1, 6)(2, 6)(2, 5))_1, ((1, 7)(1, 5)(3, 5)(0, 6)(0, 4)(2, 4))_2,$
 $((1, 4)(2, 4)(2, 7)(3, 5)(0, 5)(0, 6))_1, ((3, 4)(3, 6)(2, 6)(1, 5)(1, 7)(0, 7))_2,$
 $((2, 7)(1, 7)(1, 4)(3, 5)(0, 5)(0, 6))_1, ((3, 7)(3, 6)(1, 6)(2, 5)(2, 4)(0, 4))_2,$
 $((2, 7)(0, 7)(0, 5)(3, 6)(1, 6)(1, 4))_1, ((3, 7)(3, 4)(0, 4)(2, 6)(2, 5)(1, 5))_2.$

G_{16} designs

$2K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(0, 2)(0, 6)(1, 5)(2, 7)(3, 6))_1, ((5, 11)(3, 11)(7, 11)(2, 5)(1, 3)(6, 7))_2.$

$2K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$((0_1, 1_1)(0_1, 2_1)(0_1, 1_2)(1_1, 0_2)(2_1, 2_2)(3_1, 1_2))_1, ((0_2, 0_3)(3_1, 0_3)(2_2, 0_3)(2_1, 0_2)(1_1, 3_1)(1_2, 2_2))_2,$
 $((0_1, 2_2)(0_1, 0_3)(0_1, 1_3)(1_2, 2_2)(0_2, 0_3)(3_1, 1_3))_1, ((4_1, 1_2)(3_1, 4_1)(4_1, 0_2)(1_2, 0_3)(3_1, 2_2)(0_2, 1_3))_2,$
 $((0_3, 1_3)(2_2, 0_3)(1_2, 0_3)(2_1, 1_3)(2_2, 4_2)(1_2, 3_3))_1, ((2_1, \infty)(3_3, \infty)(4_2, \infty)(2_1, 2_2)(1_3, 3_3)(1_2, 4_2))_2,$
 $((1_1, \infty)(1_2, \infty)(1_3, \infty)(1_1, 3_3)(1_2, 2_3)(1_3, 4_3))_1, ((3_1, 3_3)(3_1, 4_3)(3_1, 2_3)(1_2, 3_3)(1_1, 4_3)(1_3, 2_3))_2.$

$2K_{6,6}$ Let the vertex set be $\{0, \dots, 5\} \cup \{6, \dots, 11\}$. The design is given by:

$((0, 6)(0, 7)(0, 8)(1, 6)(3, 7)(2, 8))_1, ((1, 9)(2, 9)(3, 9)(1, 7)(2, 6)(3, 8))_2,$
 $((0, 9)(0, 10)(0, 11)(4, 9)(1, 10)(5, 11))_1, ((4, 8)(5, 8)(1, 8)(4, 10)(5, 9)(1, 11))_2,$
 $((2, 7)(2, 10)(2, 11)(5, 7)(4, 10)(3, 11))_1, ((5, 6)(3, 6)(4, 6)(5, 10)(3, 7)(4, 11))_2,$
 $((2, 6)(4, 6)(5, 6)(2, 9)(4, 7)(5, 8))_1, ((0, 9)(0, 8)(0, 7)(4, 9)(2, 8)(5, 7))_2,$
 $((1, 8)(4, 8)(3, 8)(1, 7)(4, 11)(3, 10))_1, ((2, 7)(2, 10)(2, 11)(4, 7)(1, 10)(3, 11))_2,$
 $((1, 9)(5, 9)(3, 9)(1, 11)(5, 10)(3, 6))_1, ((0, 11)(0, 6)(0, 10)(5, 11)(1, 6)(3, 10))_2.$

$2K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$((0, 6)(0, 7)(0, 8)(1, 6)(4, 7)(5, 8))_1, ((1, 12)(5, 12)(4, 12)(1, 7)(5, 6)(4, 8))_2,$
 $((0, 9)(0, 10)(0, 11)(1, 9)(3, 10)(2, 11))_1, ((1, 6)(2, 6)(3, 6)(1, 10)(2, 9)(3, 11))_2,$
 $((1, 7)(1, 8)(1, 10)(2, 7)(3, 8)(5, 10))_1, ((2, 14)(5, 14)(3, 14)(2, 8)(5, 7)(3, 10))_2,$
 $((2, 6)(2, 12)(2, 13)(3, 6)(1, 12)(4, 13))_1, ((3, 9)(4, 9)(1, 9)(3, 12)(4, 6)(1, 13))_2,$
 $((5, 6)(5, 13)(5, 14)(4, 6)(1, 13)(0, 14))_1, ((4, 11)(0, 11)(1, 11)(4, 13)(0, 6)(1, 14))_2,$
 $((2, 9)(3, 9)(5, 9)(2, 8)(3, 13)(5, 12))_1, ((0, 8)(0, 12)(0, 13)(3, 8)(2, 12)(5, 13))_2,$
 $((3, 11)(4, 11)(5, 11)(3, 14)(4, 9)(5, 7))_1, ((0, 14)(0, 7)(0, 9)(4, 14)(3, 7)(5, 9))_2,$
 $((0, 12)(3, 12)(4, 12)(0, 13)(3, 7)(4, 10))_1, ((2, 13)(2, 10)(2, 7)(3, 13)(0, 10)(4, 7))_2,$
 $((1, 14)(2, 14)(4, 14)(1, 11)(2, 10)(4, 8))_1, ((5, 11)(5, 8)(5, 10)(2, 11)(1, 8)(4, 10))_2.$

$4K_{10}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2\}$. Cycle the following mod 5:

$((0_1, 1_1), (0_1, 2_1), (0_1, 4_1), (1_1, 3_1), (2_1, 1_2), (4_1, 0_2))_1, ((3_1, 2_2), (0_2, 2_2), (1_2, 2_2), (2_1, 3_1), (1_1, 0_2), (4_1, 1_2))_2,$
 $((0_1, 0_2), (0_1, 1_2), (0_1, 2_2), (1_1, 0_2), (1_2, 4_2), (4_1, 2_2))_1, ((1_1, 2_1), (2_1, 4_1), (2_1, 4_2), (1_1, 1_2), (4_1, 0_2), (2_2, 4_2))_2,$
 $((0_2, 1_2), (2_1, 0_2), (0_2, 3_2), (1_2, 2_2), (2_1, 4_2), (3_1, 3_2))_1, ((1_1, 2_2), (1_1, 3_1), (1_1, 4_2), (2_1, 2_2), (3_1, 1_2), (3_2, 4_2))_2.$

$4K_{19}$ Let the vertex set be \mathbb{Z}_{19} . Cycle the following mod 19:

$((0, 1)(0, 2)(0, 4)(1, 3)(2, 6)(4, 5))_1, ((3, 7)(5, 7)(6, 7)(2, 3)(1, 5)(4, 6))_2,$
 $((0, 3)(0, 5)(0, 6)(3, 8)(5, 11)(6, 9))_1, ((1, 8)(1, 9)(1, 11)(5, 8)(3, 9)(6, 11))_2,$
 $((0, 7)(0, 8)(0, 10)(7, 14)(8, 18)(2, 10))_1, ((11, 14)(2, 11)(11, 18)(8, 14)(2, 7)(10, 18))_2.$

$4K_{22}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_{11}, j = 1, 2\}$. Cycle the following mod 11:

$((0_1, 1_1)(0_1, 2_1)(0_1, 4_1)(1_1, 3_1)(2_1, 6_1)(4_1, 5_1))_1,$
 $((3_1, 7_1)(5_1, 7_1)(6_1, 7_1)(2_1, 3_1)(1_1, 5_1)(4_1, 6_1))_2,$
 $((0_1, 3_1)(0_1, 5_1)(0_1, 0_2)(3_1, 8_1)(2_1, 5_1)(6_1, 0_2))_1,$
 $((8_1, 1_2)(6_1, 1_2)(2_1, 1_2)(5_1, 8_1)(3_1, 6_1)(2_1, 0_2))_2,$
 $((0_1, 0_2)(0_1, 1_2)(0_1, 2_2)(1_1, 0_2)(4_1, 1_2)(3_1, 2_2))_1,$
 $((1_1, 3_2)(3_1, 3_2)(4_1, 3_2)(1_1, 1_2)(3_1, 0_2)(4_1, 2_2))_2,$
 $((0_1, 1_2)(0_1, 2_2)(0_1, 3_2)(5_1, 1_2)(7_1, 2_2)(6_1, 3_2))_1,$
 $((5_1, 8_2)(6_1, 8_2)(7_1, 8_2)(5_1, 2_2)(6_1, 1_2)(7_1, 3_2))_2,$
 $((0_1, 3_2)(0_1, 4_2)(0_1, 5_2)(8_1, 3_2)(0_2, 4_2)(9_1, 5_2))_1,$
 $((8_1, 1_2)(9_1, 1_2)(0_2, 1_2)(8_1, 4_2)(9_1, 3_2)(0_2, 5_2))_2,$
 $((0_2, 1_2)(0_2, 2_2)(0_2, 4_2)(1_2, 3_2)(2_2, 10_2)(4_2, 5_2))_1,$
 $((3_2, 7_2)(5_2, 7_2)(7_2, 10_2)(2_2, 3_2)(1_2, 5_2)(4_2, 10_2))_2,$
 $((0_2, 5_2)(2_1, 0_2)(0_2, 8_2)(7_1, 5_2)(2_1, 6_2)(2_2, 8_2))_1,$
 $((1_1, 7_1)(1_1, 2_2)(1_1, 6_2)(2_1, 7_1)(2_2, 5_2)(6_2, 8_2))_2.$

$6K_8$ Let the vertex set be $\mathbb{Z}_7 \cup \{\infty\}$. Cycle the following mod 7:

$((0, \infty)(1, \infty)(3, \infty)(0, 2)(1, 5)(3, 4))_1, ((2, 6)(4, 6)(5, 6)(1, 2)(0, 4)(3, 5))_2,$
 $((0, 1)(0, 2)(0, 4)(1, 3)(2, 6)(4, 5))_1, ((3, \infty)(5, \infty)(6, \infty)(2, 3)(1, 5)(4, 6))_2.$

$6K_9$ Let the vertex set be \mathbb{Z}_9 . Cycle the following mod 9:

$$(((0, 1)(0, 2)(0, 4)(1, 3)(2, 6)(4, 5))_1, ((3, 8)(5, 8)(6, 8)(2, 3)(1, 5)(4, 6))_2),$$

$$(((0, 1)(0, 3)(0, 2)(1, 4)(3, 8)(2, 5))_1, ((4, 7)(5, 7)(7, 8)(3, 4)(1, 5)(2, 8))_2).$$

$6K_{12}$ Let the vertex set be $\mathbb{Z}_{11} \cup \{\infty\}$. Cycle the following mod 11:

$$(((0, 1)(0, 2)(0, 4)(1, 3)(2, 6)(4, 5))_1, ((3, 7)(5, 7)(6, 7)(2, 3)(1, 5)(4, 6))_2),$$

$$(((1, \infty)(2, \infty)(4, \infty)(1, 3)(2, 10)(4, 5))_1, ((3, 8)(5, 8)(8, 10)(2, 3)(1, 5)(4, 10))_2),$$

$$(((0, 3)(0, 6)(0, 5)(3, 9)(2, 6)(5, 8))_1, ((9, \infty)(8, \infty)(2, \infty)(6, 9)(3, 8)(2, 5))_2).$$

$6K_{4,4}$ Let the vertex set be $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$. The design is given by:

$$(((2, 6)(2, 7)(2, 4)(1, 6)(0, 7)(3, 4))_1, ((1, 5)(3, 5)(0, 5)(1, 7)(3, 6)(0, 4))_2),$$

$$(((0, 6)(3, 6)(2, 6)(0, 7)(3, 4)(2, 5))_1, ((1, 7)(1, 5)(1, 4)(3, 7)(0, 5)(2, 4))_2),$$

$$(((2, 7)(3, 7)(1, 7)(2, 4)(3, 6)(1, 5))_1, ((0, 4)(0, 5)(0, 6)(3, 4)(2, 5)(1, 6))_2),$$

$$(((0, 4)(0, 5)(0, 6)(2, 4)(3, 5)(1, 6))_1, ((2, 7)(1, 7)(3, 7)(2, 5)(1, 4)(3, 6))_2),$$

$$(((0, 5)(0, 7)(0, 6)(3, 5)(2, 7)(1, 6))_1, ((3, 4)(1, 4)(2, 4)(3, 7)(1, 5)(2, 6))_2),$$

$$(((1, 4)(1, 7)(1, 5)(0, 4)(3, 7)(2, 5))_1, ((0, 6)(2, 6)(3, 6)(0, 7)(2, 4)(3, 5))_2),$$

$$(((0, 4)(3, 4)(1, 4)(0, 5)(3, 6)(1, 7))_1, ((2, 5)(2, 7)(2, 6)(3, 5)(0, 7)(1, 6))_2),$$

$$(((1, 5)(3, 5)(2, 5)(1, 4)(3, 7)(2, 6))_1, ((0, 4)(0, 6)(0, 7)(3, 4)(1, 6)(2, 7))_2).$$

G_{17} designs

$2K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$$(((0, 1)(1, 3)(3, 12)(7, 12)(4, 7)(2, 9))_1, ((2, 3)(0, 2)(0, 4)(4, 9)(9, 12)(1, 7))_2).$$

$2K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$$(((0_1, 1_1)(1_1, 3_1)(3_1, 3_2)(2_2, 3_2)(0_2, 2_2)(2_1, 1_2))_1, ((2_1, 3_1)(0_1, 2_1)(0_1, 0_2)(0_2, 1_2)(1_2, 3_2)(1_1, 2_2))_2),$$

$$(((0_1, 1_2)(3_1, 1_2)(3_1, 0_2)(0_2, 0_3)(3_2, 0_3)(2_2, 1_3))_1, ((3_1, 2_2)(0_1, 2_2)(0_1, 3_2)(3_2, 1_3)(0_2, 1_3)(1_2, 0_3))_2),$$

$$(((0_1, 0_3)(1_1, 0_3)(1_1, \infty)(3_3, \infty)(2_3, 3_3)(3_1, 1_3))_1, ((1_1, 1_3)(0_1, 1_3)(0_1, 2_3)(3_1, 2_3)(3_1, \infty)(0_3, 3_3))_2),$$

$$(((0_2, \infty)(0_2, 3_3)(1_1, 3_3)(1_1, 2_3)(0_3, 2_3)(3_2, 4_3))_1, ((3_2, 3_3)(3_2, \infty)(0_3, \infty)(0_3, 4_3)(1_1, 4_3)(0_2, 2_3))_2).$$

$2K_{6,6}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_6, j = 1, 2\}$. Cycle the following mod 6:

$$(((0, 6)(1, 6)(1, 9)(2, 9)(2, 8)(3, 7))_1, ((1, 7)(0, 7)(0, 8)(3, 8)(3, 9)(2, 6))_2),$$

$$(((0, 7)(1, 7)(1, 8)(3, 8)(3, 9)(2, 6))_1, ((1, 6)(0, 6)(0, 9)(2, 9)(2, 8)(3, 7))_2),$$

$$(((1, 10)(4, 10)(4, 9)(0, 9)(0, 11)(5, 8))_1, ((4, 8)(1, 8)(1, 11)(5, 11)(5, 9)(0, 10))_2),$$

$$(((1, 11)(4, 11)(4, 8)(0, 8)(0, 10)(5, 9))_1, ((4, 9)(1, 9)(1, 10)(5, 10)(5, 8)(0, 11))_2),$$

$$(((4, 6)(5, 6)(5, 11)(3, 11)(3, 10)(2, 7))_1, ((5, 7)(4, 7)(4, 10)(2, 10)(2, 11)(3, 6))_2),$$

$$(((4, 7)(5, 7)(5, 10)(2, 10)(2, 11)(3, 6))_1, ((5, 6)(4, 6)(4, 11)(3, 11)(3, 10)(2, 7))_2).$$

$2K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$$\begin{aligned} &(((0_1, 0_3)(1_1, 0_3)(1_1, 0_4)(0_2, 0_4)(0_2, 2_3)(1_2, 1_3))_1, ((1_1, 1_3)(0_1, 1_3)(0_1, 2_3)(1_2, 2_3)(1_2, 0_4)(0_2, 0_3))_2), \\ &(((0_1, 1_3)(0_2, 1_3)(0_2, 2_5)(2_2, 2_5)(2_2, 0_5)(1_1, 1_4))_1, ((0_2, 1_4)(0_1, 1_4)(0_1, 0_5)(1_1, 0_5)(1_1, 2_5)(2_2, 1_3))_2), \\ &(((0_2, 1_4)(0_1, 1_4)(0_1, 2_5)(2_1, 2_5)(2_1, 0_5)(1_2, 0_4))_1, ((0_1, 0_4)(0_2, 0_4)(0_2, 0_5)(1_2, 0_5)(1_2, 2_5)(2_1, 1_4))_2). \end{aligned}$$

$4K_{10}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2\}$. Cycle the following mod 5:

$$\begin{aligned} &(((0_1, 1_1), (1_1, 3_1), (3_1, 2_2), (0_2, 2_2), (4_1, 0_2), (2_1, 1_2))_1, ((2_1, 3_1), (0_1, 2_1), (0_1, 4_1), (4_1, 1_2), (1_2, 2_2), (1_1, 0_2))_2), \\ &(((0_1, 1_1), (1_1, 1_2), (4_1, 1_2), (4_1, 2_2), (2_2, 3_2), (2_1, 0_2))_1, ((2_1, 1_2), (0_1, 2_1), (0_1, 3_2), (0_2, 3_2), (4_1, 0_2), (1_1, 2_2))_2), \\ &(((0_1, 2_2), (2_2, 4_2), (4_1, 4_2), (2_1, 4_1), (2_1, 3_2), (0_2, 1_2))_1, ((0_2, 4_2), (0_1, 0_2), (0_1, 3_2), (1_2, 3_2), (4_1, 1_2), (2_1, 2_2))_2). \end{aligned}$$

$4K_{19}$ Let the vertex set be \mathbb{Z}_{19} . Cycle the following mod 19:

$$\begin{aligned} &(((0, 1)(1, 3)(3, 7)(5, 7)(4, 5)(2, 6))_1, ((2, 3)(0, 2)(0, 4)(4, 6)(6, 7)(1, 5))_2), \\ &(((0, 3)(3, 8)(1, 8)(1, 9)(6, 9)(5, 11))_1, ((5, 8)(0, 5)(0, 6)(6, 11)(1, 11)(3, 9))_2), \\ &(((0, 5)(5, 12)(6, 12)(6, 16)(7, 16)(3, 14))_1, ((3, 12)(0, 3)(0, 7)(7, 14)(6, 14)(5, 16))_2). \end{aligned}$$

$4K_{22}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_{11}, j = 1, 2\}$. Cycle the following mod 11:

$$\begin{aligned} &(((0_1, 1_1)(1_1, 3_1)(3_1, 7_1)(5_1, 7_1)(4_1, 5_1)(2_1, 6_1))_1, \\ & \quad \quad \quad ((2_1, 3_1)(0_1, 2_1)(0_1, 4_1)(4_1, 6_1)(6_1, 7_1)(1_1, 5_1))_2), \\ &(((0_1, 3_1)(3_1, 8_1)(2_1, 8_1)(2_1, 0_2)(6_1, 0_2)(5_1, 1_2))_1, \\ & \quad \quad \quad ((5_1, 8_1)(0_1, 5_1)(0_1, 6_1)(6_1, 1_2)(2_1, 1_2)(3_1, 0_2))_2), \\ &(((0_1, 3_1)(3_1, 0_2)(1_1, 0_2)(1_1, 2_2)(1_2, 2_2)(8_1, 3_2))_1, \\ & \quad \quad \quad ((8_1, 0_2)(0_1, 8_1)(0_1, 1_2)(1_2, 3_2)(1_1, 3_2)(3_1, 2_2))_2), \\ &(((0_1, 0_2)(1_1, 0_2)(1_1, 4_2)(2_1, 4_2)(2_1, 2_2)(4_1, 1_2))_1, \\ & \quad \quad \quad ((1_1, 1_2)(0_1, 1_2)(0_1, 2_2)(4_1, 2_2)(4_1, 4_2)(2_1, 0_2))_2), \\ &(((0_1, 1_2)(6_1, 1_2)(6_1, 10_2)(8_1, 10_2)(8_1, 4_2)(0_2, 3_2))_1, \\ & \quad \quad \quad ((6_1, 3_2)(0_1, 3_2)(0_1, 4_2)(0_2, 4_2)(0_2, 10_2)(8_1, 1_2))_2), \\ &(((0_1, 3_2)(9_1, 3_2)(9_1, 2_2)(0_2, 2_2)(0_2, 6_2)(1_2, 5_2))_1, \\ & \quad \quad \quad ((9_1, 5_2)(0_1, 5_2)(0_1, 6_2)(1_2, 6_2)(1_2, 2_2)(0_2, 3_2))_2), \\ &(((0_1, 9_2)(8_2, 9_2)(6_2, 8_2)(0_2, 6_2)(0_2, 7_2)(2_2, 5_2))_1, \\ & \quad \quad \quad ((5_2, 8_2)(0_1, 5_2)(0_1, 7_2)(2_2, 7_2)(2_2, 6_2)(0_2, 9_2))_2). \end{aligned}$$

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$2K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$$(((0, 1)(0, 2)(0, 7)(1, 6)(2, 5)(6, 10))_1, ((4, 10)(4, 7)(2, 4)(5, 10)(6, 7)(1, 5))_2).$$

$2K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$$\begin{aligned} &(((0_1, 1_1)(0_1, 2_1)(0_1, 1_2)(1_1, 3_2)(2_1, 0_2)(3_2, 4_2))_1, ((3_1, 4_2)(3_1, 1_2)(2_1, 3_1)(0_2, 4_2)(1_2, 3_2)(1_1, 0_2))_2), \\ &(((0_1, 0_2)(0_1, 4_2)(0_1, 1_3)(0_2, 2_3)(4_2, 0_3)(4_1, 2_3))_1, ((2_1, 4_1)(2_1, 1_3)(2_1, 4_2)(4_1, 0_3)(1_3, 2_3)(0_2, 0_3))_2), \\ &(((0_2, 0_3)(0_2, 3_3)(0_2, 2_2)(0_3, 1_3)(4_2, 3_3)(4_1, 1_3))_1, ((4_1, \infty)(2_2, \infty)(3_3, \infty)(4_1, 4_2)(2_2, 1_3)(4_2, 0_3))_2), \\ &(((0_3, \infty)(2_1, \infty)(1_2, \infty)(0_3, 3_3)(2_1, 2_3)(4_1, 3_3))_1, ((4_1, 4_3)(1_2, 4_3)(2_1, 4_3)(4_1, 2_3)(1_2, 3_3)(0_3, 2_3))_2). \end{aligned}$$

$2K_{6,6}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_6, j = 1, 2\}$. Cycle the following mod 6:

$((0, 6)(0, 7)(0, 8)(2, 6)(1, 7)(2, 9))_1, ((3, 9)(3, 8)(3, 7)(1, 9)(2, 8)(1, 6))_2,$
 $((0, 9)(0, 10)(0, 11)(1, 9)(2, 10)(1, 6))_1, ((3, 6)(3, 11)(3, 10)(2, 6)(1, 11)(2, 9))_2,$
 $((1, 8)(1, 10)(1, 11)(5, 8)(4, 10)(5, 6))_1, ((0, 6)(0, 11)(0, 10)(4, 6)(5, 11)(4, 8))_2,$
 $((3, 9)(3, 7)(3, 8)(5, 9)(4, 7)(5, 10))_1, ((1, 10)(1, 8)(1, 7)(4, 10)(5, 8)(4, 9))_2,$
 $((3, 11)(3, 6)(3, 10)(2, 11)(4, 6)(2, 7))_1, ((5, 7)(5, 10)(5, 6)(4, 7)(2, 10)(4, 11))_2,$
 $((4, 11)(4, 8)(4, 9)(5, 11)(2, 8)(5, 7))_1, ((0, 7)(0, 9)(0, 8)(2, 7)(5, 9)(2, 11))_2.$

$2K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$((0_1, 0_3)(0_1, 1_3)(0_1, 0_4)(0_2, 0_3)(2_1, 1_3)(0_2, 0_5))_1, ((1_1, 0_5)(1_1, 0_4)(1_1, 1_3)(2_1, 0_5)(0_2, 0_4)(2_1, 0_3))_2,$
 $((0_1, 1_4)(0_1, 0_5)(0_1, 2_4)(0_2, 1_4)(1_1, 0_5)(0_2, 1_5))_1, ((1_2, 1_5)(1_2, 2_4)(1_2, 0_5)(1_1, 1_5)(0_2, 2_4)(1_1, 1_4))_2,$
 $((0_2, 2_4)(0_2, 2_5)(0_2, 2_3)(2_2, 2_4)(1_1, 2_5)(2_2, 0_3))_1, ((1_2, 0_3)(1_2, 2_3)(1_2, 2_5)(1_1, 0_3)(2_2, 2_3)(1_1, 2_4))_2.$

$4K_{10}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2\}$. Cycle the following mod 5:

$((0_1, 1_1), (0_1, 2_1), (0_1, 0_2), (1_1, 4_1), (2_1, 3_1), (4_1, 2_2))_1, ((1_2, 2_2), (0_2, 1_2), (2_1, 1_2), (3_1, 2_2), (4_1, 0_2), (1_1, 3_1))_2,$
 $((0_1, 0_2), (0_1, 1_2), (0_1, 2_2), (0_2, 4_2), (3_1, 1_2), (2_1, 4_2))_1, ((1_1, 2_1), (1_1, 2_2), (1_1, 1_2), (2_1, 3_1), (2_2, 4_2), (3_1, 0_2))_2,$
 $((0_2, 3_2), (0_2, 4_2), (4_1, 0_2), (1_2, 3_2), (0_1, 4_2), (2_1, 1_2))_1, ((2_1, 2_2), (4_1, 2_2), (2_2, 4_2), (0_1, 2_1), (4_1, 1_2), (0_1, 3_2))_2.$

$4K_{19}$ Let the vertex set be \mathbb{Z}_{19} . Cycle the following mod 19:

$((0, 1)(0, 2)(0, 4)(1, 5)(2, 3)(5, 7))_1, ((6, 7)(4, 6)(2, 6)(3, 7)(4, 5)(1, 3))_2,$
 $((0, 3)(0, 5)(0, 6)(3, 9)(5, 8)(1, 9))_1, ((1, 11)(6, 11)(5, 11)(1, 8)(6, 9)(3, 8))_2,$
 $((0, 5)(0, 7)(0, 9)(5, 12)(7, 16)(4, 12))_1, ((4, 17)(9, 17)(7, 17)(4, 16)(9, 12)(5, 16))_2.$

$4K_{22}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_{11}, j = 1, 2\}$. Cycle the following mod 11:

$((0_1, 1_1)(0_1, 2_1)(0_1, 4_1)(1_1, 5_1)(2_1, 3_1)(5_1, 7_1))_1,$
 $((6_1, 7_1)(4_1, 6_1)(2_1, 6_1)(3_1, 7_1)(4_1, 5_1)(1_1, 3_1))_2,$
 $((0_1, 3_1)(0_1, 5_1)(0_1, 6_1)(3_1, 0_2)(5_1, 8_1)(2_1, 0_2))_1,$
 $((2_1, 1_2)(6_1, 1_2)(5_1, 1_2)(2_1, 8_1)(6_1, 0_2)(3_1, 8_1))_2,$
 $((0_1, 0_2)(0_1, 1_2)(0_1, 2_2)(2_1, 0_2)(1_1, 1_2)(2_1, 4_2))_1,$
 $((4_1, 4_2)(4_1, 2_2)(4_1, 1_2)(1_1, 4_2)(2_1, 2_2)(1_1, 0_2))_2,$
 $((0_1, 1_2)(0_1, 3_2)(0_1, 4_2)(2_1, 1_2)(4_1, 3_2)(2_1, 5_2))_1,$
 $((1_1, 5_2)(1_1, 4_2)(1_1, 3_2)(4_1, 5_2)(2_1, 4_2)(4_1, 1_2))_2,$
 $((0_1, 4_2)(0_1, 5_2)(0_1, 6_2)(0_2, 4_2)(8_1, 5_2)(0_2, 2_2))_1,$
 $((1_2, 2_2)(1_2, 6_2)(1_2, 5_2)(8_1, 2_2)(0_2, 6_2)(8_1, 4_2))_2,$
 $((0_2, 1_2)(4_1, 0_2)(0_2, 2_2)(1_2, 4_2)(4_1, 9_2)(4_2, 5_2))_1,$
 $((1_1, 5_2)(1_1, 2_2)(1_1, 4_1)(5_2, 9_2)(2_2, 4_2)(1_2, 9_2))_2,$
 $((0_2, 8_2)(0_2, 5_2)(0_2, 4_2)(3_2, 8_2)(10_1, 5_2)(7_1, 3_2))_1,$
 $((7_1, 2_2)(2_2, 4_2)(2_2, 5_2)(7_1, 10_1)(3_2, 4_2)(10_1, 8_2))_2.$

G_{19} designs

$2K_{13}$ Let the vertex set be \mathbb{Z}_{13} . Cycle the following mod 13:

$((0, 1)(0, 2)(0, 8)(1, 4)(4, 11)(7, 11))_1, ((5, 8)(1, 5)(5, 11)(7, 8)(2, 7)(2, 4))_2.$

$2K_{16}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2, 3\} \cup \{\infty\}$. Cycle the following mod 5:

$$\begin{aligned} &(((0_1, 1_1)(0_1, 2_1)(0_1, 1_2)(1_1, 0_2)(0_2, 3_2)(2_2, 3_2))_1, ((4_1, 1_2)(1_1, 4_1)(4_1, 3_2)(1_2, 2_2)(2_1, 2_2)(2_1, 0_2))_2), \\ &(((0_1, 0_2)(0_1, 2_2)(0_1, 3_2)(0_2, 0_3)(1_1, 0_3)(1_1, 3_3))_1, ((3_2, 2_3)(0_2, 2_3)(1_1, 2_3)(3_2, 3_3)(2_2, 3_3)(2_2, 0_3))_2), \\ &(((0_3, 1_3)(0_1, 0_3)(2_1, 0_3)(1_3, 3_3)(1_2, 3_3)(1_2, 2_3))_1, ((2_1, \infty)(1_3, \infty)(1_2, \infty)(2_1, 2_3)(0_1, 2_3)(0_1, 3_3))_2), \\ &(((0_3, \infty)(4_2, \infty)(2_1, \infty)(1_2, 0_3)(1_2, 4_3)(3_1, 4_3))_1, ((2_1, 1_3)(0_3, 1_3)(1_3, 4_3)(2_1, 3_1)(3_1, 4_2)(1_2, 4_2))_2). \end{aligned}$$

$2K_{6,6}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_6, j = 1, 2\}$. Cycle the following mod 6:

$$\begin{aligned} &(((0, 6)(0, 7)(0, 8)(1, 6)(1, 9)(3, 9))_1, ((2, 8)(2, 6)(2, 9)(3, 8)(3, 7)(1, 7))_2), \\ &(((0, 9)(0, 10)(0, 11)(2, 9)(2, 6)(3, 6))_1, ((1, 11)(1, 9)(1, 6)(3, 11)(3, 10)(2, 10))_2), \\ &(((1, 7)(1, 8)(1, 10)(4, 7)(4, 6)(5, 6))_1, ((0, 10)(0, 7)(0, 6)(5, 10)(5, 8)(4, 8))_2), \\ &(((2, 8)(2, 7)(2, 11)(4, 8)(4, 9)(5, 9))_1, ((0, 11)(0, 8)(0, 9)(5, 11)(5, 7)(4, 7))_2), \\ &(((3, 10)(2, 10)(4, 10)(3, 7)(5, 7)(5, 11))_1, ((4, 6)(3, 6)(5, 6)(4, 11)(2, 11)(2, 7))_2), \\ &(((3, 11)(1, 11)(4, 11)(3, 8)(5, 8)(5, 10))_1, ((4, 9)(3, 9)(5, 9)(4, 10)(1, 10)(1, 8))_2). \end{aligned}$$

$2K_{6,9}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_3, j = 1, 2\} \cup \{i_j \mid i \in \mathbb{Z}_3, j = 1, 2, 3\}$. Cycle the following mod 3:

$$\begin{aligned} &(((0_1, 0_3)(0_1, 1_3)(0_1, 0_4)(1_1, 0_3)(1_1, 2_4)(0_2, 2_4))_1, ((2_1, 0_4)(2_1, 0_3)(2_1, 2_4)(0_2, 0_4)(0_2, 1_3)(1_1, 1_3))_2), \\ &(((0_1, 0_5)(0_1, 2_4)(0_1, 1_5)(0_2, 0_5)(0_2, 1_4)(1_2, 1_4))_1, ((2_1, 1_5)(2_1, 0_5)(2_1, 1_4)(1_2, 1_5)(1_2, 2_4)(0_2, 2_4))_2), \\ &(((0_2, 2_3)(0_2, 0_3)(0_2, 1_5)(1_2, 2_3)(1_2, 0_5)(1_1, 0_5))_1, ((2_2, 1_5)(2_2, 2_3)(2_2, 0_5)(1_1, 1_5)(1_1, 0_3)(1_2, 0_3))_2). \end{aligned}$$

$4K_{10}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_5, j = 1, 2\}$. Cycle the following mod 5:

$$\begin{aligned} &(((0_1, 1_1), (0_1, 2_1), (0_1, 4_1), (1_1, 3_1), (3_1, 2_2), (1_2, 2_2))_1, ((4_1, 0_2), (1_1, 0_2), (0_2, 2_2), (4_1, 1_2), (2_1, 1_2), (2_1, 3_1))_2), \\ &(((0_1, 0_2), (0_1, 1_2), (0_1, 2_2), (1_1, 0_2), (1_1, 4_2), (4_1, 4_2))_1, ((2_2, 3_2), (0_2, 3_2), (3_2, 4_2), (4_1, 2_2), (4_1, 1_2), (1_1, 1_2))_2), \\ &(((0_2, 2_2), (3_1, 0_2), (4_1, 0_2), (1_2, 2_2), (1_2, 3_2), (0_1, 3_2))_1, ((2_1, 4_1), (2_1, 2_2), (2_1, 3_2), (0_1, 4_1), (0_1, 3_1), (3_1, 1_2))_2). \end{aligned}$$

$4K_{19}$ Let the vertex set be \mathbb{Z}_{19} . Cycle the following mod 19:

$$\begin{aligned} &(((0, 1)(0, 2)(0, 4)(1, 3)(3, 7)(6, 7))_1, ((4, 5)(1, 5)(5, 7)(4, 6)(2, 6)(2, 3))_2), \\ &(((0, 3)(0, 5)(0, 6)(3, 8)(1, 8)(1, 11))_1, ((6, 9)(3, 9)(1, 9)(6, 11)(5, 11)(5, 8))_2), \\ &(((0, 3)(0, 6)(0, 9)(3, 15)(7, 15)(7, 18))_1, ((9, 14)(3, 14)(7, 14)(9, 18)(6, 18)(6, 15))_2). \end{aligned}$$

$4K_{22}$ Let the vertex set be $\{i_j \mid i \in \mathbb{Z}_{11}, j = 1, 2\}$. Cycle the following mod 11:

$$\begin{aligned} &(((0_1, 1_1)(0_1, 2_1)(0_1, 4_1)(1_1, 3_1)(3_1, 7_1)(6_1, 7_1))_1, ((4_1, 5_1)(1_1, 5_1)(5_1, 7_1)(4_1, 6_1)(2_1, 6_1)(2_1, 3_1))_2), \\ &(((0_1, 3_1)(0_1, 5_1)(0_1, 0_2)(3_1, 8_1)(8_1, 1_2)(10_1, 1_2))_1, \\ & \quad ((6_1, 0_2)(3_1, 6_1)(6_1, 1_2)(10_1, 0_2)(5_1, 10_1)(5_1, 8_1))_2), \\ &(((0_1, 3_1)(0_1, 0_2)(0_1, 2_2)(3_1, 1_2)(1_2, 4_2)(1_1, 4_2))_1, ((8_1, 2_2)(3_1, 8_1)(8_1, 4_2)(1_1, 2_2)(1_1, 0_2)(0_2, 1_2))_2), \\ &(((0_1, 1_2)(0_1, 3_2)(0_1, 4_2)(3_1, 1_2)(3_1, 10_2)(5_1, 10_2))_1, \\ & \quad ((1_1, 4_2)(1_1, 1_2)(1_1, 10_2)(5_1, 4_2)(5_1, 3_2)(3_1, 3_2))_2), \\ &(((0_1, 1_2)(0_1, 5_2)(0_1, 6_2)(2_1, 1_2)(2_1, 8_2)(9_1, 8_2))_1, ((4_1, 6_2)(4_1, 1_2)(4_1, 8_2)(9_1, 6_2)(9_1, 5_2)(2_1, 5_2))_2), \\ &(((0_2, 1_2)(3_1, 0_2)(0_2, 2_2)(1_2, 5_2)(5_2, 8_2)(7_2, 8_2))_1, ((2_2, 3_2)(1_2, 3_2)(3_2, 8_2)(2_2, 7_2)(3_1, 7_2)(3_1, 5_2))_2), \\ &(((0_2, 4_2)(0_2, 2_2)(0_2, 6_2)(4_2, 10_2)(2_1, 10_2)(2_1, 9_2))_1, \\ & \quad ((6_2, 8_2)(4_2, 8_2)(2_1, 8_2)(6_2, 9_2)(2_2, 9_2)(2_2, 10_2))_2). \end{aligned}$$