Abstract

Instructions: Please set the following. Experiment with \( \subseteq, \cdots, \cup, \cap, \bigcup, \bigcap, \left( \ldots \right) \)

1 Properties of probability

We review some properties of the probability measure.

1.1 Continuity of \( P \)

The following very pleasing property of \( P \) is of considerable practical import-
tance. If \( \{A_n\} \) is an increasing sequence of events, that is, \( A_1 \subseteq A_2 \subseteq \cdots \),
then \( \lim_{n \to \infty} A_n \), given by \( \lim_{n \to \infty} A_n = \bigcup_{i=1}^{\infty} A_i \), satisfies

\[
P \left( \lim_{n \to \infty} A_n \right) = \lim_{n \to \infty} P(A_n).
\]

Similarly, if \( \{A_n\} \) is a decreasing sequence of events, that is, \( A_1 \supseteq A_2 \supseteq \cdots \),
then \( \lim_{n \to \infty} A_n \), given by \( \lim_{n \to \infty} A_n = \bigcap_{i=1}^{\infty} A_i \), satisfies

\[
P \left( \lim_{n \to \infty} A_n \right) = \lim_{n \to \infty} P(A_n).
\]

This property is often used in calculating probabilities. It is used, for exam-
ple, in proving that the limits of a distribution function near plus and minus
infinity are, respectively, 1 and 0.