Simple unbounded integrand

The integrand $f$ in this example is simple but unbounded and hence not Riemann integrable. For $x = 2^{-n}$ the function value $f(x) = n + 1$, otherwise $f(x) = 0$. A suitable gauge would be equal to $\frac{\varepsilon}{(2^n)(n+1)}$ if $x = 1/2^n$ and 1 otherwise. The definition of the Kurzweil integral requires that for all tagged divisions which are gauge-fine the Riemann sum is within $\varepsilon$ of the expected value of the integral. Unfortunately and inevitably the computer will display only one sum. With the above choice our program will produce 0 for the Riemann sum, a rather uninteresting result. Therefore we aim below for the worst possible scenario for the Riemann sum. For a good display we choose a fairly large $\varepsilon$. This might cause the contribution from the subinterval which is tagged by 1 becoming too large. In order to prevent this we make an adjustment in our definition of gauge. Our choice of $\varepsilon$ below causes a fairly large error. However we are interested in a good graphical display rather than in accuracy. First we denote $n = \log_2(x)$ and then we define the integrand as

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ \left(\lfloor n + 1 \rfloor - \lceil n \rceil \right)(-n + 1) & \text{otherwise}. \end{cases}$$

The gauge $\delta$ is defined as follows:

$$\delta(x) = \begin{cases} \eta & \text{if } x = 0 \\ \min(0.1, \left| \lfloor n + 1 \rfloor - \lceil n \rceil \right|) - n + 1 + 0.9 \left(\sigma[n] - x\right) & \text{otherwise}. \end{cases}$$

For the display we choose

$$\varepsilon = 0.9 \quad \text{and} \quad \eta = 0.05.$$