Assignment 5

Insert author

Insert date

Abstract

Instructions: Please set the following. Make sure to load the amsmath package. Experiment with

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Suppose that $\{X_n\}$ is a sequence of random variables which converges (pointwise) to a random variable X.

Theorem 1 (**The Monotone Convergence Theorem**) *If, for each n, $X_n(\omega) \geq 0$ and $X_n(\omega) \leq X_{n+1}(\omega)$, $\omega \in \Omega$, then $\{\mathrm{E}(X_n)\}$ converges to $\mathrm{E}(X)$.*

Now suppose that $\{X_n\}$ is a sequence of *non-negative* random variables. Define $\{Y_n\}$ by

$$Y_n(\omega) = \inf_{m \geq n} \{X_m(\omega)\}, \qquad \omega \in \Omega.$$

On applying Theorem 1 to $\{Y_n\}$ we get Fatou's lemma:

Corollary 1 (**Fatou's lemma**)

$$\liminf_{n \to \infty} \mathrm{E}(X_n) \geq \mathrm{E}\left(\liminf_{n \to \infty} X_n\right).$$

Both of these results can be found in Williams [1].

References

[1] D. Williams, *Probability with Martingales*. Cambridge University Press (1991).