Appendix to
Decompositions of complete graphs into theta graphs with fewer than ten edges

Andrew Blinco
Centre for Discrete Mathematics and Computing,
Department of Mathematics, The University of Queensland,
Queensland 4072, Australia

1 A $\Theta(1,2,3)$-design of order 24

Example 1.1 *A* $\Theta(1,2,3)$*-design of order 24.*

Let $V = \mathbb{Z}_{24}$ and let $B =$

$$
\begin{array}{llll}
\{[0,1;2,3,4], & [0,3;5,1,6], & [0,7;8,9,1], & [0,10;11,1,12], \\
[0,13;14,1,15], & [0,16;17,2,18], & [0,19;21,1,23], & [1,3;7,2,4], \\
[1,10;13,2,16], & [1,17;22,0,30], & [2,5;6,3,8], & [2,9;10,3,11], \\
[2,12;14,3,15], & [2,19;20,3,21], & [3,9;12,4,13], & [3,16;18,1,19], \\
[3,17;23,2,22], & [4,5;7,9,6], & [4,8;9,5,10], & [4,11;14,5,15], \\
[4,16;19,5,17], & [4,18;20,5,21], & [5,8;11,6,12], & [5,13;16,6,22], \\
[6,8;10,7,13], & [6,14;15,7,17], & [6,18;7,20,23], & [6,20;21,7,19], \\
[7,11;12,8,14], & [7,16;22,4,23], & [8,13;15,9,16], & [8,17;18,5,23], \\
[9,11;13,12,17], & [9,14;18,10,19], & [10,12;15,11,16], & [10,14;17,11,20], \\
[10,21;22,9,23], & [11,18;19,8,21], & [12,16;20,8,22], & [13,17;19,12,21], \\
[14,16;21,9,20], & [14,19;22,11,23], & [15,16;23,13,20], & [15,21;17,20,22], \\
[18,12;23,19,15], & [18,13;22,23,21]\}. & &
\end{array}
$$

Then (V,B) is a $\Theta(1,2,3)$-design of order 24. □

2 Examples of $\Theta(1,2,4)$-designs

Example 2.1 *A* $\Theta(1,2,4)$*-design of order 7.*

Let $V = \mathbb{Z}_7$ and let B contain the following copies of $\Theta(1,2,4)$.

$$\{[0,1;4,2,3,6],[0,3;5,1,6,2],[4,6;5,2,1,3]\}$$

Then (V,B) is a $\Theta(1,2,4)$-design of order 7. □

Example 2.2 *A* $\Theta(1,2,4)$*-design of order 8.*

Let $V=\mathbb{Z}_8$ and let B contain the following copies of $\Theta(1,2,4)$.

$$\{[6,3;0,1,2,4],[3,7;1,4,0,5],[7,0;2,5,1,6,],[5,7;4,3,2,6]\}$$

Then (V,B) is a $\Theta(1,2,4)$-design of order 8. □

Example 2.3 *A* $\Theta(1,2,4)$*-design of order 14.*

Let $V=\mathbb{Z}_{14}$ and let $B=$

$$\begin{array}{llll}\{[0,1;2,3,5,4], & [0,3;8,5,7,11], & [0,6;10,2,4,12], & [1,5;9,12,10,8],\\ [2,8;13,4,1,6], & [2,11;5,6,8,12], & [3,1;12,5,0,13], & [4,3;11,1,13,7],\\ [6,3;7,8,4,9], & [9,3;10,1,7,2], & [9,8;11,10,7,0], & [10,5;13,12,6,4],\\ [11,6;13,9,7,12]\}. & & & \end{array}$$

Then (V,B) is a $\Theta(1,2,4)$-design of order 14. □

Example 2.4 *A* $\Theta(1,2,4)$*-design of order 15.*

Let $V=\mathbb{Z}_{15}$ and let B contain the copies of $\Theta(1,2,4)$ arising from the following set, cycled modulo 15.

$$\{[7,5;14,9,8,11]\}$$

Then (V,B) is a $\Theta(1,2,4)$-design of order 15. □

Example 2.5 *A* $\Theta(1,2,4)$*-design of order 28.*

Let $V=(\mathbb{Z}_9\times\mathbb{Z}_3)\cup\infty$ and let B contain the copies of $\Theta(1,2,4)$ arising from the following set, with the first components all cycled modulo 9, and the second components fixed.

$$\begin{array}{ll}\{[(0,0),(2,0);(1,2),(0,2),(6,2),(3,0)], & [(0,1),(3,1);(4,2),(2,2),(4,0),(8,1)],\\ [(1,1),(7,0);(7,2),(3,0),(0,1),(8,0)], & [(3,1),(7,1);(5,2),(0,1),(2,0),(1,1)],\\ [(4,1),(3,2);(7,2),(2,0),(3,0),(8,0)], & [(0,0),(0,1);\infty,(0,2),(3,0),(5,2)]\}.\end{array}$$

Then (V,B) is a $\Theta(1,2,4)$-design of order 28. □

Example 2.6 *A* $\Theta(1,2,4)$*-design of order 29.*

Let $V=\mathbb{Z}_{29}$ and let B contain the copies of $\Theta(1,2,4)$ arising from the following set, cycled modulo 29.

$$\{[0,4;12,18,5,14],[5,8;10,11,4,15]\}$$

Then (V,B) is a $\Theta(1,2,4)$-design of order 29. □

Example 2.7 *A* $\Theta(1,2,4)$*-decomposition of* $K_{3(7)}$*.*

Let $V=\{i\equiv 0 \pmod 3\colon i\in\mathbb{Z}_{21}\}\cup\{i\equiv 1 \pmod 3\colon i\in\mathbb{Z}_{21}\}\cup\{i\equiv 2 \pmod 3\colon i\in\mathbb{Z}_{21}\}$ and let B contain the copies of $\Theta(1,2,4)$ arising from the following set, cycled modulo 21.

$$\{[4,6;11,10,18,8]\}$$

Then (V,B) is a $\Theta(1,2,4)$-decomposition of $K_{3(7)}$. □

Example 2.8 *A* $\Theta(1,2,4)$*-decomposition of* $K_{5(7)}$.

Let $V = \{i \equiv 0 \pmod 5)\colon i \in \mathbb{Z}_{35}\} \cup \{i \equiv 1 \pmod 5)\colon i \in \mathbb{Z}_{35}\} \cup \{i \equiv 2 \pmod 5)\colon i \in \mathbb{Z}_{35}\} \cup \{i \equiv 3 \pmod 5)\colon i \in \mathbb{Z}_{35}\} \cup \{i \equiv 4 \pmod 5)\colon i \in \mathbb{Z}_{35}\}$ and let B contain the copies of $\Theta(1,2,4)$ arising from the following set, cycled modulo 35.

$$\{[0,1;7,11,29,13],[18,7;21,12,24,16]\}$$

Then (V,B) is a $\Theta(1,2,4)$-decomposition of $K_{5(7)}$. □

3 Examples of $\Theta(2,2,3)$-designs

Example 3.1 *A* $\Theta(2,2,3)$*-design of order 8.*

Let $V = \mathbb{Z}_8$ and let B contain the following copies of $\Theta(2,2,3)$.

$$\{[0:1;2;3,4:5],[1:6;0;5,7:4],[2:0;7;6,5:3],[6:2;4;1,3:7]\}$$

Then (V,B) is a $\Theta(2,2,3)$-design of order 8. □

Example 3.2 *A* $\Theta(2,2,3)$*-design of order 14.*

Let $V = \mathbb{Z}_{14}$ and let $B =$

$\{[0:1;2;3,5:4]$, $[0:4;5;7,2:6]$, $[0:10;13;6,9:1]$, $[2:1;9;5,10:7]$,
$[3:8;12;13,9:4]$, $[4:7;10;3,1:6]$, $[5:9;13;1,11:12]$, $[6:3;11;13,10:9]$,
$[8:0;1;5,7:12]$, $[8:2;12;7,3:10]$, $[8:10;13;9,0:1]$, $[11:2;5;8,6:12]$,
$[11:4;7;3,2:13]\}$.

Then (V,B) is a $\Theta(2,2,3)$-design of order 14. □

Example 3.3 *A* $\Theta(2,2,3)$*-design of order 15.*

Let $V = \mathbb{Z}_{15}$ and let B contain the copies of $\Theta(2,2,3)$ arising from the following set, cycled modulo 15.

$$\{[7:2;11;6,3:9]\}$$

Then (V,B) is a $\Theta(2,2,3)$-design of order 15. □

Example 3.4 *A* $\Theta(2,2,3)$*-design of order 21.*

Let $V = \mathbb{Z}_3 \times \mathbb{Z}_7$ and let B contain the copies of $\Theta(2,2,3)$ arising from the following set, with the first components all cycled modulo 3, and the second components fixed.

$\{[(0,0):(1,0);(0,1);(1,1),(2,1):(0,2)]$, $[(0,0):(0,2);(0,3);(2,5),(0,1):(1,1)]$,
$[(0,0):(1,3);(2,4);(1,5),(2,2):(2,3)]$, $[(0,0):(2,3);(1,4);(1,6),(2,4):(2,1)]$,
$[(0,2):(1,2);(2,6);(2,4),(2,0):(2,5)]$, $[(0,3):(2,1);(1,2);(2,5),(0,5):(1,6)]$,
$[(0,4):(2,1);(0,2);(1,3),(2,4):(0,5)]$, $[(1,4):(2,4);(1,6);(0,2),(2,1):(2,5)]$,
$[(1,6):(2,0);(1,1);(0,6),(2,3):(2,6)]$, $[(2,5):(2,3);(0,4);(1,3),(0,2):(1,6)]\}$.

Then (V,B) is a $\Theta(2,2,3)$-design of order 21. □

Example 3.5 *A* $\Theta(2,2,3)$*-design of order 28.*

Let $V = (\mathbb{Z}_9 \times \mathbb{Z}_3) \cup \infty$ and let B contain the copies of $\Theta(2,2,3)$ arising from the following set, with the first components all cycled modulo 9, and the second components fixed.

$$\{[(0,0):(2,0);(3,0);(4,0),(3,2):(7,2)],\ [(0,0):(1,1);(3,2);(0,1),(5,0):(8,1)],$$
$$[(1,1):(2,0);(0,2);(5,0),(6,0):(3,2)],\ [(1,1):(3,0);(8,0);(7,1),(1,0):(1,2)],$$
$$[(5,1):(4,1);(1,2);(5,2),(6,2):(8,1)],\ [(0,2):(8,1);(7,2);(6,1),(3,2):\infty]\}.$$

Then (V,B) is a $\Theta(2,2,3)$-design of order 28. □

Example 3.6 *A* $\Theta(2,2,3)$*-design of order 29.*

Let $V = \mathbb{Z}_{29}$ and let B contain the copies of $\Theta(2,2,3)$ arising from the following set, cycled modulo 29.

$$\{[0:1;2;3,13:8],[4:15;19;20,3:28]\}$$

Then (V,B) is a $\Theta(2,2,3)$-design of order 29. □

Example 3.7 *A* $\Theta(2,2,3)$*-design of order 35.*

Let $V = (\mathbb{Z}_{14} \times \mathbb{Z}_2) \cup \infty$ and let B contain the copies of $\Theta(2,2,3)$ arising from the following set, with the first components all cycled modulo 14, and the second components fixed.

$$\{[(0,0):(4,0);(8,0);(3,1),(1,1):(2,1)],\ [(0,0):(8,1);(13,1);(16,1),(6,1):(0,1)],$$
$$[(5,0):(12,0);(15,1);(6,1),(9,0):(15,1)],\ [(9,0):(11,0);(1,1);(14,0),(2,1):(12,1)],$$
$$[(9,1):(14,0);(6,1);(7,1),\infty:(1,1)]\}.$$

Then (V,B) is a $\Theta(2,2,3)$-design of order 35. □

Example 3.8 *A* $\Theta(2,2,3)$*-design of order 49.*

Let $V = \mathbb{Z}_7 \times \mathbb{Z}_7$ and let B contain the copies of $\Theta(2,2,3)$ arising from the following set, with the first components all cycled modulo 7, and the second components fixed.

$$\{[(0,0):(1,0);(2,0);(3,0),(0,1):(0,2)],\ [(0,0):(1,1);(2,1);(5,1),(6,0):(2,2)],$$
$$[(0,0):(0,2);(4,2);(0,3),(2,0):(5,3)],\ [(0,0):(1,3);(2,3);(0,4),(1,0):(4,4)],$$
$$[(0,0):(1,4);(4,4);(5,4),(1,1):(5,2)],\ [(0,0):(0,5);(2,5);(3,5),(5,0):(4,6)],$$
$$[(0,0):(6,5);(1,6);(4,6),(0,1):(2,1)],\ [(0,1):(3,1);(3,2);(5,2),(1,2):(5,4)],$$
$$[(0,1):(3,3);(4,3);(6,3),(3,2):(3,5)],\ [(0,1):(0,5);(1,5);(2,5),(3,2):(4,2)],$$
$$[(0,1):(6,5);(0,6);(5,6),(4,2):(5,4)],\ [(0,2):(0,3);(5,4);(2,4),(3,5):(0,5)],$$
$$[(0,4):(1,4);(0,6);(4,5),(6,2):(5,6)],\ [(0,5):(3,5);(5,6);(0,4),(1,6):(2,6)],$$
$$[(1,3):(4,3);(1,6);(6,1),(5,4):(0,5)],\ [(2,4):(2,1);(2,2);(4,1),(2,3):(4,6)],$$
$$[(2,5):(4,1);(0,2);(4,3),(5,0):(0,6)],\ [(2,6):(5,3);(3,4);(3,3),(6,0):(6,6)],$$
$$[(3,0):(4,2);(0,5);(5,4),(4,5):(5,5)],\ [(3,2):(1,2);(0,3);(5,2),(3,0):(4,5)],$$
$$[(3,3):(4,2);(5,3);(0,4),(5,5):(1,6)],\ [(3,3):(4,3);(2,4);(3,1),(2,1):(5,4)],$$
$$[(4,1):(2,0);(4,0);(5,3),(0,5):(0,6)],\ [(6,4):(5,1);(1,3);(4,6),(1,2):(6,6)]\}.$$

Then (V,B) is a $\Theta(2,2,3)$-design of order 49. □

Example 3.9 *A* $\Theta(2,2,3)$*-decomposition of* $K_{3(7)}$.

Let $V = \{i \equiv 0 \pmod 3 \colon i \in \mathbb{Z}_{21}\} \cup \{i \equiv 1 \pmod 3 \colon i \in \mathbb{Z}_{21}\} \cup \{i \equiv 2 \pmod 3 \colon i \in \mathbb{Z}_{21}\}$ and let B contain the copies of $\Theta(2,2,3)$ arising from the following set, cycled modulo 21.

$$\{[9:4;16;11,7:15]\}.$$

Then (V,B) is a $\Theta(2,2,3)$-decomposition of $K_{3(7)}$. □

Example 3.10 *A* $\Theta(2,2,3)$*-decomposition of* $K_{5(7)}$.

Let $V = \{i \equiv 0 \pmod 5 \colon i \in \mathbb{Z}_{35}\} \cup \{i \equiv 1 \pmod 5 \colon i \in \mathbb{Z}_{35}\} \cup \{i \equiv 2 \pmod 5 \colon i \in \mathbb{Z}_{35}\} \cup \{i \equiv 3 \pmod 5 \colon i \in \mathbb{Z}_{35}\} \cup \{i \equiv 4 \pmod 5 \colon i \in \mathbb{Z}_{35}\}$ and let B contain the copies of $\Theta(2,2,3)$ arising from the following set, cycled modulo 35.

$$\{[0:1;2;6,19:5],[26:7;8;15,22:34]\}.$$

Then (V,B) is a $\Theta(2,2,3)$-decomposition of $K_{5(7)}$. □

4 Examples of $\Theta(1,2,5)$-designs

Example 4.1 *A* $\Theta(1,2,5)$*-design of order 16.*

Let $V = \mathbb{Z}_{16}$ and let $B =$

$$\begin{array}{lll}
\{[0,1;3,4,5,6,8], & [0,2;7,4,1,5,11], & [0,4;9,6,3,8,10], \\
[1,10;12,7,8,4,15], & [2,6;15,10,7,11,4], & [4,10;14,7,5,0,12], \\
[5,10;9,7,13,3,2], & [6,12;11,9,14,3,7], & [8,1;14,2,9,3,5], \\
[9,15;12,2,1,11,13], & [10,2;11,14,5,12,3], & [11,3;15,0,13,2,8], \\
[12,8;13,10,6,0,14], & [13,5;15,7,1,6,4], & [13,6;14,15,8,9,1]\}.
\end{array}$$

Then (V,B) is a $\Theta(1,2,5)$-design of order 16. □

Example 4.2 *A* $\Theta(1,2,5)$*-design of order 17.*

Let $V = \mathbb{Z}_{17}$ and let B contain the copies of $\Theta(1,2,5)$ arising from the following set, cycled modulo 17

$$\{[7,1;11,12,3,5,2]\}.$$

Then (V,B) is a $\Theta(1,2,5)$-design of order 17. □

Example 4.3 *A* $\Theta(1,2,5)$*-design of order 32.*

Let $V = \mathbb{Z}_{32}$ and let $B =$

$$\begin{array}{lll}
\{[0,1;2,3,4,5,6], & [0,3;5,1,4,2,7], & [0,4;8,1,3,6,9], \\
[0,10;11,4,26,12,24], & [0,12;13,1,6,2,14], & [0,15;16,1,7,3,17], \\
[0,18;19,1,9,2,20], & [0,21;22,1,10,2,23], & [0,25;26,1,11,2,27], \\
[1,12;14,3,8,2,15], & [1,17;18,2,5,7,20], & [1,21;23,3,9,4,24], \\
[1,25;27,3,10,4,28], & [1,29;30,0,28,2,31], & [2,12;16,3,11,5,13], \\
[2,17;19,3,12,4,21], & [2,22;24,3,13,4,25], & [2,26;29,0,31,3,30], \\
[3,15;18,4,6,7,21], & [3,20;22,4,7,8,25], & [3,26;28,5,8,6,29], \\
[4,14;15,5,9,7,16], & [4,17;20,5,10,6,19], & [4,23;27,5,12,6,30], \\
[5,14;16,6,11,7,17], & [5,18;21,6,13,7,19], & [5,22;23,6,14,7,24], \\
[5,25;29,4,31,6,26], & [6,15;17,8,9,10,18], & [6,20;24,8,10,7,22],
\end{array}$$

$[6,25;28,7,12,8,27]$,	$[7,15;23,8,11,9,18]$,	$[7,25;30,5,31,8,26]$,
$[7,27;29,8,13,9,31]$,	$[8,14;18,11,12,9,15]$,	$[8,16;19,9,14,10,20]$,
$[9,16;17,10,12,15,20]$,	$[9,21;24,10,13,11,22]$,	$[9,23;25,10,15,11,26]$,
$[10,16;21,8,22,12,19]$,	$[10,22;26,13,14,11,23]$,	$[10,27;28,8,30,9,29]$,
$[11,16;20,12,17,13,19]$,	$[11,17;21,12,18,13,24]$,	$[11,25;31,10,30,12,27]$,
$[11,28;29,12,23,13,30]$,	$[12,28;31,13,15,19,25]$,	$[13,16;22,14,17,23,20]$,
$[13,21;25,14,19,20,27]$,	$[14,20;21,15,22,17,24]$,	$[14,23;26,15,24,16,27]$,
$[15,27;30,14,28,16,25]$,	$[15,29;31,16,18,20,28]$,	$[17,26;27,9,28,13,29]$,
$[18,22;25,17,28,19,23]$,	$[18,24;26,16,23,28,30]$,	$[19,21;26,20,25,24,27]$,
$[19,22;29,14,31,17,30]$,	$[20,31;30,22,27,18,29]$,	$[21,27;31,23,29,16,30]$,
$[24,19;31,18,28,21,29]$,	$[24,23;30,26,31,22,28]\}$.	

Then (V,B) is a $\Theta(1,2,5)$-design of order 32. □

Example 4.4 *A* $\Theta(1,2,5)$*-design of order 33.*

Let $V=\mathbb{Z}_{33}$ and let B contain the copies of $\Theta(1,2,5)$ arsing from the following set, cycled modulo 33.

$$\{[0,2;12,1,7,4,17],[17,32;18,9,16,21,13]:\}.$$

Then (V,B) is a $\Theta(1,2,5)$-design of order 33. □

Example 4.5 *A* $\Theta(1,2,5)$*-decomposition of* $K_{3(16)}$.

Let $V=\{i\equiv 0 \pmod 3\colon i\in\mathbb{Z}_{48}\}\cup\{i\equiv 1 \pmod 3\colon i\in\mathbb{Z}_{48}\}\cup\{i\equiv 2 \pmod 3\colon i\in\mathbb{Z}_{48}\}$ and let B contain the copies of $\Theta(1,2,5)$ arising from the following set, cycled modulo 48.

$$\{[0,7;17,1,27,13,5],[24,23;43,32,36,34,11]\}.$$

Then (V,B) is a $\Theta(1,2,5)$-decomposition of $K_{3(16)}$. □

Example 4.6 *A* $\Theta(1,2,5)$*-decomposition of* $K_{5(16)}$.

Let $V=\{i\equiv 0 \pmod 5\colon i\in\mathbb{Z}_{80}\}\cup\{i\equiv 1 \pmod 5\colon i\in\mathbb{Z}_{80}\}\cup\{i\equiv 2 \pmod 5\colon i\in\mathbb{Z}_{80}\}\cup\{i\equiv 3 \pmod 5\colon i\in\mathbb{Z}_{80}\}\cup\{i\equiv 4 \pmod 5\colon i\in\mathbb{Z}_{80}\}$ and let B contain the copies of $\Theta(1,2,5)$ arising from the following set, cycled modulo 80.

$$\{[0,1;8,4,2,11,23],[0,14;31,3,19,1,22],$$
$$[0,19;53,5,31,70,33],[73,37;79,28,25,38,49]\}.$$

Then (V,B) is a $\Theta(1,2,5)$-decomposition of $K_{5(16)}$. □

5 Examples of $\Theta(1,3,4)$-designs

Example 5.1 *A* $\Theta(1,3,4)$*-design of order 16.*

Let $V = \mathbb{Z}_{16}$ and let $B =$

$\{[0,1,2;3,4,5,6]$, $[0,2,4;7,3,9,5]$, $[0,4,6;8,2,5,10]$,
$[0,9,2;11,8,7,12]$, $[0,15,5;14,1,11,13]$, $[1,10,11;5,8,15,7]$,
$[2,6,9;15,10,7,13]$, $[4,15,6;11,12,3,13]$, $[6,1,4;14,15,11,3]$,
$[9,7,6;10,4,8,12]$, $[10,3,5;13,15,12,2]$, $[10,8,9;14,7,5,12]$,
$[11,7,2;14,8,1,9]$, $[12,4,9;13,8,3,1]$, $[13,6,12;14,3,15,1]\}$.

Then (V,B) is a $\Theta(1,3,4)$-design of order 16. □

Example 5.2 *A* $\Theta(1,3,4)$*-design of order 17.*

Let $V = \mathbb{Z}_{17}$ and let B contain the copies of $\Theta(1,3,4)$ arising from the following set, cycled modulo 17.

$$\{[6,5,12;10,2,14,3]\}.$$

Then (V,B) is a $\Theta(1,3,4)$-design of order 17. □

Example 5.3 *A* $\Theta(1,3,4)$*-design of order 32.*

Let $V = \mathbb{Z}_{32}$ and let $B =$

$\{[0,1,2;3,4,5,6]$, $[0,2,4;7,1,3,5]$, $[0,4,1;8,2,5,9]$,
$[0,10,1;13,9,29,28]$, $[0,11,1;12,2,6,14]$, $[0,15,1;16,2,7,17]$,
$[0,18,1;19,2,9,20]$, $[0,21,1;22,2,10,23]$, $[0,24,1;25,2,11,26]$,
$[1,5,7;6,3,8,9]$, $[1,14,2;17,3,7,20]$, $[1,23,2;26,3,9,27]$,
$[2,13,3;15,4,6,18]$, $[2,20,3;21,4,8,24]$, $[2,27,0;29,1,28,30]$,
$[3,10,4;11,5,8,12]$, $[3,14,4;16,5,10,18]$, $[3,19,4;22,5,12,23]$,
$[3,24,4;25,5,13,27]$, $[4,9,6;12,7,8,13]$, $[4,17,5;18,7,9,23]$,
$[4,20,5;26,6,8,27]$, $[5,14,7;15,6,10,19]$, $[5,21,6;23,7,10,24]$,
$[5,27,6;28,2,31,29]$, $[6,11,7;13,10,8,16]$, $[6,17,8;19,7,16,20]$,
$[6,22,7;24,9,10,25]$, $[6,29,3;30,4,28,31]$, $[7,21,8;25,9,11,27]$,
$[7,26,8;28,3,31,30]$, $[8,11,10;14,9,12,15]$, $[8,18,9;22,10,12,20]$,
$[9,15,10;16,11,12,17]$, $[9,19,11;21,10,17,26]$, $[9,28,10;30,8,23,31]$,
$[10,20,11;29,4,31,26]$, $[11,13,12;14,15,16,17]$, $[11,15,13;18,12,16,22]$,
$[11,23,13;24,12,19,25]$, $[11,28,12;30,13,14,31]$, $[12,21,13;22,14,16,25]$,
$[13,16,18;17,14,19,20]$, $[13,19,15;25,14,18,26]$, $[13,28,14;29,12,27,31]$,
$[14,20,15;21,16,19,23]$, $[15,17,19;18,20,21,22]$, $[15,23,16;24,14,26,27]$,
$[15,26,12;31,0,30,29]$, $[16,26,19;27,17,20,28]$, $[16,29,7;31,10,27,30]$,
$[17,21,18;22,19,24,23]$, $[17,24,18;25,20,22,28]$, $[17,29,8;31,18,23,30]$,
$[19,21,23;28,18,27,29]$, $[20,23,22;24,21,25,26]$, $[21,26,22;27,23,25,29]$,
$[21,30,1;31,20,27,28]$, $[24,30,5;31,22,29,26]$, $[25,28,24;27,14,30,22]$,
$[25,31,19;30,18,29,24]$, $[26,28,15;30,20,29,23]\}$.

Then (V,B) is a $\Theta(1,3,4)$-design of order 32. □

Example 5.4 *A* $\Theta(1,3,4)$*-design of order 33.*

Let $V = \mathbb{Z}_{33}$ and let B contain the copies of $\Theta(1,3,4)$ arising from the following set, cycled modulo 33.

$$\{[0,3;7,15,4,11,21],[24,23;32,30,13,18,5]\}.$$

Then (V,B) is a $\Theta(1,3,4)$-design of order 33. □

Example 5.5 *A* $\Theta(1,3,4)$*-decomposition of* $K_{3(16)}$.

Let $V = \{i \equiv 0 \pmod 3 \colon i \in \mathbb{Z}_{48}\} \cup \{i \equiv 1 \pmod 3 \colon i \in \mathbb{Z}_{48}\} \cup \{i \equiv 2 \pmod 3 \colon i \in \mathbb{Z}_{48}\}$ and let B contain the copies of $\Theta(1,3,4)$ arising from the following set, cycled modulo 48.

$$\{[0,1,3;22,6,13,5],[34,11,31;44,40,6,17]\}.$$

Then (V,B) is a $\Theta(1,3,4)$-decomposition of $K_{3(16)}$. □

Example 5.6 *A* $\Theta(1,3,4)$*-decomposition of* $K_{5(16)}$.

Let $V = \{i \equiv 0 \pmod 5 \colon i \in \mathbb{Z}_{80}\} \cup \{i \equiv 1 \pmod 5 \colon i \in \mathbb{Z}_{80}\} \cup \{i \equiv 2 \pmod 5 \colon i \in \mathbb{Z}_{80}\} \cup \{i \equiv 3 \pmod 5 \colon i \in \mathbb{Z}_{80}\} \cup \{i \equiv 4 \pmod 5 \colon i \in \mathbb{Z}_{80}\}$ and let B contain the copies of $\Theta(1,3,4)$ arising from the following set, cycled modulo 80.

$$\{[0,1,3;6,2,10,23],[0,9,23;39,1,19,43],$$
$$[0,17,48;22,49,1,29],[28,35,16;75,39,50,62]\}.$$

Then (V,B) is a $\Theta(1,2,5)$-decomposition of $K_{5(16)}$. □

6 Examples of $\Theta(2,2,4)$-designs

Example 6.1 *A* $\Theta(2,2,4)$*-design of order 16.*

Let $V = \mathbb{Z}_{16}$ and let $B =$

$$\begin{array}{lll} \{[0:1;2;3,4,7:6], & [0:4;5;6,3,1:8], & [0:7;8;11,3,5:2], \\ [0:9;15;13,5,14:8], & [0:12;14;10,13,6:9], & [1:2;5;13,11,12:10], \\ [3:9;12;8,7,5:4], & [5:6;11;9,2,4:14], & [7:1;3;13,12,2:15], \\ [8:12;13;6,4,1:14], & [10:4;6;1,12,5:15], & [10:7;9;14,3,13:15], \\ [11:2;15;1,9,7:14], & [11:4;9;10,3,2:13], & [11:6;7;8,10,15:12]\}. \end{array}$$

Then (V,B) is a $\Theta(2,2,4)$-design of order 16. □

Example 6.2 *A* $\Theta(2,2,4)$*-design of order 17.*

Let $V = \mathbb{Z}_{17}$ and let B contain the copies of $\Theta(2,2,4)$ arising from the following set, cycled modulo 17.

$$\{[3:11;15;9,10,6:8]\}.$$

Then (V,B) is a $\Theta(2,2,4)$-design of order 17. □

Example 6.3 *A* $\Theta(2,2,4)$*-decomposition of* $K_{16,16}$.

Let $V = \mathbb{Z}_{16} \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(2,2,4)$ arising from the following set, with the first components all cycled modulo 8, and the second components fixed.

$$\{[(1,0):(3,1);(5,1);(6,1),(0,0),(9,1):(2,0)],$$

$$[(6,0):(4,1);(6,1);(0,1),(8,0),(3,1):(7,0)]\}.$$

Then (V, B) is a $\Theta(2,2,4)$-decomposition of $K_{16,16}$, where V is partitioned in the obvious way. □

7 Examples of $\Theta(2,3,3)$-designs

Example 7.1 *A* $\Theta(2,3,3)$*-design of order 16.*

Let $V = \mathbb{Z}_{16}$ and let $B =$

$\{[0:1;2,3;4,6:5]$,	$[0:3;5,7;6,2:1]$,	$[0:7;8,2;11,3:9]$,
$[2:7;4,8;10,5:13]$,	$[3:15;4,1;12,0:14]$,	$[4:7;10,9;13,1:12]$,
$[6:1;3,7;8,11:10]$,	$[7:8;6,15;11,5:9]$,	$[8:3;1,11;12,2:13]$,
$[9:11;0,13;6,10:12]$,	$[11:6;2,15;4,5:12]$,	$[13:6;9,4;10,3:14]$,
$[14:7;5,8;10,0:15]$,	$[14:11;2,5;8,10:15]$,	$[14:13;9,1;12,4:15]\}$.

Then (V, B) is a $\Theta(2,3,3)$-design of order 16. □

Example 7.2 *A* $\Theta(2,3,3)$*-design of order 17.*

Let $V = \mathbb{Z}_{17}$ and let B contain the copies of $\Theta(2,3,3)$ arising from the following set, cycled modulo 17.

$$\{[13:14;5,11;9,6:16]\}.$$

Then (V, B) is a $\Theta(2,3,3)$-design of order 17. □

Example 7.3 *A* $\Theta(2,3,3)$*-design of order 32.*

Let $V = \mathbb{Z}_{32}$ and let $B =$

$\{[0:1;2,3;4,5:6]$,	$[0,3,5,1,6,2,4]$,	$[0:7;8,1;9,2:10]$,
$[0:10;11,4;15,18:20]$,	$[0:12;13,1;14,2:7]$,	$[0:16;17,1;18,2:11]$,
$[0:19;20,1;21,2:12]$,	$[0:22;23,1;24,2:15]$,	$[0:25;26,1;27,2:16]$,
$[1:2;3,5;9,4:8]$,	$[1:14;18,3;19,2:13]$,	$[1:21;22,2;24,3:17]$,
$[1:25;27,3;28,0:29]$,	$[1:29;30,2;31,3:20]$,	$[2:5;23,3;25,4:7]$,
$[3:8;9,5;10,4:12]$,	$[3:11;12,6;14,4:13]$,	$[3:15;16,4;19,5:17]$,
$[3:21;22,4;25,5:15]$,	$[3:26;28,2;30,0:31]$,	$[4:6;18,5;19,7:11]$,
$[4:21;23,5;24,6:10]$,	$[4:26;27,5;28,6:14]$,	$[4:29;30,5;31,6:16]$,
$[5:13;20,6;21,7:8]$,	$[5:22;24,7;26,2:29]$,	$[5:28;29,6;31,7:9]$,
$[6:7;15,8;17,9;14]$,	$[6:18;19,8;21,9:10]$,	$[6:22;23,7;25,8:16]$,
$[6:26;27,7;30,8:17]$,	$[7:13;15,9;18,8:20]$,	$[7:20;22,8;25,9:11]$,
$[7:26;28,8;30,9:23]$,	$[8:9;21,11;24,10:12]$,	$[9:13;16,10;18,11:15]$,
$[9:19;22,10;24,11:14]$,	$[10:11;13,12;17,14:22]$,	$[10:19;23,11;25,12:17]$,

$[10:26;27,8;28,11:29]$, $[10:29;30,11;31,9:27]$, $[12:14;15,16;18,13:21]$,
$[12:16;20,14;21,18:23]$, $[12:23;24,13;26,8:31]$, $[12:27;28,13;29,9:26]$,
$[13:16;17,18;19,11:26]$, $[13:22;23,15;25,11:31]$, $[13:27;29,14;30,12:31]$,
$[14:15;16,17;18,19:20]$, $[14:24;25,15;27,16:19]$, $[15:24;26,19;27,17:22]$,
$[15:28;29,17;30,16:24]$, $[17:23;25,18;28,14:30]$, $[18:16;22,20;24,21:28]$,
$[19:21;23,20;25,22:26]$, $[19:27;28,22;29,21:23]$, $[20:16;21,25;24,29:31]$,
$[22:21;27,18;30,17:31]$, $[24:25;27,20;31,19:30]$, $[24:30;23,29;26,25:28]$,
$[25:20;23,28;27,30:31]$, $[28:26;18,29;27,21:30]\}$.

Then (V,B) is a $\Theta(2,3,3)$-design of order 32. □

Example 7.4 *A* $\Theta(2,3,3)$*-design of order 33.*

Let $V=\mathbb{Z}_{33}$ and let B contain the copies of $\Theta(2,3,3)$ arising from the following set, cycled modulo 33.

$$\{[0:3;5,27;13,1:18],[3:4;28,22;29,6:8]\}.$$

Then (V,B) is a $\Theta(2,3,3)$-design of order 33. □

Example 7.5 *A* $\Theta(2,3,3)$*-decomposition of* $K_{3(16)}$.

Let $V=\{i\equiv 0 \pmod 3\colon i\in\mathbb{Z}_{48}\}\cup\{i\equiv 1 \pmod 3\colon i\in\mathbb{Z}_{48}\}\cup\{i\equiv 2 \pmod 3\colon i\in\mathbb{Z}_{48}\}$ and let B contain the copies of $\Theta(2,3,3)$ arising from the following set, cycled modulo 48.

$$\{[0:1;5,28;14,34:18],[44:3;4,26;33,35:22]\}.$$

Then (V,B) is a $\Theta(2,3,3)$-decomposition of $K_{3(16)}$. □

Example 7.6 *A* $\Theta(2,3,3)$*-decomposition of* $K_{5(16)}$.

Let $V=\{i\equiv 0 \pmod 5\colon i\in\mathbb{Z}_{80}\}\cup\{i\equiv 1 \pmod 5\colon i\in\mathbb{Z}_{80}\}\cup\{i\equiv 2 \pmod 5\colon i\in\mathbb{Z}_{80}\}\cup\{i\equiv 3 \pmod 5\colon i\in\mathbb{Z}_{80}\}\cup\{i\equiv 4 \pmod 5\colon i\in\mathbb{Z}_{80}\}$ and let B contain the copies of $\Theta(2,3,3)$ arising from the following set, cycled modulo 80.

$$\{[0:1;2,6;11,5:18],[0:16;18,37;22,46:73],$$

$$[0:26;37,3;39,6:55],[41:44;3,17;48,56:65]\}.$$

Then (V,B) is a $\Theta(2,3,3)$-decomposition of $K_{5(16)}$. □

8 Isolated cases for $\Theta(1,2,5)$-designs

In this section we give examples of $\Theta(1,2,5)$-designs of orders 64 and 65.

Example 8.1 *A* $\Theta(1,2,5)$*-decomposition of* $K_{4(4)}$.

Let $V = \{0,1,2,3\} \cup \{4,5,6,7\} \cup \{8,9,10,11\} \cup \{12,13,14,15\}$ and let B contain the following copies of $\Theta(1,2,5)$.

$$\begin{array}{lll}
\{[0,4;9,1,6,2,8], & [0,5;11,1,7,3,13], & [0,6;12,1,8,3,14], \\
[0,15;10,13,4,12,7], & [2,5;9,12,11,14,7], & [2,11;4,8,13,9,14], \\
[3,12;10,4,1,14,5], & [3,15;4,14,10,7,9], & [5,1;15,11,3,6,10], \\
[5,8;12,2,10,1,13], & [6,9;15,2,13,7,8], & [6,13;11,7,15,8,14]\}.
\end{array}$$

Then (V,B) is a $\Theta(1,2,5)$-decomposition of $K_{4(4)}$. □

Example 8.2 *A* $\Theta(1,2,5)$*-design of order 64.*

Let $V = \mathbb{Z}_4 \times \mathbb{Z}_{16}$. Let $(Q,*_1)$ and $(Q,*_2)$ be the quasigroups arising from two MOLS(4). Let B contain the copies of $\Theta(1,2,5)$ from the following two types of $\Theta(1,2,5)$-decompositions.
Type 1: For each i, $0 \le i \le 4$, place a $\Theta(1,2,5)$-design of order 9 on $\{i\} \times \mathbb{Z}_{16}$; such a design exists by Example 4.1.
Type 2: For each $(i,j) \in \mathbb{Z}_4 \times \mathbb{Z}_4$ place an $\Theta(1,2,5)$-decomposition of $K_{4(4)}$ on $(\{0\}\times\{4i,4i+1,4i+2,4i+3\})\cup(\{1\}\times\{4j,4j+1,4j+2,4j+3\})\cup(\{2\}\times\{4(i*_1 j),4(i*_1 j)+1,4(i*_1 j)+2,4(i*_1 j)+3\})\cup(\{3\}\times\{4(i*_2 j),4(i*_2 j)+1,4(i*_2 j)+2,4(i*_2 j)+3\})$.
Then (V,B) is a $\Theta(1,2,5)$-design of order 64. □

Example 8.3 *A* $\Theta(1,2,5)$*-design of order 65.*

Let $V = \mathbb{Z}_{65}$ and let B contain the copies of $\Theta(1,2,5)$ arising from the following set, cycled modulo 65.

$$\begin{array}{ll}
\{[0,1;6,3,11,2,12], & [0,13;27,3,18,1,19], \\
[0,16;36,1,24,2,33], & [11,36;15,4,30,32,39]\}.
\end{array}$$

Then (V,B) is a $\Theta(1,2,5)$-design of order 65. □

9 Isolated cases for $\Theta(1,3,4)$-designs

In this section we give examples of $\Theta(1,3,4)$-designs of orders 64 and 65.

Example 9.1 *A* $\Theta(1,3,4)$*-decomposition of* $K_{4(4)}$.

Let $V = \{0,1,2,3\} \cup \{4,5,6,7\} \cup \{8,9,10,11\} \cup \{12,13,14,15\}$ and let B contain the following copies of $\Theta(1,3,4)$.

$$\begin{array}{lll} \{[0,4,1;6,3,5,8], & [0,5,2;7,8,1,9], & [0,10,2;11,3,4,12], \\ [0,13,1;15,8,3,14], & [6,12,3;15,5,1,14], & [7,11,4;10,12,2,13], \\ [8,4,9;12,1,11,6], & [8,14,10;13,6,9,2], & [9,5,11;15,2,14,7], \\ [9,14,4;13,5,10,3], & [10,1,7;15,4,2,6], & [11,14,5;12,7,3,13]\}. \end{array}$$

Then (V,B) is a $\Theta(1,3,4)$-decomposition of $K_{4(4)}$. □

Example 9.2 *A* $\Theta(1,3,4)$*-design of order 64.*

Let $V = \mathbb{Z}_4 \times \mathbb{Z}_{16}$. Let $(Q,*_1)$ and $(Q,*_2)$ be the quasigroups arising from two MOLS(4). Let B contain the copies of $\Theta(1,3,4)$ from the following two types of $\Theta(1,3,4)$-decompositions.

Type 1: For each i, $0 \le i \le 4$, place a $\Theta(1,3,4)$-design of order 9 on $\{i\} \times \mathbb{Z}_{16}$; such a design exists by Example 5.1.

Type 2: For each $(i,j) \in \mathbb{Z}_4 \times \mathbb{Z}_4$ place an $\Theta(1,3,4)$-decomposition of $K_{4(4)}$ on $(\{0\}\times\{4i,4i+1,4i+2,4i+3\})\cup(\{1\}\times\{4j,4j+1,4j+2,4j+3\})\cup(\{2\}\times\{4(i*_1j),4(i*_1 j)+1,4(i*_1j)+2,4(i*_1j)+3\})\cup(\{3\}\times\{4(i*_2j),4(i*_2j)+1,4(i*_2j)+2,4(i*_2j)+3\})$.

Then (V,B) is a $\Theta(1,3,4)$-design of order 64. □

Example 9.3 *A* $\Theta(1,3,4)$*-design of order 65.*

Let $V = \mathbb{Z}_{65}$ and let B contain the copies of $\Theta(1,3,4)$ arising from the following set, cycled modulo 65.

$$\begin{array}{ll} \{[0,2,5;10,1,8,19], & [0,8,20;33,4,18,38], \\ [0,15,39;22,40,3,26], & [60,54,29;64,63,32,16]\}. \end{array}$$

Then (V,B) is a $\Theta(1,3,4)$-design of order 65. □

10 Isolated cases for $\Theta(2,3,3)$-designs

In this section we give examples of $\Theta(2,3,3)$-designs of orders 64 and 65.

Example 10.1 *A* $\Theta(2,3,3)$*-decomposition of* $K_{4(4)}$.

Let $V = \{0,1,2,3\} \cup \{4,5,6,7\} \cup \{8,9,10,11\} \cup \{12,13,14,15\}$ and let B contain the following copies of $\Theta(2,3,3)$.

$$\begin{array}{lll}
\{[0:4;5,1;6,2:9], & [0:7;8,2;9,3:11], & [1:4;6,9;7,2:14], \\
[1:11;12,2;14,3:13], & [4:8;11,6;15,1:13], & [4:12;2,15;13,0:10], \\
[7:8;13,9;15,0:12], & [7:14;3,15;12,5:11], & [8:1;5,13;6,3:10], \\
[8:14;3,4;15,5:10], & [9:7;5,2;15,6:10], & [12:6;3,5;11,0:14]\}.
\end{array}$$

Then (V,B) is a $\Theta(2,2,3)$-decomposition of $K_{4(4)}$. □

Example 10.2 *A* $\Theta(2,3,3)$*-design of order 64.*

Let $V = \mathbb{Z}_4 \times \mathbb{Z}_{16}$. Let $(Q,*_1)$ and $(Q,*_2)$ be the quasigroups arising from two MOLS(4). Let B contain the copies of $\Theta(2,3,3)$ from the following two types of $\Theta(2,3,3)$-decompositions.
Type 1: For each i, $0 \le i \le 4$, place a $\Theta(2,3,3)$-design of order 9 on $\{i\} \times \mathbb{Z}_{16}$; such a design exists by Example 7.1.
Type 2: For each $(i,j) \in \mathbb{Z}_4 \times \mathbb{Z}_4$ place an $\Theta(2,3,3)$-decomposition of $K_{4(4)}$ on $(\{0\}\times\{4i,4i+1,4i+2,4i+3\})\cup(\{1\}\times\{4j,4j+1,4j+2,4j+3\})\cup(\{2\}\times\{4(i*_1 j),4(i*_1 j)+1,4(i*_1 j)+2,4(i*_1 j)+3\})\cup(\{3\}\times\{4(i*_2 j),4(i*_2 j)+1,4(i*_2 j)+2,4(i*_2 j)+3\})$.
Then (V,B) is a $\Theta(2,3,3)$-design of order 64. □

Example 10.3 *A* $\Theta(2,3,3)$*-design of order 65.*

Let $V = \mathbb{Z}_{65}$ and let B contain the copies of $\Theta(2,3,3)$ arising from the following set, cycled modulo 65.

$$\begin{array}{ll}
\{[0:1;5,12;8,17:29], & [0:11;13,27;16,34:53], \\
[0:20;30,8;36,11:52], & [42:48;15,11;32,47:45]\}.
\end{array}$$

Then (V,B) is a $\Theta(2,3,3)$-design of order 65. □

11 Examples of $\Theta(1,2,6)$-designs

Example 11.1 *A* $\Theta(1,2,6)$*-design of order 9.*

Let $V = \mathbb{Z}_9$ and let $B =$

$$\{[0,5;7,6,8,2,4,3],[0,8;4,5,6,2,3,1],[1,6;4,7,3,8,5,2],[1,8;7,2,0,6,3,5]\}.$$

Then (V, B) is a $\Theta(1,2,6)$-design of order 9. □

Example 11.2 *A* $\Theta(1,2,6)$*-design of order 10.*

Let $V = \mathbb{Z}_5 \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(1,2,6)$ arising from the following set, with the first components all cycled modulo 5, and the second components fixed.

$$\{[(0,0),(1,0);(0,1),(3,0),(1,1),(4,1),(3,1),(2,0)]\}.$$

Then (V, B) is a $\Theta(1,2,6)$-design of order 10. □

Example 11.3 *A* $\Theta(1,2,6)$*-design of order 18.*

Let $V = \mathbb{Z}_{18}$ and let $B =$

$\{[0,1;3,4,5,2,6,7]$,	$[0,2;16,1,4,6,8,9]$,	$[0,4;10,1,5,3,7,11]$,
$[0,5;14,10,7,4,11,6]$,	$[0,8;12,1,6,15,3,13]$,	$[1,8;11,3,12,6,16,15]$,
$[4,2;13,6,14,11,16,17]$,	$[5,9;10,11,12,15,4,16]$,	$[7,8;13,4,12,5,13,16]$,
$[8,10;15,0,17,14,16,3]$,	$[9,2;17,1,14,12,13,7]$,	$[9,3;14,2,1,13,8,4]$,
$[6,3;10,2,7,5,11,17]$,	$[9,16;12,2,3,17,7,1]$,	$[12,10;17,13,11,9,15,7]$,
$[15,5;17,8,16,10,13,14]$,	$[15,11;2,8,5,6,9,13]\}$.	

Then (V, B) is a $\Theta(1,2,6)$-design of order 18. □

Example 11.4 *A* $\Theta(1,2,6)$*-design of order 19.*

Let $V = \mathbb{Z}_{19}$ and let B contain the copies of $\Theta(1,2,6)$ arising from the following set, cycled modulo 19.

$$\{[0,1;3,7,2,8,16,9].$$

Then (V, B) is a $\Theta(1,2,6)$-design of order 19. □

Example 11.5 *A* $\Theta(1,2,6)$*-decomposition of* $K_{3(3)}$.

Let $V = \{0,1,2\} \cup \{3,4,5\} \cup \{6,7,8\}$ and let $B = \{[0,3;6,1,7,2,8,5],[0,7;4,1,5,2,3,8],[4,2;6,5,7,3,1,8]\}$.
Then (V, B) is a $\Theta(1,2,6)$-decomposition of $K_{3(3)}$. □

12 Examples of $\Theta(1,3,5)$-designs

Example 12.1 *A $\Theta(1,3,5)$-design of order 9.*

Let $V = \mathbb{Z}_9$ and let $B = \{[0,5,6;7,4,3,2,1],\ [0,8,5;2,4,1,6,3],\ [2,6,4;8,3,5,1,7],\ [7,3,1;8,6,0,5,4]\}$.

Then (V,B) is a $\Theta(1,3,5)$-design of order 9. □

Example 12.2 *A $\Theta(1,3,5)$-design of order 10.*

Let $V = \mathbb{Z}_5 \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(1,3,5)$ arising from the following set, with the first components all cycled modulo 5, and the second components fixed.

$$\{[(0,0),(0,1),(4,1);(1,1),(4,0),(2,1),(3,0),(1,0)]\}.$$

Then (V,B) is a $\Theta(1,3,5)$-design of order 10. □

Example 12.3 *A $\Theta(1,3,5)$-decomposition of $K_{9,9}$.*

Let $V = \mathbb{Z}_9 \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(1,3,5)$ arising from the following set, with the first components all cycled modulo 9, and the second components fixed.

$$\{[(0,0),(3,1),(1,0);(0,1),(5,0),(6,1),(8,0),(5,1)]\}.$$

Then (V,B) is a $\Theta(1,3,5)$-decomposition of $K_{9,9}$, where V is partitioned in the obvious way. □

13 Examples of $\Theta(1,4,4)$-designs

Example 13.1 *A $\Theta(1,4,4)$-design of order 10.*

Let $V = \mathbb{Z}_5 \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, with the first components all cycled modulo 5, and the second components fixed.

$$\{[(0,0),(1,0),(3,0),(0,1);(1,1),(4,1),(4,0),(3,1)]\}.$$

Then (V,B) is a $\Theta(1,4,4)$-design of order 10.

Example 13.2 *A $\Theta(1,4,4)$-design of order 18.*

Let $V = \mathbb{Z}_{18}$ and let $B =$

$\{[0,1,2,3;4,5,7,6]$, $[0,2,4,6;8,1,3,5]$, $[0,3,7,2;9,1,5,10]$,
$[0,7,4,8;12,1,6,13]$, $[0,11,1,10;14,2,5,17]$, $[1,13,2,6;14,4,11,15]$,
$[2,8,5,9;11,7,10,12]$, $[3,11,5,6;15,9,14,12]$, $[5,12,4,16;14,7,8,13]$,
$[6,9,17,3;10,11,13,16]$, $[7,13,4,15;12,9,3,16]$, $[8,9,16,1;17,12,11,14]$,
$[8,10,9,7;15,5,16,11]$, $[10,2,15,0;16,17,14,13]$, $[10,4,1,7;17,2,16,15]$,
$[12,6,11,17;13,3,8,16]$, $[15,13,9,4;17,6,3,14]\}$.

Then (V,B) is a $\Theta(1,4,4)$-design of order 18. □

Example 13.3 *A $\Theta(1,4,4)$-design of order 19.*

Let $V = \mathbb{Z}_{19}$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, cycled modulo 19.

$$\{[0,1,3,6;10,4,12,5]\}.$$

Then (V,B) is a $\Theta(1,4,4)$-design of order 19. □

Example 13.4 *A $\Theta(1,4,4)$-design of order 27.*

Let $V = \mathbb{Z}_3 \times \mathbb{Z}_9$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, with the first components all cycled modulo 3, and the second components fixed.

$$\begin{array}{l}
\{[(0,0),(2,2),(0,3),(1,0);(0,4),(2,0),(2,3),(1,6)],\\
[(0,0),(1,6),(2,5),(0,7);(2,6),(2,2),(1,5),(2,8)],\\
[(0,4),(1,5),(2,3),(2,1);(0,7),(0,6),(1,8),(1,2)],\\
[(0,4),(1,7),(1,0),(0,7);(2,8),(0,6),(1,5),(0,1)],\\
[(0,6),(0,3),(0,8),(2,7);(2,2),(2,4),(1,4),(0,1)],\\
[(0,8),(2,8),(0,2),(1,7);(1,5),(1,8),(1,0),(1,1)],\\
[(1,0),(0,1),(1,8),(1,6);(2,1),(1,4),(0,5),(2,0)],\\
[(1,0),(2,3),(0,7),(2,7);(1,5),(2,5),(2,1),(2,8)],\\
[(1,1),(0,5),(1,0),(1,2);(2,1),(2,2),(1,3),(2,3)],\\
[(1,2),(2,2),(0,8),(1,4);(1,3),(2,8),(2,7),(0,1)],\\
[(1,4),(0,6),(1,6),(1,3);(1,5),(1,2),(2,1),(0,7)],\\
[(2,1),(1,3),(1,4),(1,3);(2,7),(1,6),(0,4),(0,6)],\\
[(2,3),(2,7),(0,2),(2,6);(2,5),(2,2),(0,4),(1,8)]\}.
\end{array}$$

Then (V,B) is a $\Theta(1,4,4)$-design of order 27. □

Example 13.5 *A $\Theta(1,4,4)$-design of order 45.*

Let $V = (\mathbb{Z}_{11} \times \mathbb{Z}_4) \cup \{\infty\}$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, with the first components all cycled modulo 11, and the second components fixed.

$$\begin{array}{l}
\{[(0,0),(3,0),(8,0),(0,1);(6,1),(7,0),(3,1),(5,2)],\\
[(0,0),(0,2),(1,0),(4,2);(6,2),(9,0),(5,2),(6,3)],\\
[(0,1),(2,1),(1,2),(0,2);(4,1),(5,2),(2,2),(6,2)],\\
[(0,1),(4,2),(10,1),(1,3);(7,3),(9,0),(6,3),(5,3)],\\
[(1,1),(10,0),(0,1),(0,2);(2,1),(8,0),(4,0),(4,1)],\\
[(5,0),(5,3),(9,0),(0,2);(4,3),(10,2),(4,2),(6,3)],\\
[(5,0),(9,3),(3,1),(3,3);(2,1),(1,3),(3,2),(10,3)],\\
[(8,3),(10,1),(6,0),(4,3);(10,3),(10,2),(9,0),(0,3)],\\
[(3,1),(5,0),(3,0),(4,0);(7,3),(0,3),\infty,(0,2)],\\
[(4,2),(1,3),(9,2),(6,1);(3,3),(0,1),\infty,(0,0)]\}.
\end{array}$$

Then (V,B) is a $\Theta(1,4,4)$-design of order 45. □

Example 13.6 *A* $\Theta(1,4,4)$*-design of order 63.*

Let $V = (\mathbb{Z}_{31} \times \mathbb{Z}_2) \cup \{\infty\}$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, with the first components all cycled modulo 31, and the second components fixed.

$$\begin{array}{l}\{[(0,0),(1,0),(3,0),(6,0);(10,0),(4,0),(9,0),(16,0)],\\ [(0,0),(9,0),(20,0),(1,0);(14,0),(2,1),(2,0),(5,1)],\\ [(0,0),(4,1),(5,0),(0,1);(8,1),(1,0),(7,1),(9,1)],\\ [(0,0),(14,1),(18,0),(2,1);(24,1),(19,1),(15,1),(16,1)],\\ [(6,0),(18,1),(3,1),(2,0);(24,1),(30,0),(1,1),(29,1)],\\ [(13,1),(7,1),(21,0),(13,0);(26,1),(14,1),(3,1),(27,1)],\\ [(0,0),(10,1),(12,0),(1,1);(11,1),(14,0),(4,1),\infty]\}.\end{array}$$

Then (V,B) is a $\Theta(1,4,4)$-design of order 63. □

Example 13.7 *A* $\Theta(1,4,4)$*-decomposition of* $K_{3(3)}$.

Let $V = \{0,1,2\} \cup \{3,4,5\} \cup \{6,7,8\}$ and let $B = \{[0,3,1,6;4,7,2,5], [0,6,3,2;8,1,5,7], [4,1,7,3;8,5,6,2]\}$.

Then (V,B) is a $\Theta(1,4,4)$-decomposition of $K_{3(3)}$. □

Example 13.8 *A* $\Theta(1,4,4)$*-decomposition of* $K_{3(9)}$.

Let $V = \{i \equiv 0 \pmod 3 \colon i \in \mathbb{Z}_{27}\} \cup \{i \equiv 1 \pmod 3 \colon i \in \mathbb{Z}_{27}\} \cup \{i \equiv 2 \pmod 3 \colon i \in \mathbb{Z}_{27}\}$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, cycled modulo 27.

$$\{[0,1,6,17;13,20,12,10]\}.$$

Then (V,B) is a $\Theta(1,4,4)$-decomposition of $K_{3(9)}$. □

14 Examples of $\Theta(2,2,5)$-designs

Example 14.1 *A* $\Theta(2,2,5)$*-design of order 18.*

Let $V = \mathbb{Z}_{18}$ and let $B =$

$$\begin{array}{lll}\{[0:1;2;3,4,7,6:5], & [0:4;5;6,2,1,3:8], & [0:7;8;9,1,4,2:12],\\ [0:10;11;12,1,7,2:3], & [0:13;14;15,1,10,2:8], & [1:11;13;17,0,16,2:9],\\ [2:11;13;14,3,7,8:10], & [3:5;6;16,9,17,11:13], & [3:9;12;15,2,17,8:6],\\ [4:12;15;16,6,11,14:13], & [5:11;15;9,4,17,16:8], & [5:12;17;10,9,8,1:14],\\ [7:5;16;9,12,11,15:14], & [10:4;16;15,6,17,7:11], & [10:7;12;14,6,1,16:15],\\ [13:4;7;3,17,15,9:14], & [16:12;13;5,4,6,10:17]\}. & \end{array}$$

Then (V,B) is a $\Theta(2,2,5)$-design of order 18. □

Example 14.2 *A $\Theta(2,2,5)$-design of order 19.*

Let $V = \mathbb{Z}_{19}$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, cycled modulo 19.

$$\{[0:1;2;3,7,12,4:14]\}.$$

Then (V,B) is a $\Theta(2,2,5)$-design of order 19. □

Example 14.3 *A $\Theta(2,2,5)$-design of order 27.*

Let $V = \mathbb{Z}_3 \times \mathbb{Z}_9$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 3, and the second components fixed.

$$\begin{array}{l}
\{[(0,1):(1,1);(1,2);(0,5),(1,0),(0,6),(2,0):(2,7)],\\
[(0,0):(2,4);(2,7);(1,5),(1,1),(0,2),(1,2):(1,4)],\\
[(0,1):(0,2);(2,5);(1,3),(0,7),(1,5),(1,6):(2,4)],\\
[(0,1):(0,3);(0,6);(2,7),(2,5),(1,2),(2,6):(2,3)],\\
[(0,3):(1,4);(0,8);(2,6),(2,0),(2,8),(1,8):(2,5)],\\
[(0,8):(0,4);(0,7);(1,2),(0,5),(1,4),(2,6):(2,8)],\\
[(1,1):(0,4);(1,7);(1,8),(2,1),(2,0),(0,8):(1,3)],\\
[(1,2):(1,3);(1,6);(1,5),(1,8),(0,1),(1,0):(2,7)],\\
[(1,3):(0,2);(0,5);(1,0),(2,3),(2,4),(1,1):(2,6)],\\
[(1,6):(0,7);(2,8);(2,1),(0,2),(1,0),(1,4):(1,7)],\\
[(2,1):(0,1);(1,3);(2,4),(2,6),(1,6),(0,5):(1,5)],\\
[(2,2):(2,0);(0,4);(2,7),(1,5),(0,3),(1,2):(1,8)],\\
[(2,7):(0,2);(2,6);(0,4),(2,0),(1,0),(0,3):(1,8)]\}.
\end{array}$$

Then (V,B) is a $\Theta(2,2,5)$-design of order 27. □

Example 14.4 *A $\Theta(2,2,5)$-decomposition of $K_{3(9)}$.*

Let $V = \{i \equiv 0 \pmod 3\colon i \in \mathbb{Z}_{27}\} \cup \{i \equiv 1 \pmod 3\colon i \in \mathbb{Z}_{27}\} \cup \{i \equiv 2 \pmod 3\colon i \in \mathbb{Z}_{27}\}$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, cycled modulo 27.

$$\{[18:7;11;23,21,20,16:24]\}$$

Then (V,B) is a $\Theta(2,2,3)$-decomposition of $K_{3(9)}$. □

Example 14.5 *A $\Theta(2,2,5)$-design of order 28.*

Let $V = (\mathbb{Z}_9 \times \mathbb{Z}_3) \cup \{\infty\}$. Let B contain the copies of $\Theta(2,2,5)$ from the following two types of $\Theta(2,2,5)$-decompositions.
Type 1: For each $i \in \mathbb{Z}_3$, place a $\Theta(2,2,5)$-design of order 10 on $(\{i\} \times \mathbb{Z}_9) \cup \{\infty\}$.
Type 2: Place a $\Theta(2,2,5)$-decomposition of $K_{3(9)}$ on $\mathbb{Z}_9 \times \mathbb{Z}_3$.
Then (V,B) is a $\Theta(2,2,5)$-design of order 28. □

Example 14.6 *A* $\Theta(2,2,5)$*-design of order 36.*

Let $V = (\mathbb{Z}_7 \times \mathbb{Z}_5) \cup \{\infty\}$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 7, and the second components fixed.

$$\begin{array}{l}
\{[(0,0):(2,0);(3,0);(2,1),(4,0),(0,1),(3,1):(1,2)],\\
[(0,0):(4,1);(0,2);(1,2),(5,0),(1,3),(3,0):(4,3)],\\
[(0,1):(5,1);(5,3);(4,2),(6,3),(6,0),(0,4):(2,4)],\\
[(0,2):(1,1);(6,2);(4,1),(2,4),(3,0),(2,3):(1,4)],\\
[(0,2):(2,2);(6,3);(5,1),(4,3),(2,3),(6,1):(6,2)],\\
[(1,0):(1,4);(3,4);(4,4),(6,2),(0,4),(6,4):(3,3)],\\
[(3,4):(0,2);(4,2);(5,0),(6,1),(2,4),(3,1):(4,4)],\\
[(4,0):(4,1);(1,3);(1,4),(4,4),(3,3),(6,3):(5,2)],\\
[(0,3):\infty;(5,1);(1,3),(0,1),(1,1),(5,3):(0,4)],\\
[\infty:(0,0);(0,1);(0,2),(5,3),(3,0),(5,2):(1,0)]\}.
\end{array}$$

Then (V, B) is a $\Theta(2,2,5)$-design of order 36. □

Example 14.7 *A* $\Theta(2,2,5)$*-design of order 37.*

Let $V = \mathbb{Z}_{37}$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, cycled modulo 37

$$\{[0:1;2;10,22,3,19:5],[13:24;26;28,33,16,23:32]\}.$$

Then (V, B) is a $\Theta(2,2,5)$-design of order 37. □

Example 14.8 *A* $\Theta(2,2,5)$*-design of order 45.*

Let $V = (\mathbb{Z}_{11} \times \mathbb{Z}_4) \cup \{\infty\}$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 11, and the second components fixed.

$$\begin{array}{l}
\{[(0,0):(1,0);(2,0);(3,0),(8,0),(1,1),(5,0):(3,1)],\\
[(0,0):(0,1);(5,1);(0,2),(4,0),(6,2),(2,0):(8,2)],\\
[(0,0):(1,3);(7,3);(5,2),(0,1),(1,1),(3,1):(8,1)],\\
[(0,1):(3,1);(7,2);(4,1),(2,2),(2,3),(8,2):(9,3)],\\
[(0,2):(6,2);(8,3);(3,3),(6,3),(1,0),(9,1):(2,2)],\\
[(7,1):(4,0);(10,3);(9,2),(1,0),(5,0),(0,3):(10,1)],\\
[(7,2):(9,0);(6,2);(4,0),(5,2),(2,2),(7,1):(8,2)],\\
[(9,3):(1,0);(1,1);(2,2),(1,3),(7,1),(6,2):(10,3)],\\
[(0,0):\infty;(2,3);(10,3),(4,3),(5,3),(1,0):(0,1)],\\
[(0,2):\infty;(9,3);(0,1),(7,3),(4,0),(4,3):(0,3)]\}.
\end{array}$$

Then (V, B) is a $\Theta(2,2,5)$-design of order 45. □

Example 14.9 *A* $\Theta(2,2,5)$*-design of order 46.*

Let $V = \mathbb{Z}_{23} \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 23, and the second components fixed.

$$\{[(0,0):(2,0);(4,0);(5,0),(12,0),(1,0),(9,0):(1,1)],$$
$$[(0,0):(9,0);(10,0);(0,1),(4,0),(5,1),(1,0):(3,1)],$$
$$[(0,0):(3,1);(5,1);(6,1),(11,0),(0,1),(10,0):(19,1)],$$
$$[(0,0):(8,1);(10,1);(11,1),(1,1),(3,1),(7,1):(13,1)],$$
$$[(14,0):(12,1);(21,1);(15,0),(12,0),(18,0),(9,1):(20,1)]\}.$$

Then (V,B) is a $\Theta(2,2,5)$-design of order 46. □

Example 14.10 *A* $\Theta(2,2,5)$*-design of order 63.*

Let $V = (\mathbb{Z}_{31} \times \mathbb{Z}_2) \cup \{\infty\}$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 31, and the second components fixed.

$$\{[(0,0):(1,0);(4,0);(10,0),(24,0),(9,1),(3,0):(0,1)],$$
$$[(0,1):(1,1);(18,1);(5,0),(7,1),(3,0),(23,1):(13,1)],$$
$$[(5,0):(19,1);(30,1);(20,0),(8,0),(2,0),(15,1):(21,1)],$$
$$[(10,0):(13,1);(15,1);(12,0),(21,0),(0,1),(30,0):(25,0)],$$
$$[(22,0):(9,0);(30,0);(15,1),(28,0),(14,1),(6,1):(21,1)],$$
$$[(16,1):(23,1);(30,1);(7,0),(4,0),(11,1),(19,0):(27,1)],$$
$$[(0,0):\infty;(11,1);(11,0),(9,1),(9,0),(16,0):(0,1)]\}.$$

Then (V,B) is a $\Theta(2,2,5)$-design of order 63. □

Example 14.11 *A* $\Theta(2,2,5)$*-design of order 64.*

Let $V = (\mathbb{Z}_9 \times \mathbb{Z}_7) \cup \{\infty\}$. Let B contain the copies of $\Theta(2,2,5)$ from the following two types of $\Theta(2,2,5)$-decompositions.
Type 1: For each $i \in \mathbb{Z}_7$, place a $\Theta(2,2,5)$-design of order 10 on $(\{i\} \times \mathbb{Z}_9) \cup \{\infty\}$.
Type 2: For each triple, $\{a,b,c\}$ say, in an STS(7), place a $\Theta(2,2,5)$-decomposition of $K_{3(9)}$ on $\mathbb{Z}_9 \times \{a,b,c\}$.
Then (V,B) is a $\Theta(2,2,5)$-design of order 64. □

Example 14.12 *A* $\Theta(2,2,5)$*-decomposition of* $K_{3(6)}$.

Let $V = \{0,1,2,3,4,5\} \cup \{6,7,8,9,10,11\} \cup \{12,13,14,15,16,17\}$ and let $B =$

$$\begin{array}{lll}\{[0:6;7;8,3,9,12:2], & [0:9;10;11,1,15,4:16], & [0:12;16;14,7,5,10:3],\\ [0:13;17;15,11,16,1:10], & [2:9;17;11,14,10,12:4], & [3:11;14;15,5,16,7:4],\\ [6:1;14;3,7,15,2:8], & [6:4;15;17,9,14,5:8], & [10:2;4;15,9,5,11:13],\\ [12:1;11;6,5,13,7:17], & [12:5;8;7,1,13,3:17], & [13:6;8;9,1,14,2:16].\end{array}$$

Then (V,B) is a $\Theta(2,2,5)$-decompostion of $K_{3(6)}$. □

15 Examples of $\Theta(2,3,4)$-designs

Example 15.1 *A* $\Theta(2,3,4)$*-design of order 9.*

Let $V = \mathbb{Z}_9$ and let $B =$

$$\{[0:1;2,3;4,5,6:7],[0:2;4,3;1,6,8:5],[4:7;8,1;6,3,0:5],[6:4;1,3;0,7,2:8]\}.$$

Then (V,B) is a $\Theta(2,3,4)$-design of order 9. □

Example 15.2 *A* $\Theta(2,3,4)$*-design of order 10.*

noindent

Let $V = \mathbb{Z}_5 \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(2,3,4)$ arising from the following set, with the first components all cycled modulo 5, and the second components fixed.

$$\{[(0,0):(1,0);(2,0),(0,1);(1,1),(2,1),(3,0):(3,2)]\}$$

.

Then (V,B) is a $\Theta(2,3,4)$-decomposition of K_{10}. □

Example 15.3 *A* $\Theta(2,3,4)$*-design of order 18.*

Let $V = \mathbb{Z}_{18}$ and let $B =$

$\{[0:1;2,3;4,5,9:6]$, $[0:3;5,1;6,2,9:4]$, $[0:7;8,1;9,3,11:14]$,
$[0:10;11,1;12,2,16:17]$, $[0:13;14,3;15,4,6:8]$, $[1:2;3,13;12,10,6:14]$,
$[2:5;11,9;17,7,10:14]$, $[3:7;17,6;16,8,12:15]$, $[4:10;7,1;8,17,0:16]$,
$[4:17;14,8;13,6,16:9]$, $[5:3;10,9;17,13,1:15]$, $[7:11;9,1;5,15,13:10]$,
$[7:12;6,11;8,10,15:16]$, $[7:13;16,4;2,8,15:11]$, $[12:9;6,5;3,10,2:13]$,
$[12:13;17,14;11,8,5:16]$, $[12:14;4,2;5,11,17:15]\}$.

Then (V,B) is a $\Theta(2,3,4)$-design of order 18. □

Example 15.4 *A* $\Theta(2,3,4)$*-design of order 19.*

Let $V = \mathbb{Z}_{19}$ and let B contain the copies of $\Theta(2,3,4)$ arising from the following set, cycled modulo 19.

$$\{[0:1;2,5;4,9,3:12]\}$$

.

Then (V,B) is a $\Theta(2,3,4)$-design of order 19. □

Example 15.5 *A* $\Theta(2,3,4)$*-decomposition of* $K_{3(3)}$.

Let $V = \{0,1,2\} \cup \{3,4,5\} \cup \{6,7,8\}$ and let $B =$

$$\{[0:3;4,6;7,5,8:1],[0:5;6,3;8,4,7:2],[1:4;5,6;7,3,8:2]\}.$$

Then (V,B) is a $\Theta(2,3,4)$-decomposition of $K_{3(3)}$. □

16 Examples of $\Theta(3,3,3)$-designs

Example 16.1 *A $\Theta(3,3,3)$-design of order 10.*

Let $V = \mathbb{Z}_5 \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(3,3,3)$ arising from the following set, with the first components all cycled modulo 5, and the second components fixed.

$$\{[(2,0):(3,0),(0,0);(2,1),(0,1);(3,1),(1,0):(4,1)]\}.$$

Then (V,B) is a $\Theta(3,3,3)$-design of order 10. □

Example 16.2 *A $\Theta(3,3,3)$-design of order 18.*

Let $V = \mathbb{Z}_{18}$ and let $B =$

$\{[0:1,2;3,4;5,6:7]$, $[0:2,4;1,5;3,7:8]$, $[0:4,6;1,7;9,10:15]$,
$[0:8,2;5,6;11,12:7]$, $[0:13,1;9,2;15,16:14]$, $[1:10,3;6,8;11,12:9]$,
$[2:7,10;5,8;12,16:17]$, $[4:15,14;13,8;16,17:12]$, $[5:11,14;17,6;13,16:1]$,
$[7:11,3;14,4;13,15:8]$, $[8:3,15;11,4;10,17:13]$, $[9:3,1;14,7;4,10:2]$,
$[9:8,4;12,3;13,15:5]$, $[10:6,15;17,0;14,16:3]$, $[11:2,6;14,5;9,16:12]$,
$[11:10,13;16,6;12,17:7]$, $[15:12,10;17,2;13,16:9]\}$.

Then (V,B) is a $\Theta(3,3,3)$-design of order 18. □

Example 16.3 *A $\Theta(3,3,3)$-design of order 27.*

Let $V = \mathbb{Z}_3 \times \mathbb{Z}_9$ and let B contain the copies of $\Theta(3,3,3)$ arising from the following set, with the first components all cycled modulo 3, and the second components fixed.

$$\begin{aligned}
\{&[(0,0):(1,0),(0,1);(2,0),(2,1);(1,2),(0,8):(1,3)],\\
&[(0,1):(1,1),(2,4);(0,7),(1,8);(1,2),(1,3):(2,1)],\\
&[(0,2):(0,3),(1,2);(0,5),(1,1);(0,4),(1,4):(2,0)],\\
&[(0,3):(2,3),(1,5);(1,6),(1,1);(1,4),(0,5):(0,1)],\\
&[(1,2):(1,0),(2,3);(0,8),(0,7);(1,1),(0,4):(2,4)],\\
&[(1,2):(0,2),(1,0);(1,5),(0,4);(2,5),(2,7):(2,0)],\\
&[(1,6):(0,2),(2,8);(0,7),(1,4);(2,3),(0,8):(0,3)],\\
&[(2,0):(0,4),(0,8);(2,2),(2,6);(2,4),(0,7):(2,5)],\\
&[(2,0):(1,6),(2,2);(0,3),(0,4);(2,6),(0,8):(0,1)],\\
&[(2,1):(2,7),(2,4);(2,5),(0,5);(0,8),(1,8):(1,6)],\\
&[(2,5):(1,1),(0,6);(1,6),(0,3);(2,7),(1,8):(0,5)],\\
&[(2,6):(1,0),(1,3);(2,7),(0,5);(2,3),(1,7):(2,2)],\\
&[(2,7):(1,6),(2,8);(0,8),(1,4);(2,6),(0,7):(1,0)]\}.
\end{aligned}$$

Then (V,B) is a $\Theta(3,3,3)$-design of order 27. □

Example 16.4 *A $\Theta(3,3,3)$-decomposition of $K_{9,9}$.*

Let $V = \mathbb{Z}_9 \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(3,3,3)$ arising from the following set, with the first components all cycled modulo 9, and the second components fixed.

$$\{[(0,0):(0,1),(5,0);(1,1),(7,0);(2,1),(6,0):(8,1)]\}.$$

Then (V,B) is a $\Theta(3,3,3)$-decomposition of $K_{9,9}$, where V is partitioned in the obvious way. □

17 Isolated Cases for $\Theta(1,2,6)$-designs

In this section we give examples of $\Theta(1,2,6)$-designs of orders 36 and 37.

Example 17.1 *A* $\Theta(1,2,6)$*-decomposition of* $K_{4(3)}$.

Let $V = \{0,1,2\} \cup \{3,4,5\} \cup \{6,7,8\} \cup \{9,10,11\}$ and let $B = \{[0,3;6,1,4,2,5,7]$, $[1,5;8,0,4,6,2,7]$, $[2,8;10,6,9,1,3,11]$, $[3,2;9,0,10,1,11,7]$, $[4,9;7,10,5,0,11,8]$, $[5,6;11,4,10,3,8,9]\}$.
Then (V,B) is a $\Theta(1,2,6)$-decomposition of $K_{4(3)}$. □

Example 17.2 *A* $\Theta(1,2,6)$*-design of order 36.*

Let $V = \mathbb{Z}_4 \times \mathbb{Z}_9$. Let $(Q,*_1)$ and $(Q,*_2)$ be the quasigroups arising from two MOLS(3). Let B contain the copies of $\Theta(1,2,6)$ from the following two types of $\Theta(1,2,6)$-decompositions.
Type 1: For each i, $0 \leq i \leq 3$, place a $\Theta(1,2,6)$-design of order 9 on $\{i\} \times \mathbb{Z}_9$.
Type 2: For each $(i,j) \in \mathbb{Z}_3 \times \mathbb{Z}_3$ place an $\Theta(1,2,6)$-decomposition of $K_{4(3)}$ on $\{0\} \times \{3i, 3i+1, 3i+2\} \cup \{1\} \times \{3j, 3j+1, 3j+2\} \cup \{2\} \times \{3(i *_1 j), 3(i *_1 j)+1, 3(i *_1 j)+2\} \cup \{3\} \times \{(i *_2 j), 3(i *_2 j)+1, 3(i *_2 j)+2\}$.
Then (V,B) is a $\Theta(1,2,6)$-design of order 36. □

Example 17.3 *A* $\Theta(1,2,6)$*-design of order 37.*

Let $V = \mathbb{Z}_{37}$ and let B contain the copies of $\Theta(1,2,6)$ arising from the following set, cycled modulo 37.

$$\{[0,1;4,11,19,5,14,24],[23,7;29,9,27,22,33,21]\}.$$

Then (V,B) is a $\Theta(1,2,6)$-design of order 37. □

18 Isolated Cases for $\Theta(1,4,4)$-designs

In this section we give examples of $\Theta(1,4,4)$-designs of orders 36, 37, 99 and 117.

Example 18.1 *A $\Theta(1,4,4)$-design of order 36.*

Let $V = (\mathbb{Z}_7 \times \mathbb{Z}_5) \cup \{\infty\}$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, with the first components all cycled modulo 7, and the second components fixed.

$$\begin{array}{l}
\{[(0,0),(0,1),(2,0),(2,2);(3,2),(2,1),(5,1),(6,3)],\\
[(0,0),(2,3),(0,1),(0,2);(5,3),(2,1),(0,3),(0,4)],\\
[(0,2),(2,3),(1,4),(1,0);(2,4),(5,0),(1,4),(4,3)],\\
[(0,2),(3,3),(6,4),(3,1);(2,2),(1,3),(2,1),(3,4)],\\
[(1,1),(3,2),(2,0),(4,0);(1,3),(0,2),(0,3),(5,3)],\\
[(2,1),(5,0),(6,0),(5,1);(4,4),(2,0),(5,3),(6,4)],\\
[(2,4),(1,2),(4,1),(0,2);(5,4),(5,2),(3,0),(1,4)],\\
[(4,2),(6,0),(2,1),(0,1);(5,0),(2,0),(6,2),(3,4)],\\
[(0,1),(6,1),(4,2),(0,2);\infty,(0,0),(0,3),(6,0)],\\
[(2,4),(4,1),(4,4),(2,3);(3,3),(0,3),\infty,(0,4)]\}.
\end{array}$$

Then (V,B) is a $\Theta(1,4,4)$-design of order 36. □

Example 18.2 *A $\Theta(1,4,4)$-design of order 37.*

Let $V = \mathbb{Z}_{37}$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, cycled modulo 37.

$$\{[0,1,3,6;10,2,7,13],[0,7,16,2;17,28,3,19]\}.$$

Then (V,B) is a $\Theta(1,4,4)$-design of order 37. □

Example 18.3 *A $\Theta(1,4,4)$-design of order 99.*

Let $V = (\mathbb{Z}_{49} \times \mathbb{Z}_2) \cup \{\infty\}$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, with the first components all cycled modulo 49, and the second components fixed.

$$\begin{array}{l}
\{[(0,0),(1,0),(3,0),(6,0);(10,0),(2,0),(7,0),(16,0)],\\
[(0,0),(6,0),(20,0),(3,0);(24,0),(2,0),(17,0),(0,1)],\\
[(0,0),(20,0),(2,1),(1,0);(3,1),(4,0),(0,1),(5,1)],\\
[(0,0),(4,1),(6,0),(0,1);(7,1),(1,0),(9,1),(10,1)],\\
[(0,0),(9,1),(12,0),(0,1);(11,1),(16,0),(2,1),(14,1)],\\
[(0,0),(13,1),(21,0),(5,1);(15,1),(24,0),(0,1),(18,1)],\\
[(0,0),(22,1),(33,0),(11,1);(26,1),(4,1),(44,3),(36,1)],\\
[(9,0),(16,0),(47,0),(11,0);(39,1),(9,1),(7,1),(32,1)],\\
[(34,0),(11,0),(23,1),(6,1);(19,1),(42,1),(39,1),(46,0)],\\
[(31,1),(14,0),(25,0),(6,0);(35,1),(7,1),(23,1),(17,1)],\\
[(0,0),(16,1),(26,0),(1,1);(21,1),(1,0),(20,1),\infty]\}.
\end{array}$$

Then (V, B) is a $\Theta(1,4,4)$-design of order 99. □

Example 18.4 *A* $\Theta(1,4,4)$*-design of order 117.*

Let $V = (\mathbb{Z}_{29} \times \mathbb{Z}_4) \cup \{\infty\}$ and let B contain the copies of $\Theta(1,4,4)$ arising from the following set, with the first components all cycled modulo 29, and the second components fixed.

$$\begin{array}{l}
\{[(0,0),(2,0),(5,0),(1,0);(6,0),(13,0),(3,0),(11,0)],\\
[(0,0),(9,0),(22,0),(7,0);(0,1),(1,0),(2,1),(3,1)],\\
[(0,0),(2,1),(4,0),(0,1);(4,1),(9,0),(1,1),(6,1)],\\
[(0,0),(5,1),(11,0),(1,1);(7,1),(16,0),(0,1),(8,1)],\\
[(0,0),(9,1),(21,0),(2,1);(11,1),(25,0),(10,1),(6,2)],\\
[(0,0),(4,2),(5,0),(0,2);(5,2),(7,0),(1,2),(10,2)],\\
[(0,0),(7,2),(10,0),(0,2);(8,2),(12,0),(1,2),(12,2)],\\
[(0,0),(14,2),(21,0),(7,2);(17,2),(25,0),(0,3),(11,3)],\\
[(0,0),(0,3),(1,0),(2,3);(14,3),(0,1),(3,1),(5,3)],\\
[(0,0),(6,3),(1,1),(8,1);(9,3),(0,1),(10,1),(8,3)],\\
[(0,0),(10,3),(0,1),(12,1);(15,3),(3,1),(16,1),(27,3)],\\
[(0,0),(13,3),(5,1),(2,2);(20,3),(1,1),(0,2),(21,3)],\\
[(0,0),(16,3),(0,1),(2,2);(22,3),(0,1),(0,2),(21,3)],\\
[(0,1),(1,2),(3,1),(6,2);(5,2),(13,1),(2,2),(17,2)],\\
[(0,1),(6,2),(15,1),(23,2);(24,3),(27,0),(23,3),(11,2)],\\
[(0,2),(25,2),(12,1),(24,2);(26,4),(22,2),(13,1),(27,2)],\\
[(2,3),(2,2),(9,3),(28,2);(4,3),(9,0),(21,0),(8,2)],\\
[(4,2),(27,2),(8,1),(15,3);(20,3),(5,1),(22,3),(2,3)],\\
[(6,0),(19,2),(26,2),(21,3);(22,1),(22,3),(7,2),(18,3)],\\
[(6,2),(28,1),(22,2),(21,0);(15,3),(28,3),(6,1),(19,3)],\\
[(13,2),(2,0),(14,1),(25,0);(27,3),(10,2),(10,0),(11,0)],\\
[(14,1),(11,3),(17,3),(14,3);(25,1),(27,1),(12,1),(18,3)],\\
[(22,0),(12,3),(4,3),(10,1);(25,2),(9,2),(5,1),(25,3)],\\
[(22,3),(0,2),(20,0),(27,3);(23,3),(4,2),(7,2),(19,2)],\\
[(5,1),(27,2),(27,1),(1,0);(21,2),(0,3),\infty,(0,2)],\\
[(18,0),(22,2),(21,3),(11,3);(25,3),(0,1),\infty,(0,0)]\}.
\end{array}$$

Then (V, B) is a $\Theta(1,4,4)$-design of order 117. □

19 Isolated Cases for $\Theta(2,2,5)$-designs

In this section we give examples of $\Theta(2,2,5)$-designs of orders 10, 72, 73, 99, 100, 117, 118, 126 and 127.

Example 19.1 *A* $\Theta(2,2,5)$*-design of order 10.*

Let $V = \mathbb{Z}_5 \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 5, and the second components fixed.

$$\{[(1,0):(0,0);(1,1);(3,0),(0,1),(2,0),(3,1):(4,1)]\}$$

.
Then (V, B) is a $\Theta(2,2,5)$-decomposition of K_{10}. □

Example 19.2 *A* $\Theta(2,2,5)$*-design of order 72.*

Let $V = \mathbb{Z}_8 \times \mathbb{Z}_9$. Let B contain the copies of $\Theta(2,2,5)$ from the following two types of $\Theta(2,2,5)$-decompositions.
Type 1: For each $i \in \{0,1,2,3\}$, place a $\Theta(2,2,5)$-design of order 18 on $(\{i\} \times \mathbb{Z}_9) \cup (\{i+4\} \times \mathbb{Z}_9)$, where addition is taken modulo 8.
Type 2: For each $i \in \mathbb{Z}_8$, place a $\Theta(2,2,5)$-decomposition of $K_{3(9)}$ on $\{i, i+1, i+3\} \times \mathbb{Z}_9)$.
Then (V, B) is a $\Theta(2,2,5)$-design of order 72. □

Example 19.3 *A* $\Theta(2,2,5)$*-design of order 73.*

Let $V = (\mathbb{Z}_8 \times \mathbb{Z}_9) \cup \{\infty\}$. Let B contain the copies of $\Theta(2,2,5)$ from the following two types of $\Theta(2,2,5)$-decompositions.
Type 1: For each $i \in \{0,1,2,3\}$, place a $\Theta(2,2,5)$-design of order 19 on $(\{i\} \times \mathbb{Z}_9) \cup (\{i+4\} \times \mathbb{Z}_9) \cup \{\infty\}$, where addition is taken modulo 8.
Type 2: For each $i \in \mathbb{Z}_8$, place a $\Theta(2,2,5)$-decomposition of $K_{3(9)}$ on $\{i, i+1, i+3\} \times \mathbb{Z}_9)$.
Then (V, B) is a $\Theta(2,2,5)$-design of order 73. □

Example 19.4 *A* $\Theta(2,2,5)$*-design of order 99.*

Let $V = (\mathbb{Z}_{49} \times \mathbb{Z}_2) \cup \{\infty\}$ and let B contain the copis of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 49, and the second components fixed.

$$
\begin{aligned}
&\{[(0,0):(2,0);(3,0);(5,0),(11,0),(1,0),(10,0):(21,0)],\\
&[(0,0):(12,0);(14,0);(16,0),(33,0),(4,0),(26,0):(6,1)],\\
&[(0,0):(24,0);(0,1);(2,1),(3,0),(8,1),(1,0):(12,1)],\\
&[(0,0):(6,1);(9,1);(14,1),(18,0),(0,1),(5,0):(47,1)],\\
&[(0,0):(17,1);(18,1);(20,1),(31,0),(3,1),(29,0):(13,1)],\\
&[(0,0):(22,1);(30,1);(26,1),(9,1),(10,1),(25,1):(47,0)],\\
&[(44,0):(31,1);(45,1);(11,1),(14,1),(24,0),(3,0):(15,1)],\\
&[(4,1):(13,0);(34,0);(40,1),(25,0),(32,0),(6,0):(10,1)],\\
&[(15,1):(1,1);(22,1);(6,1),(33,1),(4,1),(1,0):(48,1)],\\
&[(40,1):(19,1);(34,1);(22,1),(43,0),(30,0),(34,0):(44,1)],\\
&[(0,0):\infty;(41,0);(13,1),(28,0),(29,0),(14,0):(0,1)]\}.
\end{aligned}
$$

Then (V, B) is a $\Theta(2,2,5)$-design of order 99. □

Example 19.5 *A* $\Theta(2,2,5)$*-design of order 100.*

Let $V = \mathbb{Z}_{50} \times \mathbb{Z}_2$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 50, and the second components fixed.

$$\begin{array}{l}
\{[(0,0):(1,0);(2,0);(3,0),(7,0),(12,0),(5,0):(14,0)],\\
[(0,0):(6,0);(11,0);(14,0),(29,0),(1,0),(17,0):(35,0)],\\
[(0,0):(17,0);(19,0);(20,0),(43,0),(18,0),(0,1):(2,1)],\\
[(0,0):(0,1);(1,1);(2,1),(3,0),(6,1),(1,0):(7,1)],\\
[(0,0):(8,1);(9,1);(10,1),(12,0),(0,1),(3,0):(17,1)],\\
[(0,0):(11,1);(12,1);(13,1),(17,0),(1,1),(6,0):(22,1)],\\
[(0,0):(15,1);(17,1);(18,1),(24,0),(0,1),(7,0):(30,1)],\\
[(0,0):(19,1);(20,1);(21,1),(29,0),(1,1),(10,0):(0,1)],\\
[(0,0):(24,1);(25,1);(27,1),(38,0),(16,1),(2,1):(1,1)],\\
[(0,0):(30,1);(37,1);(36,1),(3,1),(31,1),(10,1):(5,1)],\\
[(0,1):(43,0);(38,1);(46,0),(36,0),(44,0),(25,1):(22,1)]\}.
\end{array}$$

Then (V,B) is a $\Theta(2,2,5)$-design of order 100. □

Example 19.6 *A* $\Theta(2,2,5)$*-design of order 117.*

Let $V = (\mathbb{Z}_{29} \times \mathbb{Z}_4) \cup \{\infty\}$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, with the first components all cycled modulo 29, and the second components fixed.

$$\begin{array}{l}
\{[(0,0):(1,0);(2,0);(3,0),(7,0),(12,0),(4,0):(11,0)],\\
[(0,0):(6,0);(11,0);(12,0),(25,0),(10,0),(0,1):(2,1)],\\
[(0,0):(1,1);(2,1);(4,1),(5,0),(0,1),(2,0):(7,1)],\\
[(0,0):(6,1);(8,1);(9,1),(15,0),(0,1),(8,0):(19,1)],\\
[(0,0):(10,1);(12,1);(13,1),(25,0),(11,1),(24,0):(2,2)],\\
[(0,0):(0,2);(1,2);(3,2),(4,0),(2,2),(5,0):(21,2)],\\
[(0,0):(8,2);(10,2);(11,2),(15,0),(1,2),(6,0):(25,3)],\\
[(0,0):(12,2);(13,2);(18,2),(24,0),(15,2),(23,0):(7,3)],\\
[(0,0):(0,3);(2,3);(4,3),(5,0),(1,3),(3,0):(14,3)],\\
[(0,0):(5,3);(6,3);(7,3),(10,0),(2,3),(19,0):(4,3)],\\
[(0,0):(10,3);(15,3);(24,3),(1,1),(0,1),(4,1):(14,1)],\\
[(0,1):(7,1);(14,1);(1,2),(2,1),(0,2),(3,1):(18,2)],\\
[(0,1):(3,2);(6,2);(7,2),(12,1),(0,2),(15,1):(13,3)],\\
[(0,1):(8,2);(12,2);(0,3),(1,1),(3,3),(6,1):(28,3)],\\
[(0,1):(3,3);(7,3);(10,3),(6,2),(2,2),(2,3):(20,1)],\\
[(0,2):(14,2);(17,2);(3,3),(13,1),(1,3),(15,1):(28,3)],\\
[(8,0):(14,2);(27,2);(9,3),(1,1),(19,2),(18,2):(5,1)],\\
[(5,1):(14,1);(7,2);(10,2),(8,2),(19,2),(23,1):(26,1)],\\
[(10,1):(7,0);(2,3);(1,2),(11,2),(4,2),(17,1):(25,1)],\\
[(28,1):(22,2);(13,3);(6,0),(15,3),(22,0),(25,3):(20,3)],\\
[(2,2):(14,0);(28,3);(4,3),(10,3),(22,0),(7,2):(23,2)],\\
[(15,2):(20,2);(24,3);(11,0),(11,1),(4,0),(20,3):(16,3)],\\
[(22,2):(17,0);(20,0);(23,3),(1,2),(4,2),(22,3):(11,3)],\\
[(27,2):(27,1);(11,3);(17,3),(14,3),(9,1),(18,3):(2,3)],\\
[(0,0):\infty;(18,3);(22,2),(21,3),(11,3),(3,0):(0,1)],\\
[(0,2):\infty;(23,2);(21,3),(13,2),(25,3),(5,1):(0,3)]\}.
\end{array}$$

Then (V, B) is a $\Theta(2,2,5)$-design of order 117. $\square$

Example 19.7 *A* $\Theta(2,2,5)$*-design of order 118.*

Let $V = (\mathbb{Z}_9 \times \mathbb{Z}_{13}) \cup \{\infty\}$. Let B contain the copies of $\Theta(2,2,5)$ from the following two types of $\Theta(2,2,5)$-decompositions.
Type 1: For each $i \in \mathbb{Z}_{13}$, place a $\Theta(2,2,5)$-design of order 10 on $(\{i\} \times \mathbb{Z}_9) \cup \{\infty\}$.
Type 2: For each triple, $\{a,b,c\}$ say, in an STS(13), place a $\Theta(2,2,5)$-decomposition of $K_{3(9)}$ on $\mathbb{Z}_9 \times \{a,b,c\}$.
Then (V, B) is a $\Theta(2,2,5)$-design of order 118. $\square$

Example 19.8 *A* $\Theta(2,2,5)$*-decomposition of* $K_{3(18)}$.

Let $V = \{i \equiv 0 \pmod 3 \colon i \in \mathbb{Z}_{54}\} \cup \{i \equiv 1 \pmod 3 \colon i \in \mathbb{Z}_{54}\} \cup \{i \equiv 2 \pmod 3 \colon i \in \mathbb{Z}_{54}\}$ and let B contain the copies of $\Theta(2,2,5)$ arising from the following set, cycled modulo 54.

$$\{[0:7;8;14,1,21,5:30],[40:30;42;41,45,17,36:47]\}$$

Then (V, B) is a $\Theta(2,2,5)$-decomposition of $K_{3(18)}$. $\square$

Example 19.9 *A* $\Theta(2,2,5)$*-design of order 126.*

Let $V = \mathbb{Z}_{18} \times \mathbb{Z}_7$. Let B contain the copies of $\Theta(2,2,5)$ from the following two types of $\Theta(2,2,5)$-decompositions.
Type 1: For each $i \in \mathbb{Z}_7$, place a $\Theta(2,2,5)$-design of order 18 on $\{i\} \times \mathbb{Z}_{18}$.
Type 2: For each triple, $\{a,b,c\}$ say, in an STS(7), place a $\Theta(2,2,5)$-decomposition of $K_{3(9)}$ on $\mathbb{Z}_{18} \times \{a,b,c\}$.
Then (V, B) is a $\Theta(2,2,5)$-design of order 126. $\square$

Example 19.10 *A* $\Theta(2,2,5)$*-design of order 127.*

Let $V = (\mathbb{Z}_{18} \times \mathbb{Z}_7) \cup \{\infty\}$. Let B contain the copies of $\Theta(2,2,5)$ from the following two types of $\Theta(2,2,5)$-decompositions.
Type 1: For each $i \in \mathbb{Z}_7$, place a $\Theta(2,2,5)$-design of order 19 on $(\{i\} \times \mathbb{Z}_{18}) \cup \{\infty\}$.
Type 2: For each triple, $\{a,b,c\}$ say, in an STS(13), place a $\Theta(2,2,5)$-decomposition of $K_{3(9)}$ on $\mathbb{Z}_{18} \times \{a,b,c\}$.
Then (V, B) is a $\Theta(2,2,5)$-design of order 127. $\square$

20 Isolated Cases for $\Theta(2,3,4)$-designs

In this section we give examples of $\Theta(2,3,4)$-designs of orders 36 and 37.

Example 20.1 *A* $\Theta(2,3,4)$*-decomposition of* $K_{4(3)}$.

Let $V = \{0,1,2\} \cup \{3,4,5\} \cup \{6,7,8\} \cup \{9,10,11\}$ and let $B = \{[0:3;4,6;8,10,7:1]$, $[2:5;3,10;9,7,0:6]$, $[3:8;6,2;11,0,10:4]$, $[5:0;8,1;11,7,4:9]$, $[7:2;3,9;5,1,11:8]$, $[10:2;1,4;5,9,6:11]\}$.

Then (V, B) is a $\Theta(2,3,4)$-decomposition of $K_{4(3)}$. $\square$

Example 20.2 *A* $\Theta(2,3,4)$*-design of order 36.*

Let $V = \mathbb{Z}_4 \times \mathbb{Z}_9$. Let $(Q, *_1)$ and $(Q, *_2)$ be the quasigroups arising from two MOLS(3). Let B contain the copies of $\Theta(2,3,4)$ from the following two types of $\Theta(2,3,4)$-decompositions.

Type 1: For each i, $0 \le i \le 3$, place a $\Theta(1,2,6)$-design of order 9 on $\{i\} \times \mathbb{Z}_9$.

Type 2: For each $(i,j) \in \mathbb{Z}_3 \times \mathbb{Z}_3$ place an $\Theta(2,3,4)$-decomposition of $K_{4(3)}$ on $(\{0\} \times \{3i, 3i+1, 3i+2\}) \cup (\{1\} \times \{3j, 3j+1, 3j+2\}) \cup (\{2\} \times \{3(i *_1 j), 3(i *_1 j)+1, 3(i *_1 j)+2\}) \cup (\{3\} \times \{3(i *_2 j), 3(i *_2 j)+1, 3(i *_2 j)+2\})$.

Then (V, B) is a $\Theta(2,3,4)$-design of order 36. □

Example 20.3 *A* $\Theta(2,3,4)$*-design of order 37.*

Let $V = \mathbb{Z}_{37}$ and let B contain the copies of $\Theta(2,3,4)$ arising from the following set, cycled modulo 37.

$$\{[0:1;2,5;4,9,3:12],[0:8;12,2;13,28,7:25]\}.$$

Then (V, B) is a $\Theta(2,3,4)$-design of order 37. □