Interaction between habitat quality and an Allee-like effect in metapopulations

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- negative for small population density
- positive for moderate population density
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Allee Effect: The population growth rate is

- negative for small population density $(0 < x < A)$
- positive for moderate population density $(A < x < K)$
- negative for densities above carrying capacity $(x > K)$

... as exemplified by the simple model

$$\frac{dx}{dt} = rx\left(\frac{x}{A} - 1\right)\left(1 - \frac{x}{K}\right)$$

Collaborator

Ross McVinish
Department of Mathematics
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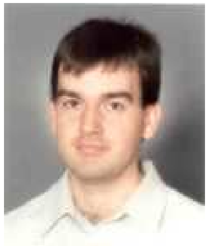
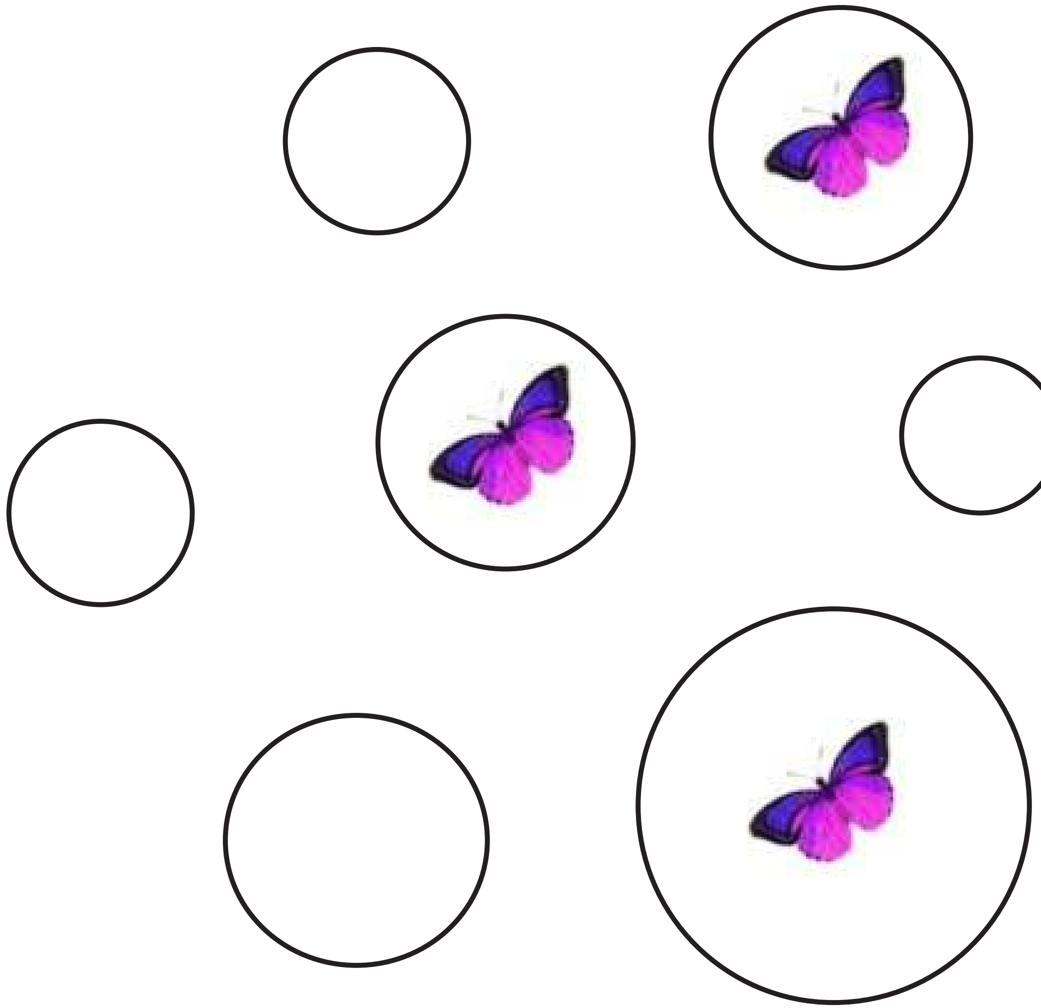


Metapopulations



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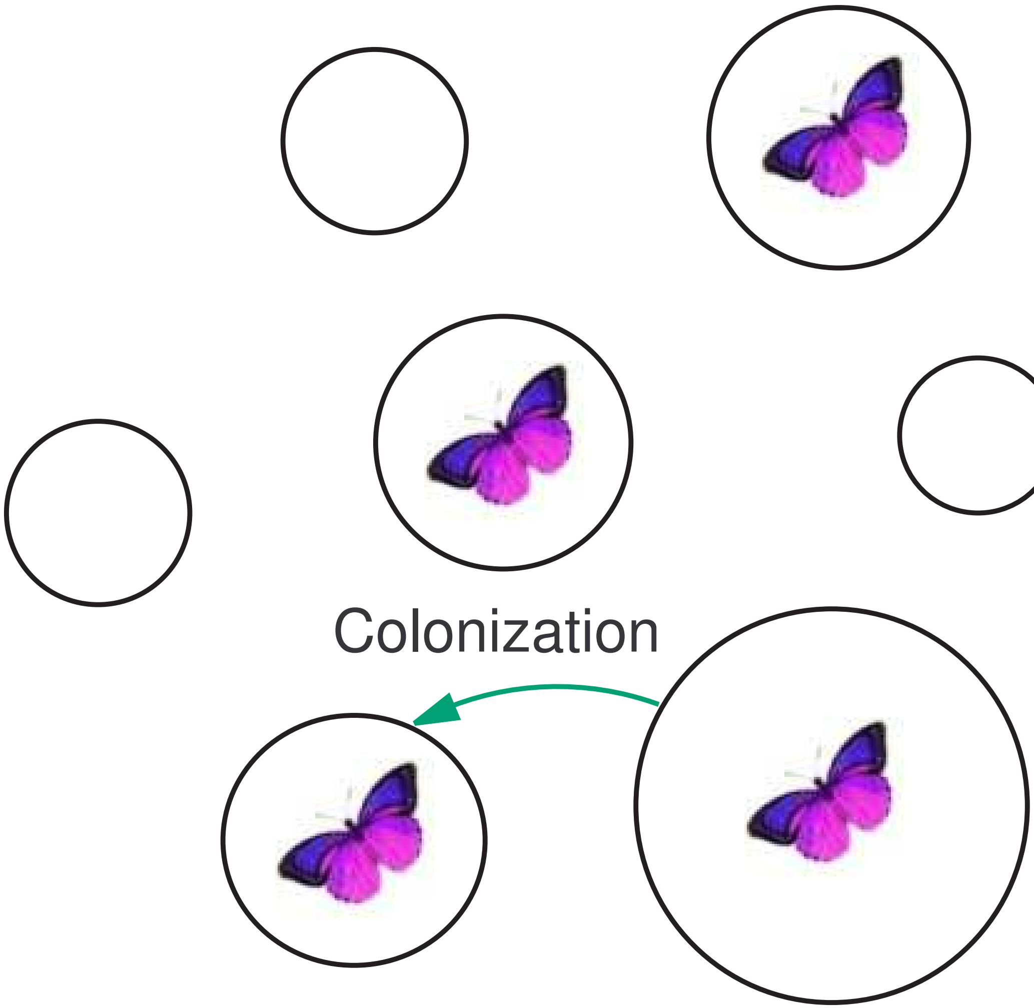

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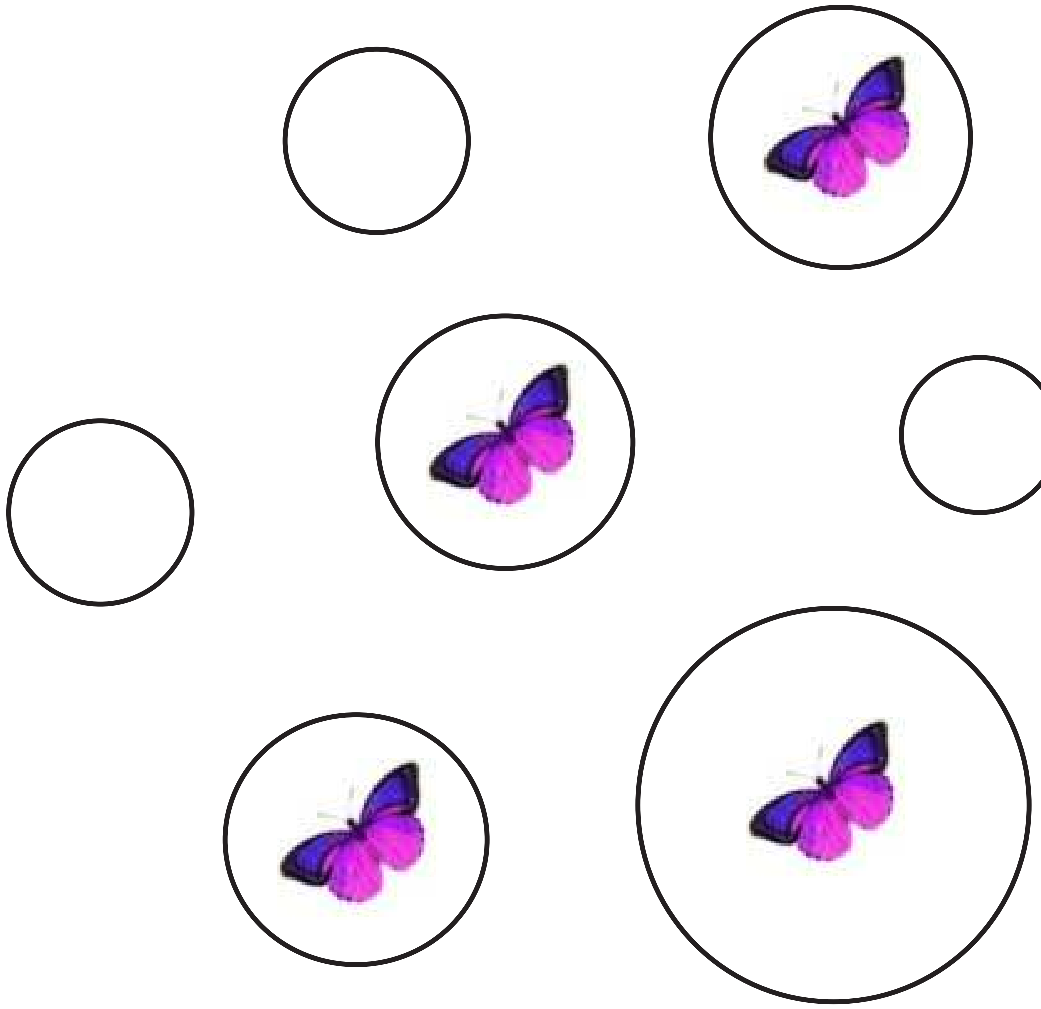

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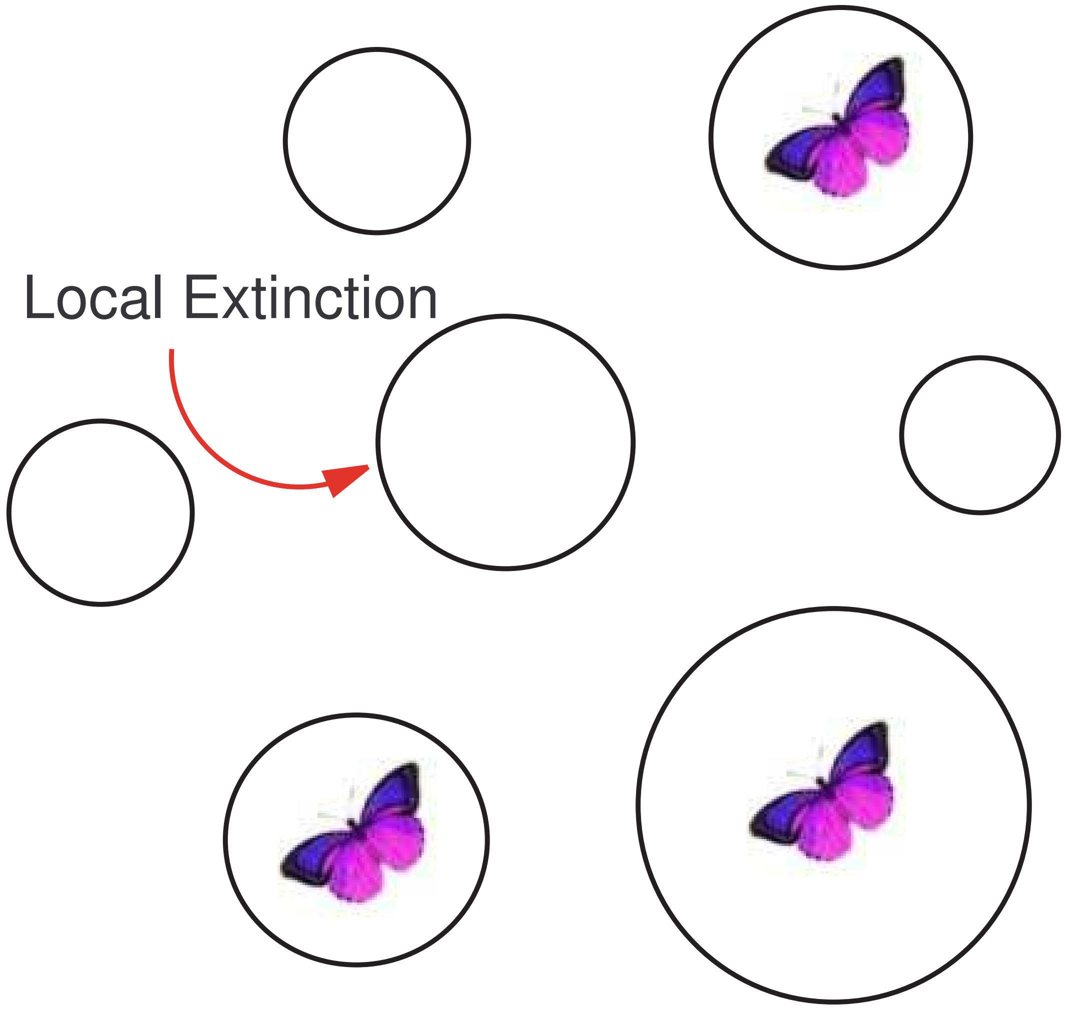

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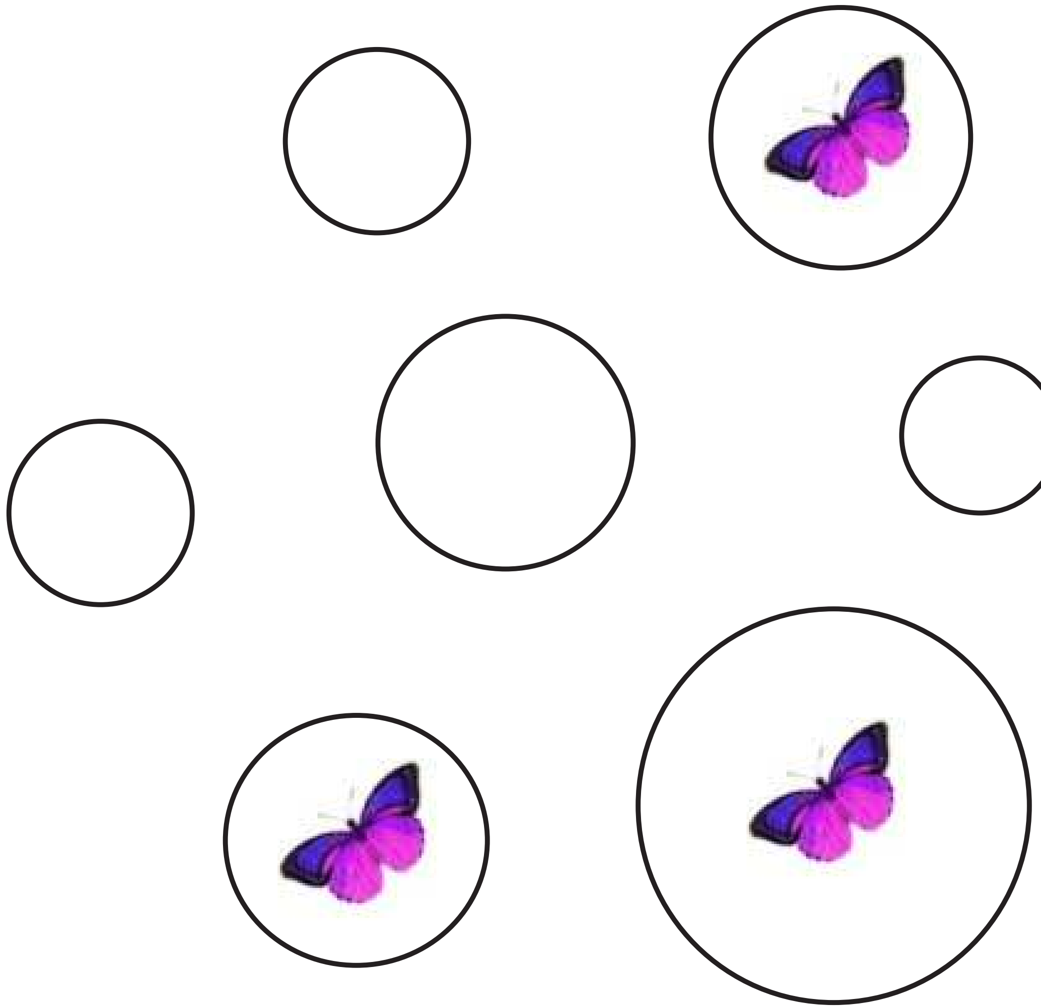

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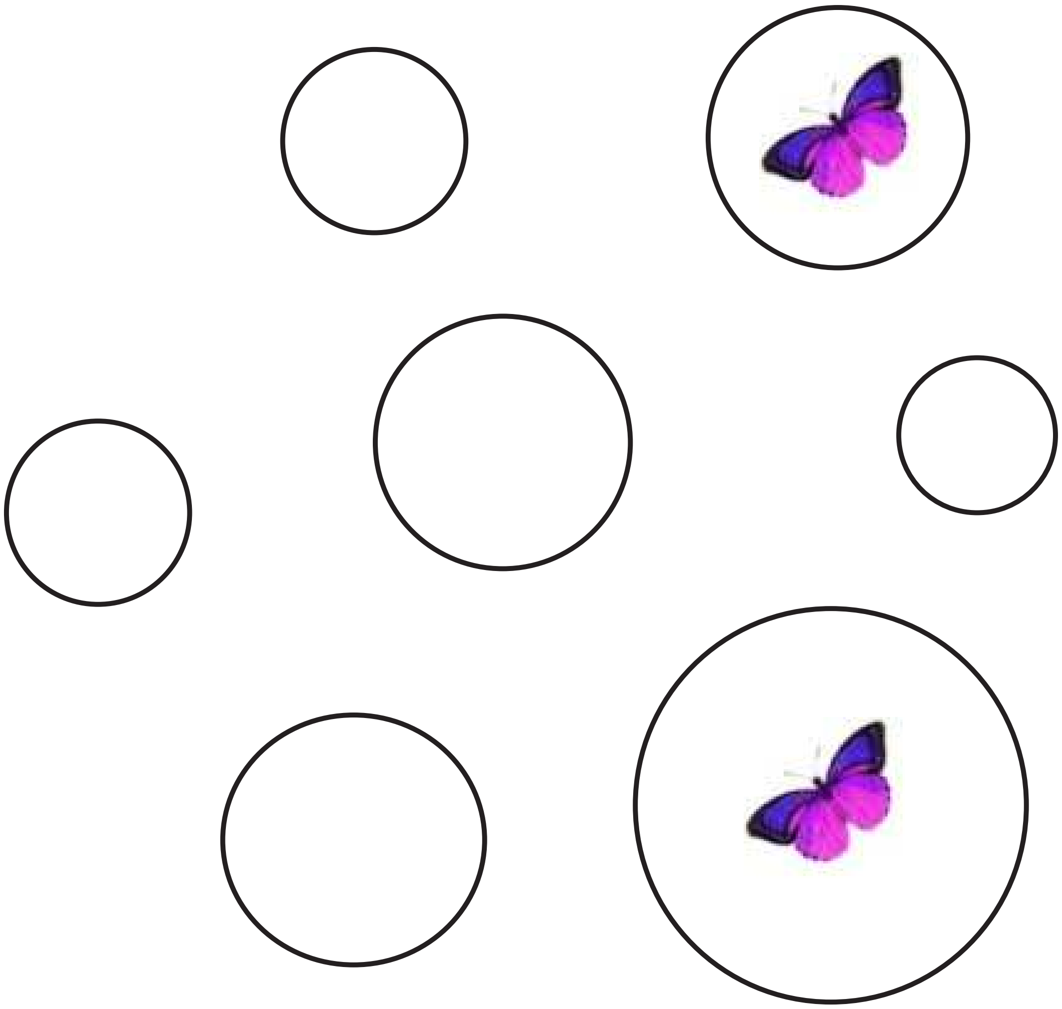

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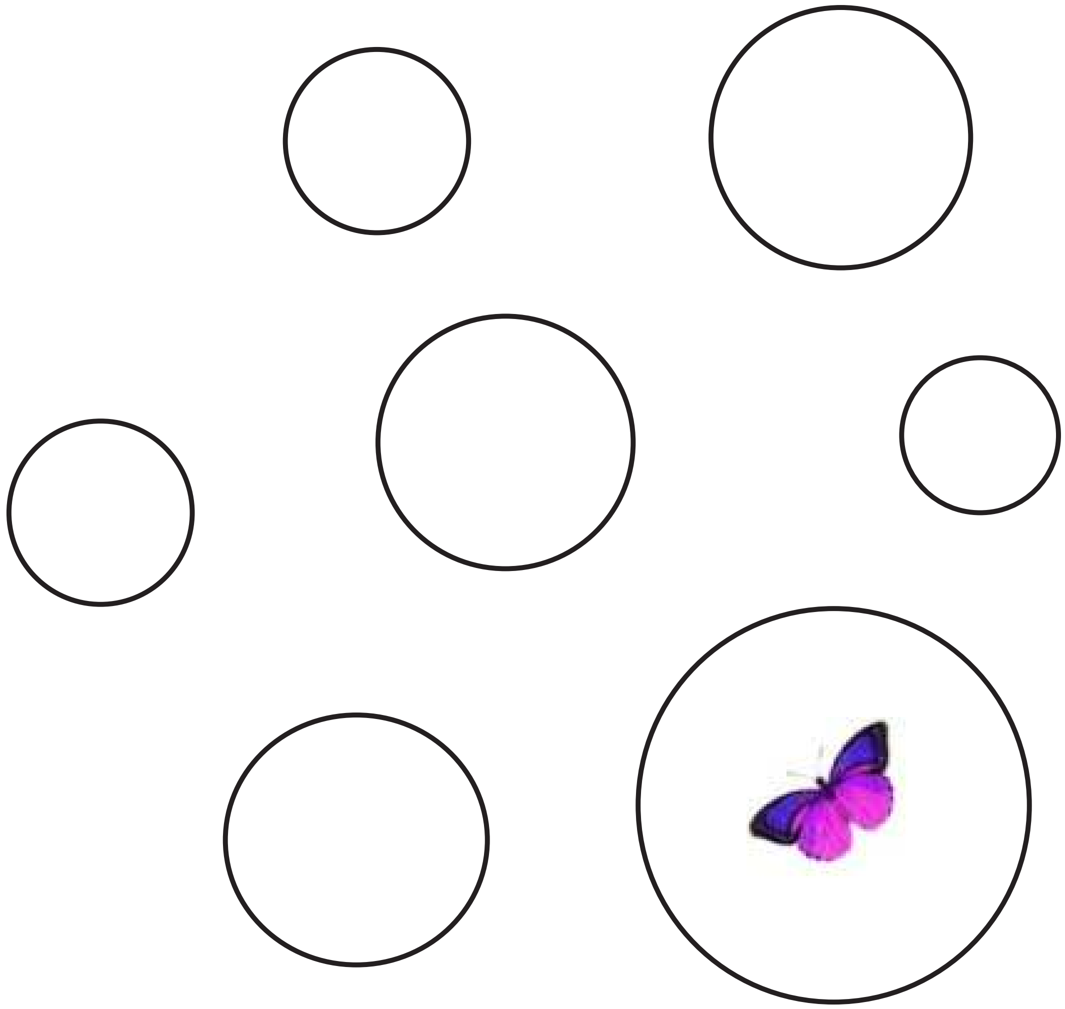

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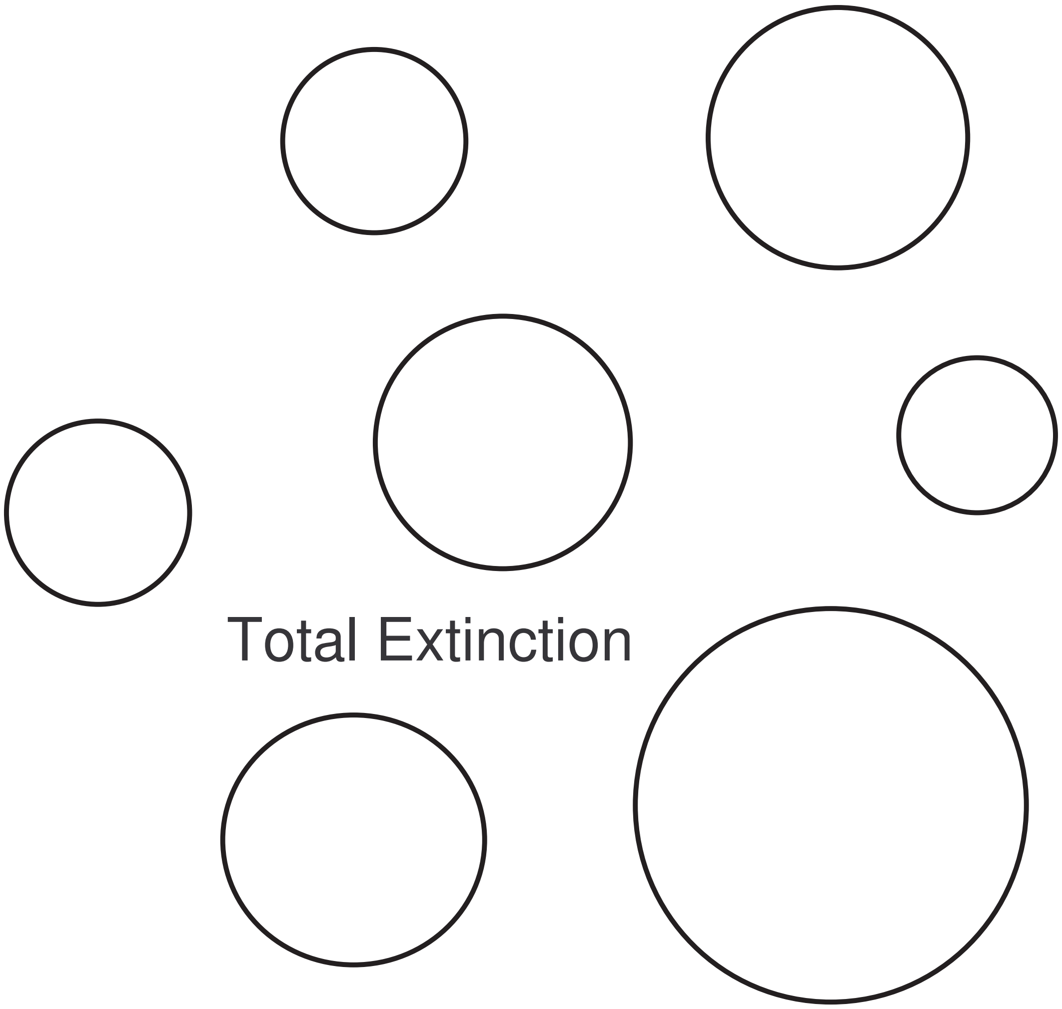

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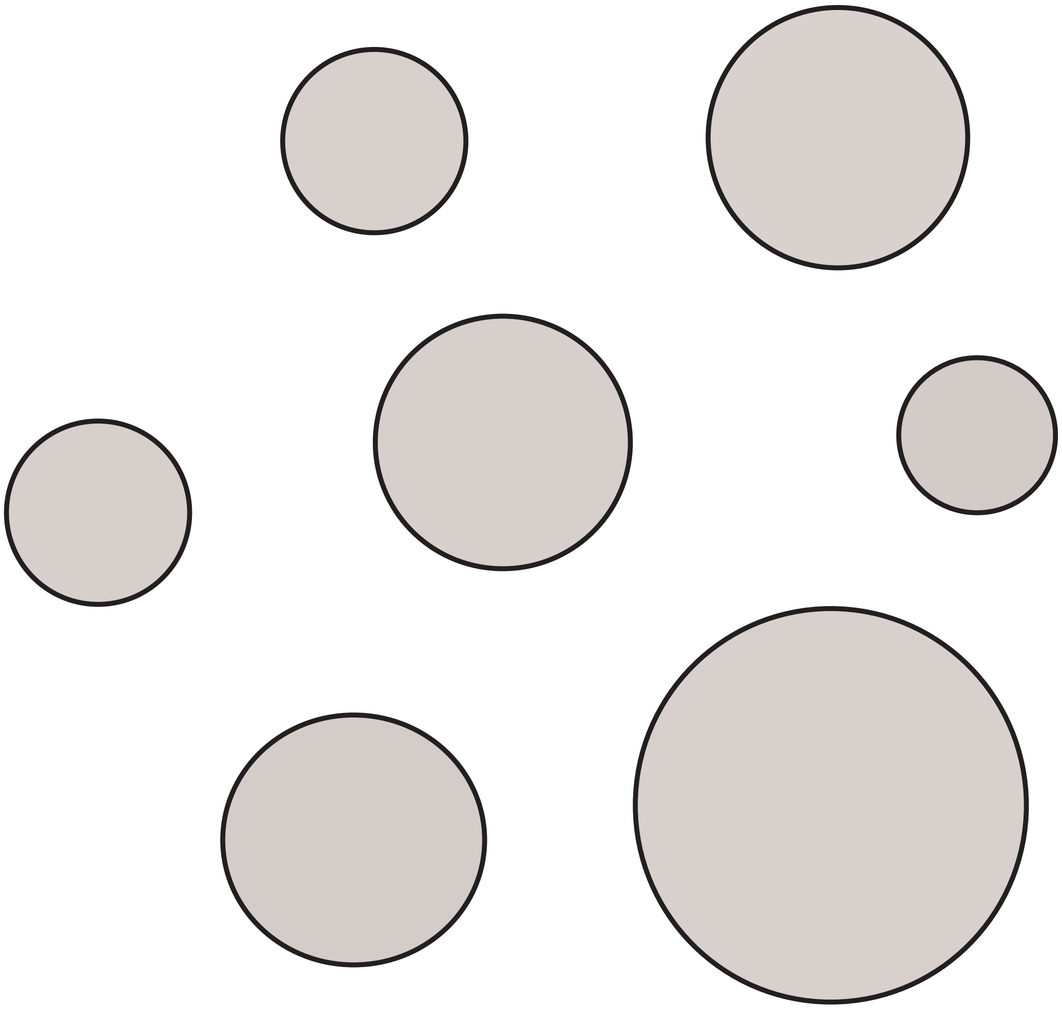

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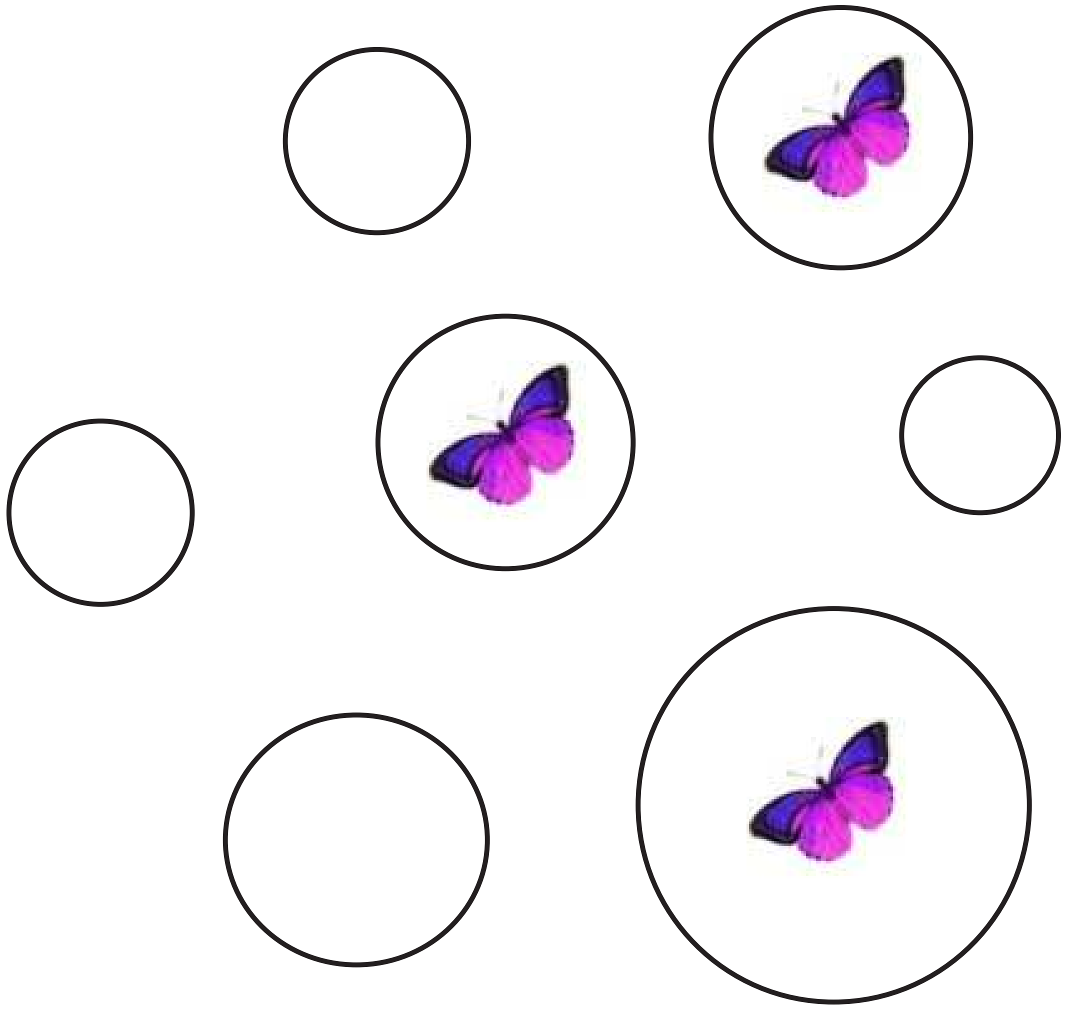

A Stochastic Patch Occupancy Model (SPOM)

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Suppose that there are n patches.

Let $X_t^{(n)} = (X_{1,t}^{(n)}, \dots, X_{n,t}^{(n)})$, where $X_{i,t}^{(n)}$ is a binary variable indicating whether or not patch i is occupied.

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SPOM - Phase structure

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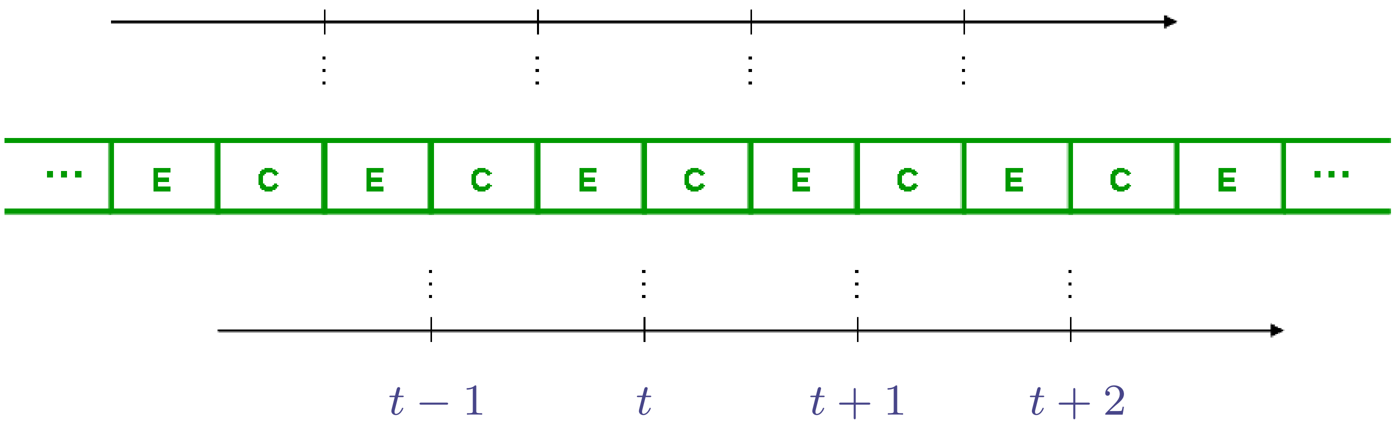

We will we assume that the population is *observed after successive extinction phases* (CE Model).

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Colonization: unoccupied patches become occupied independently with probability $f(n^{-1}\sum_{i=1}^{n} X_{i,t}^{(n)})$, where $f:[0,1]\to[0,1]$ is continuous and increasing with $f(0)=0$ and $f'(0)>0$.

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Extinction: occupied patch i remains occupied independently with probability s_i (fixed or random).

Thus, we have a *Chain Bernoulli* structure:

$$X_{i,t+1}^{(n)} \stackrel{d}{=} \mathit{Bin}\left(X_{i,t}^{(n)} + \mathit{Bin}\left(1 - X_{i,t}^{(n)}, f\left(\frac{1}{n}\sum_{j=1}^{n} X_{j,t}^{(n)}\right)\right), s_i\right)$$

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where $\bar{X}_t^{(n)} = \frac{1}{n}\sum_{j=1}^{n} X_{j,t}^{(n)}$.

Clearly, then, $X_{i,t+1}^{(n)}$ has the same distribution as the sum of two Bernoulli random variables:

$$X_{i,t+1}^{(n)} \stackrel{d}{=} \mathit{Bin}\Big(X_{i,t}^{(n)}, s_i\Big) + \mathit{Bin}\Big(1 - X_{i,t}^{(n)}, s_i\, f\big(\bar{X}_t^{(n)}\big)\Big).$$

(This is Equation (2) of our paper.)

Results for large n and then large t

If (i) the survival probabilities (s_i) are iid with distribution σ (which we call the *survival distribution*) and (ii) given the (s_i), the initial occupancies are independent with $\Pr(X_{i,0}^{(n)} = 1 | s_i) = p(s_i)$ for some function p, then (Theorem 1 of our paper)

$$\frac{1}{n}\sum_{i=1}^{n} X_{i,t}^{(n)} \xrightarrow{p} l_t \text{ as } n \to \infty,$$

where l_t is non-random.

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$$\frac{1}{n}\sum_{i=1}^{n} s_i^k X_{i,t}^{(n)} \xrightarrow{p} l_t(k) \quad \text{as} \quad n \to \infty,$$

for all $k = 0, 1, \ldots$, where $l_t(k)$ is non-random.

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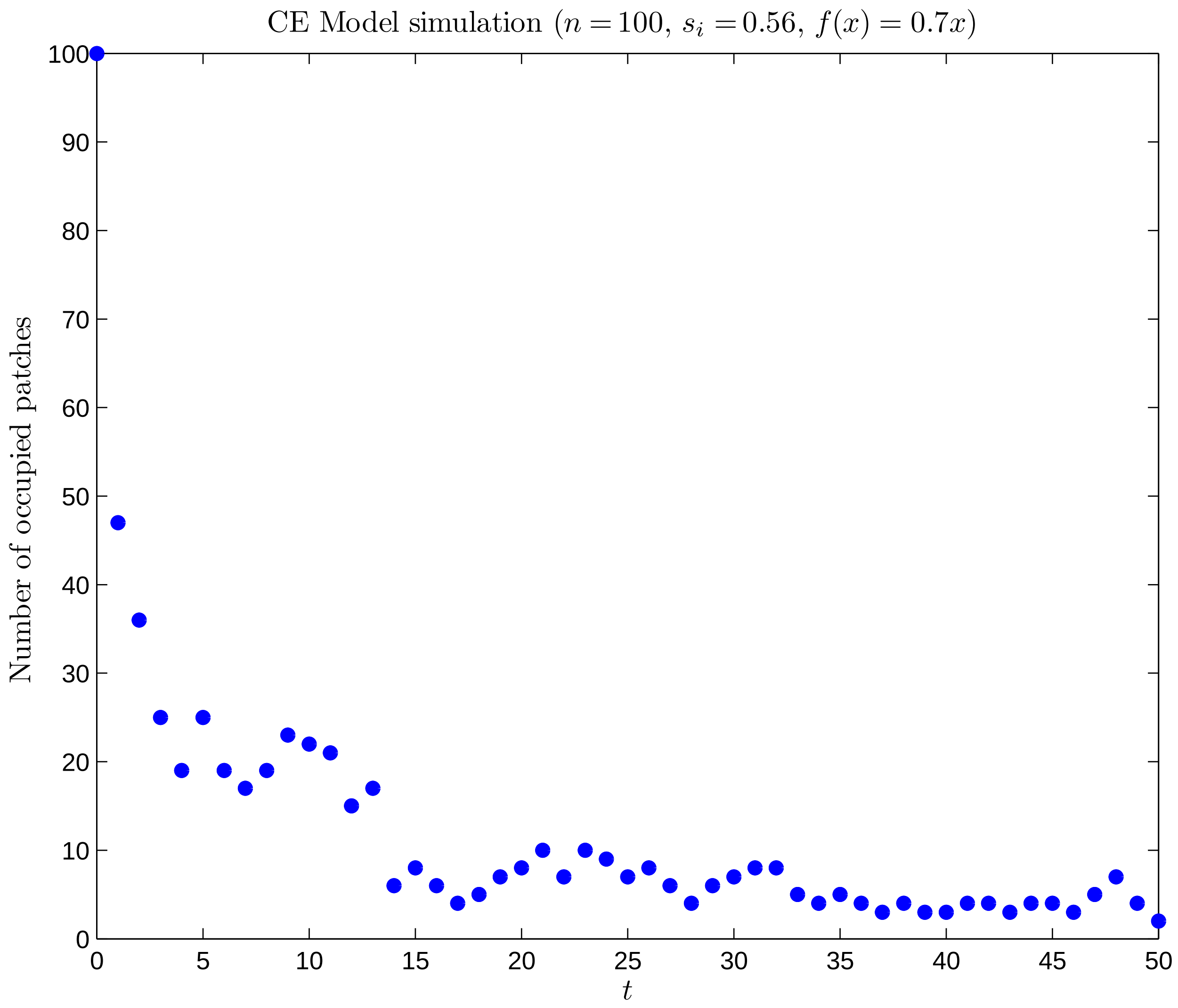
We then study long-term $(t \to \infty)$ behaviour by examining the stability of the system $(l_t(k))$. In particular, $l_t \to ?$

Concave f - zero state stable



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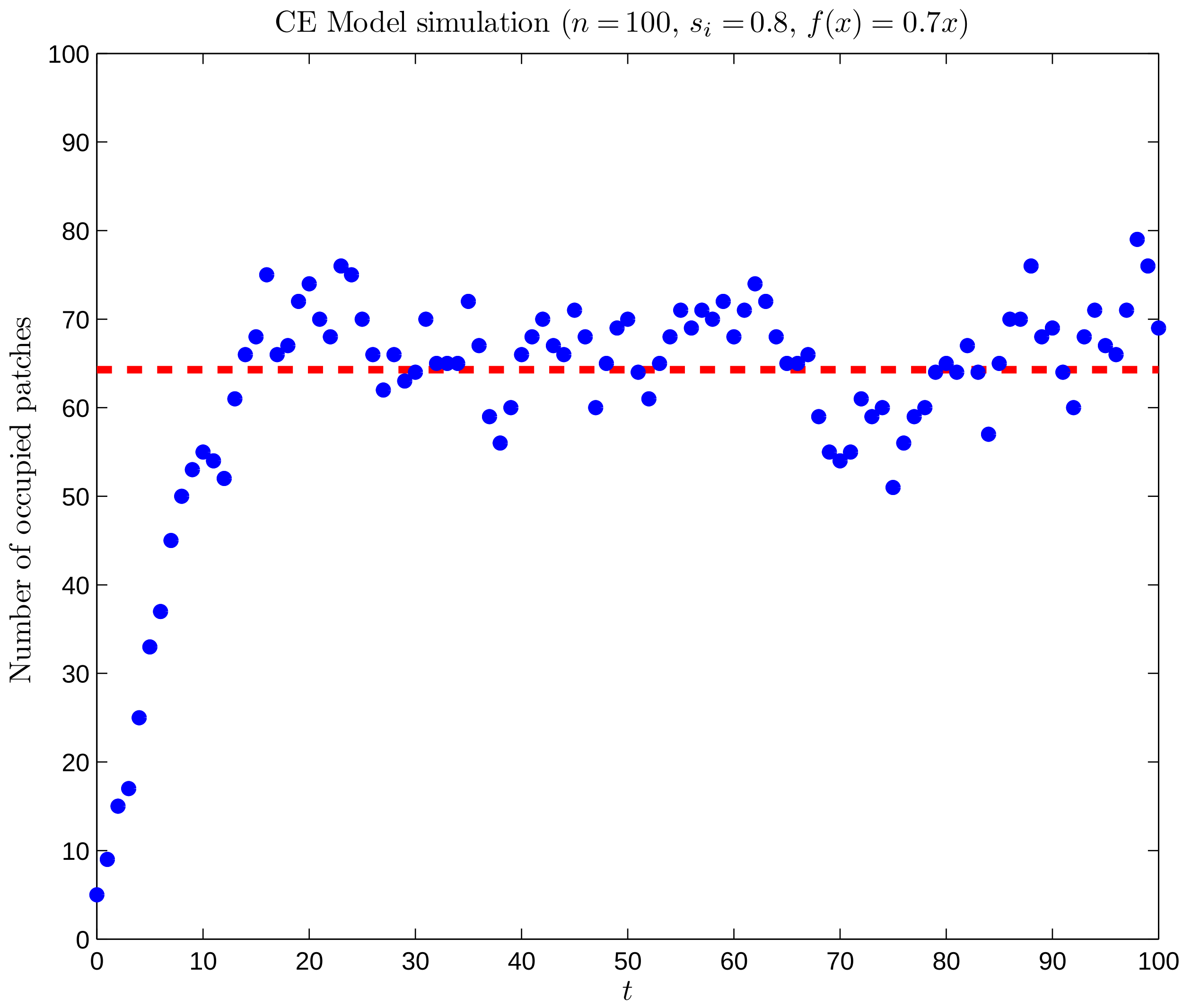
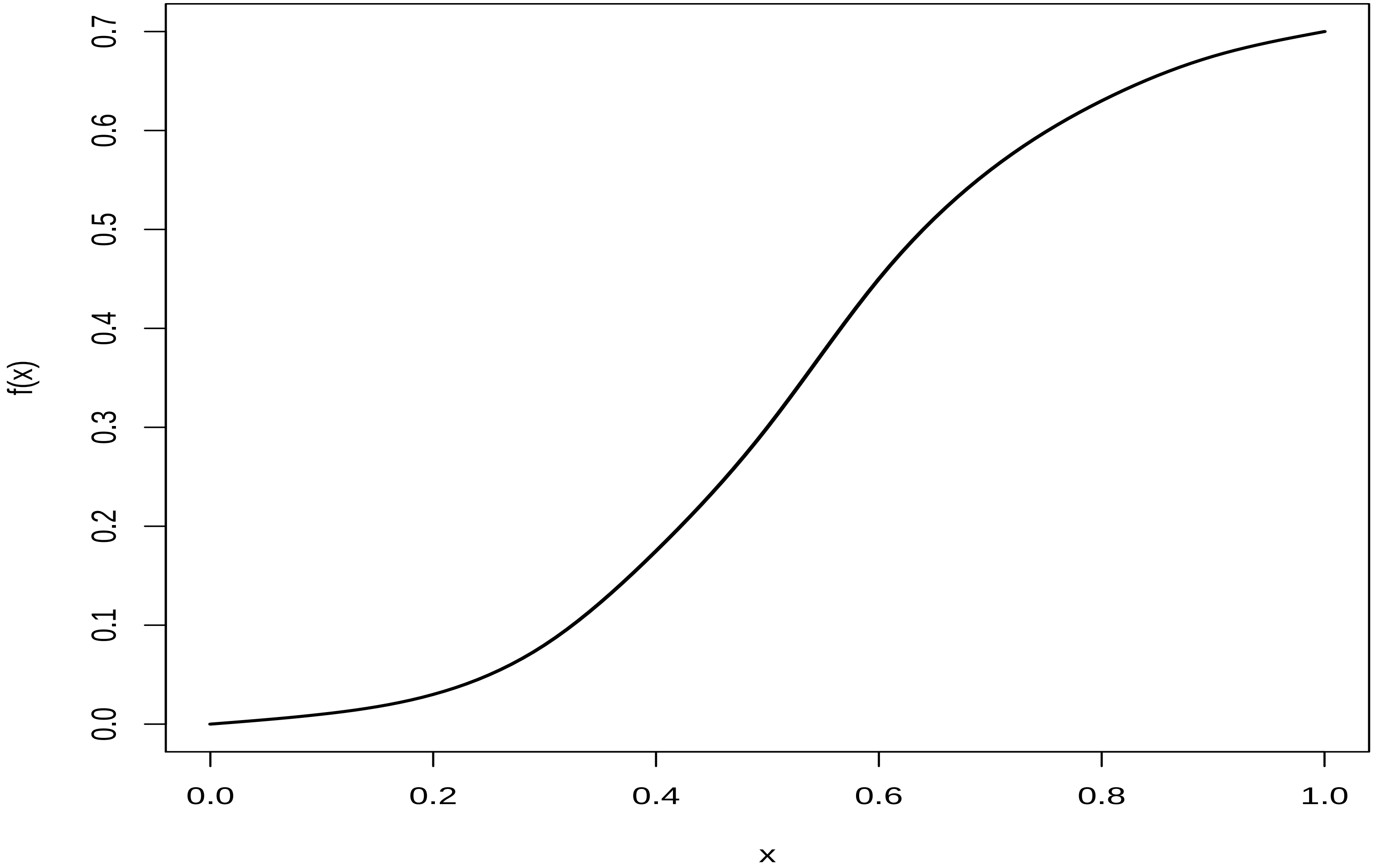


The non-concave f we used



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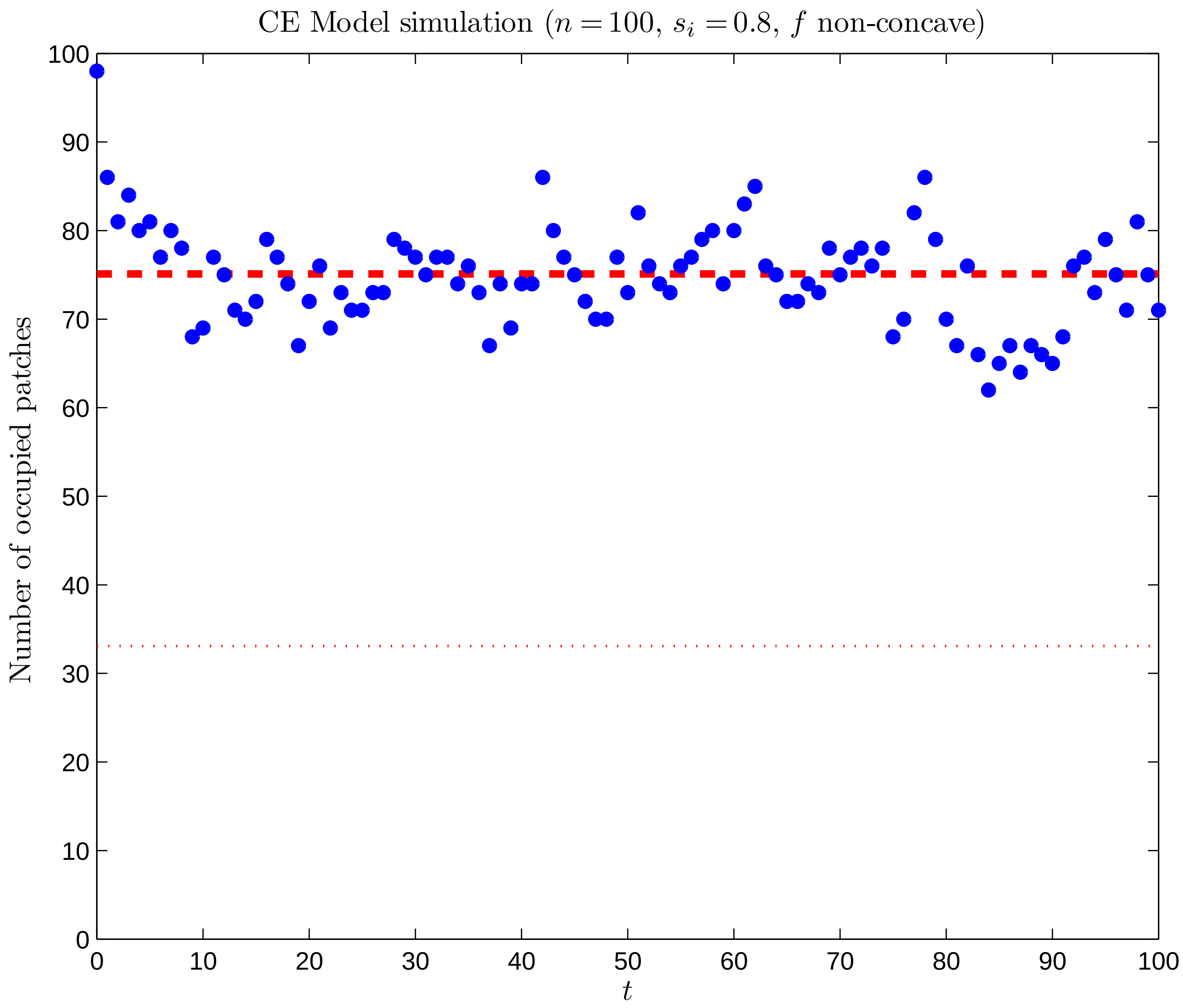

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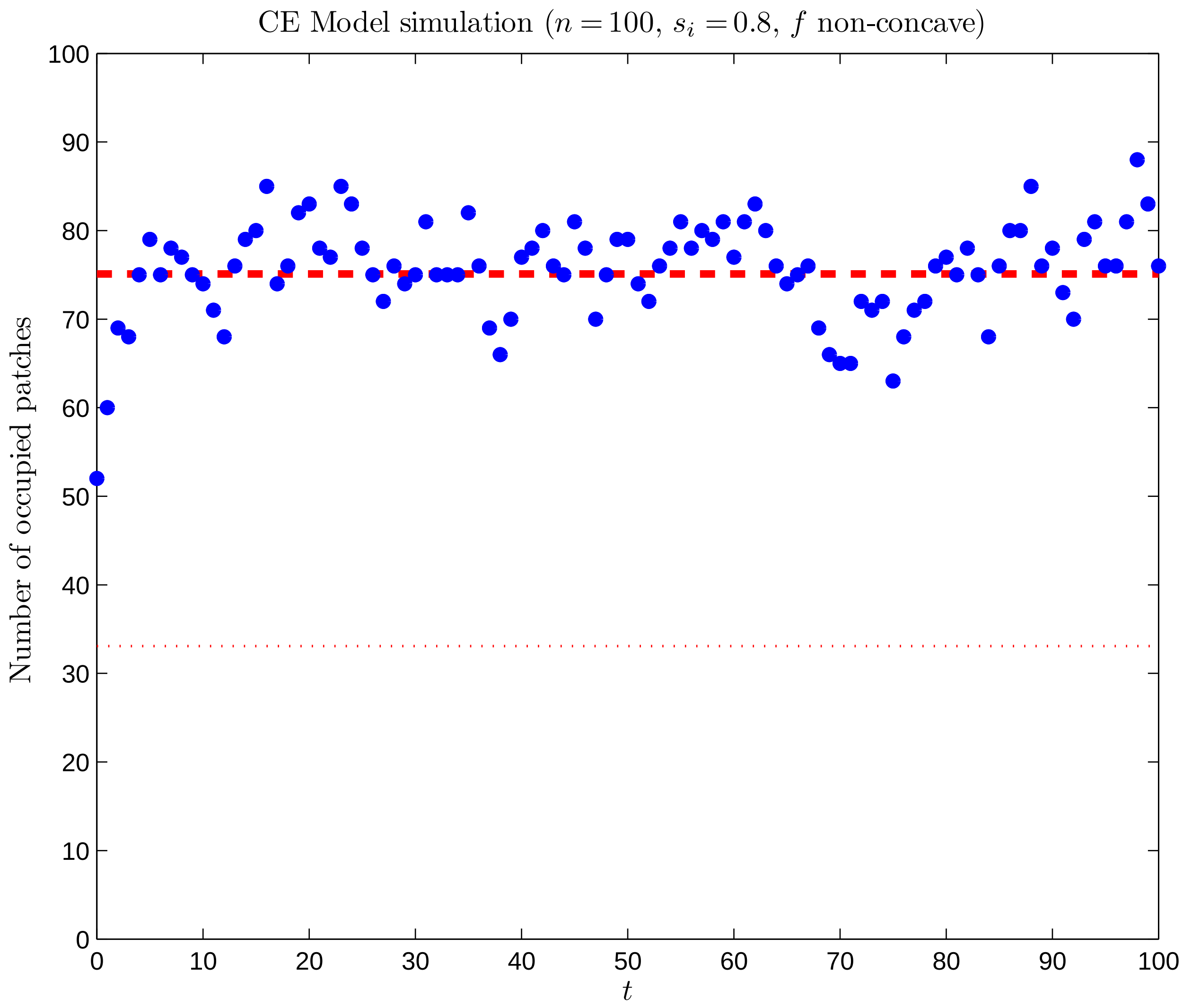

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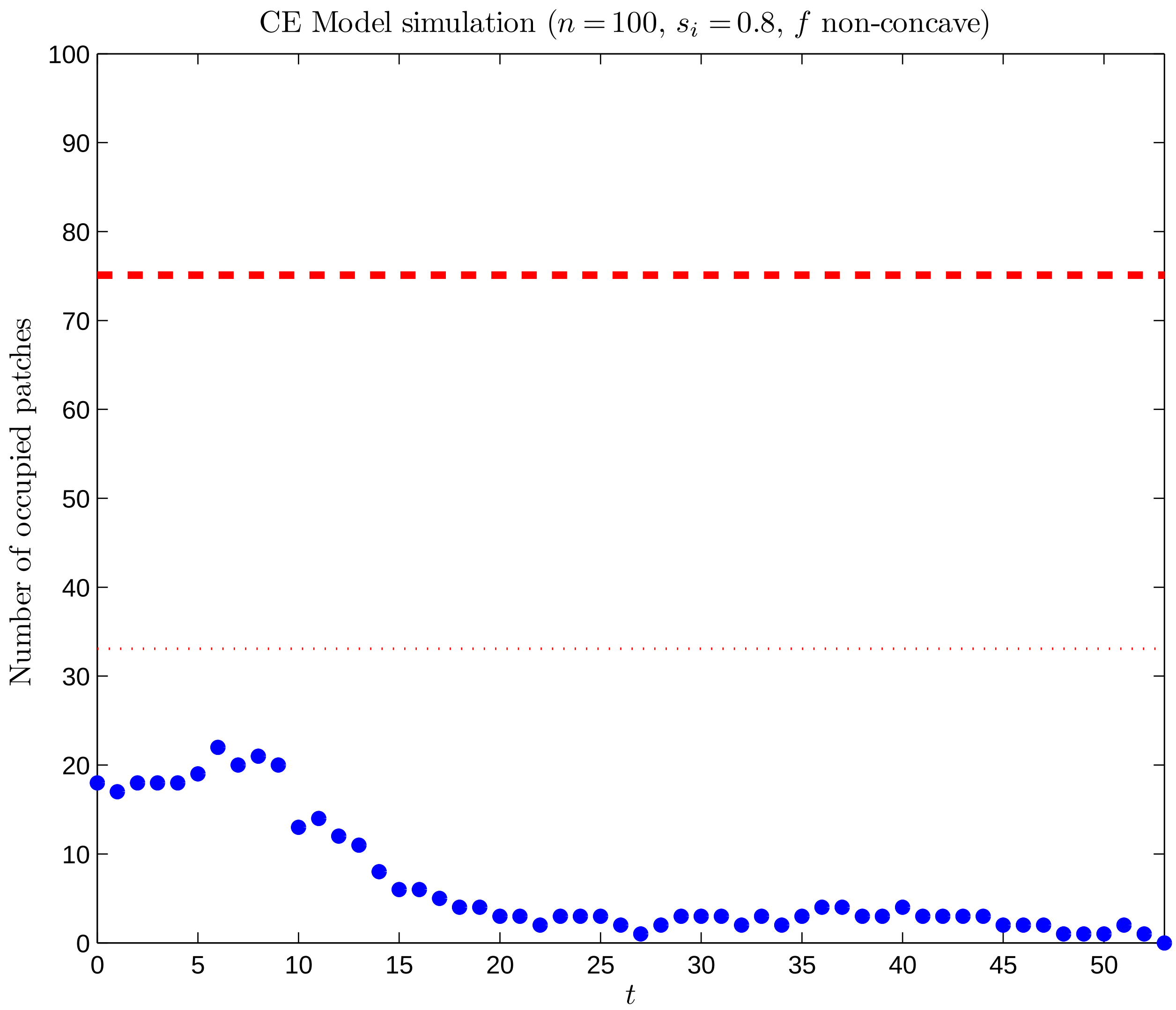

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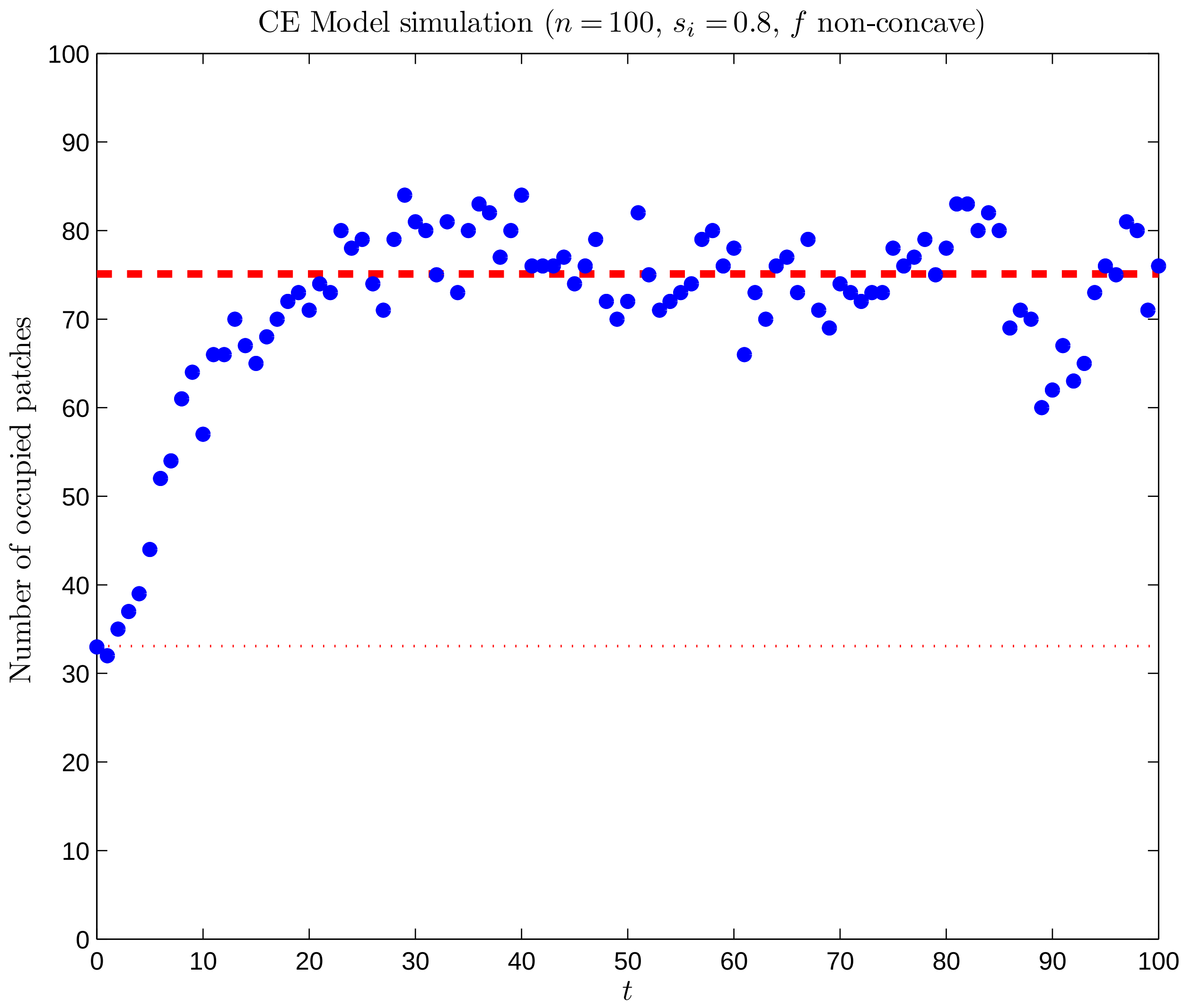

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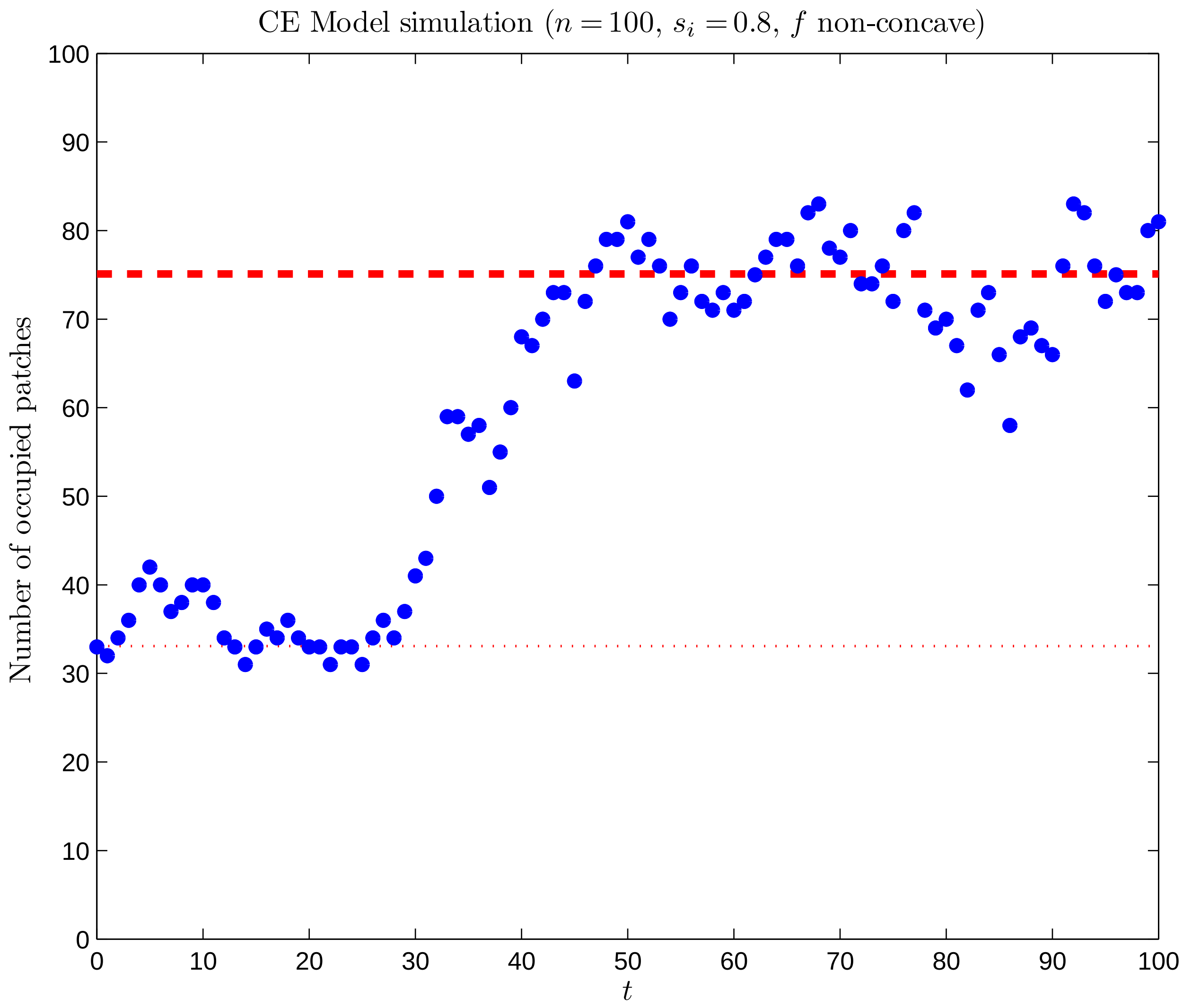

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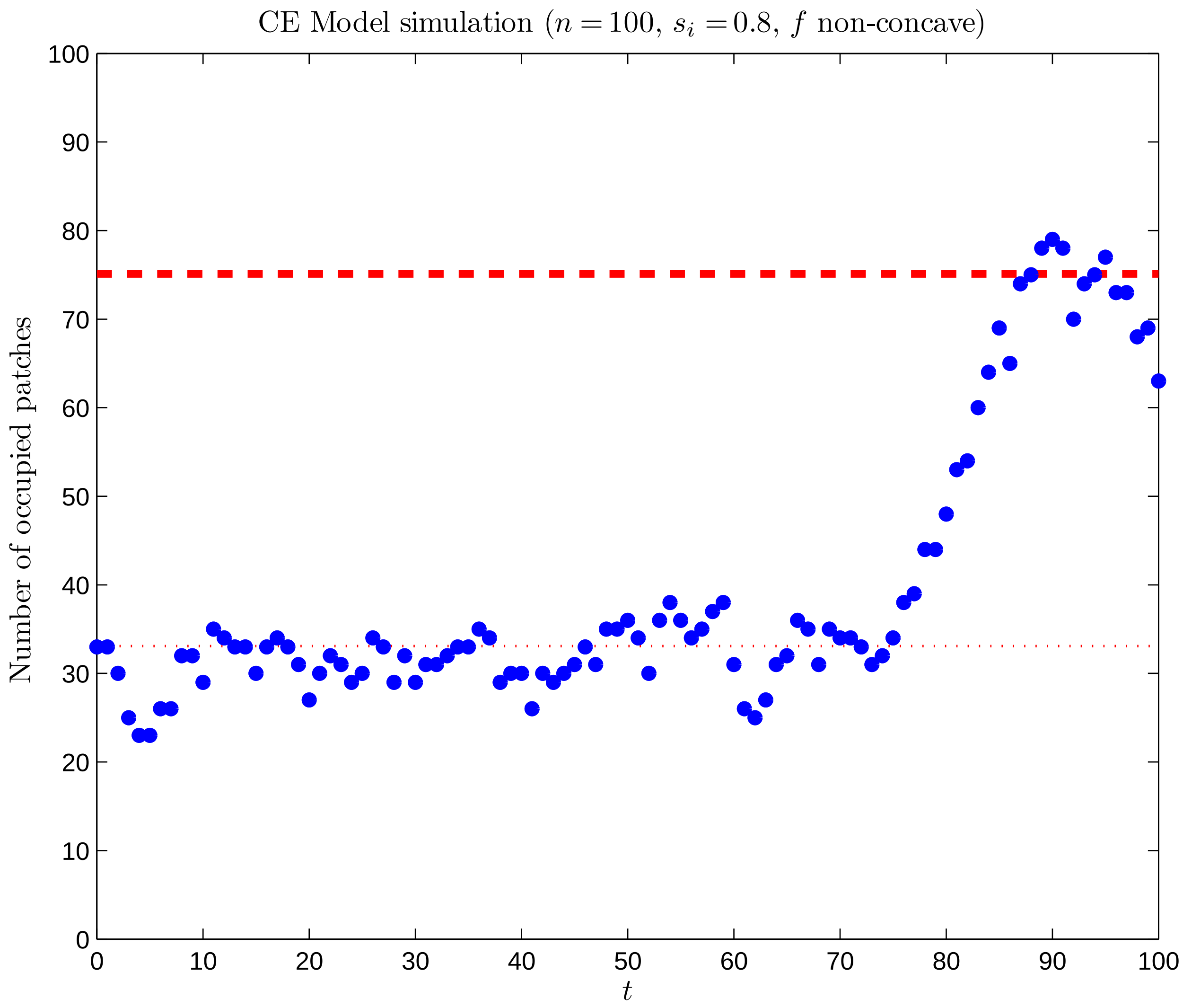

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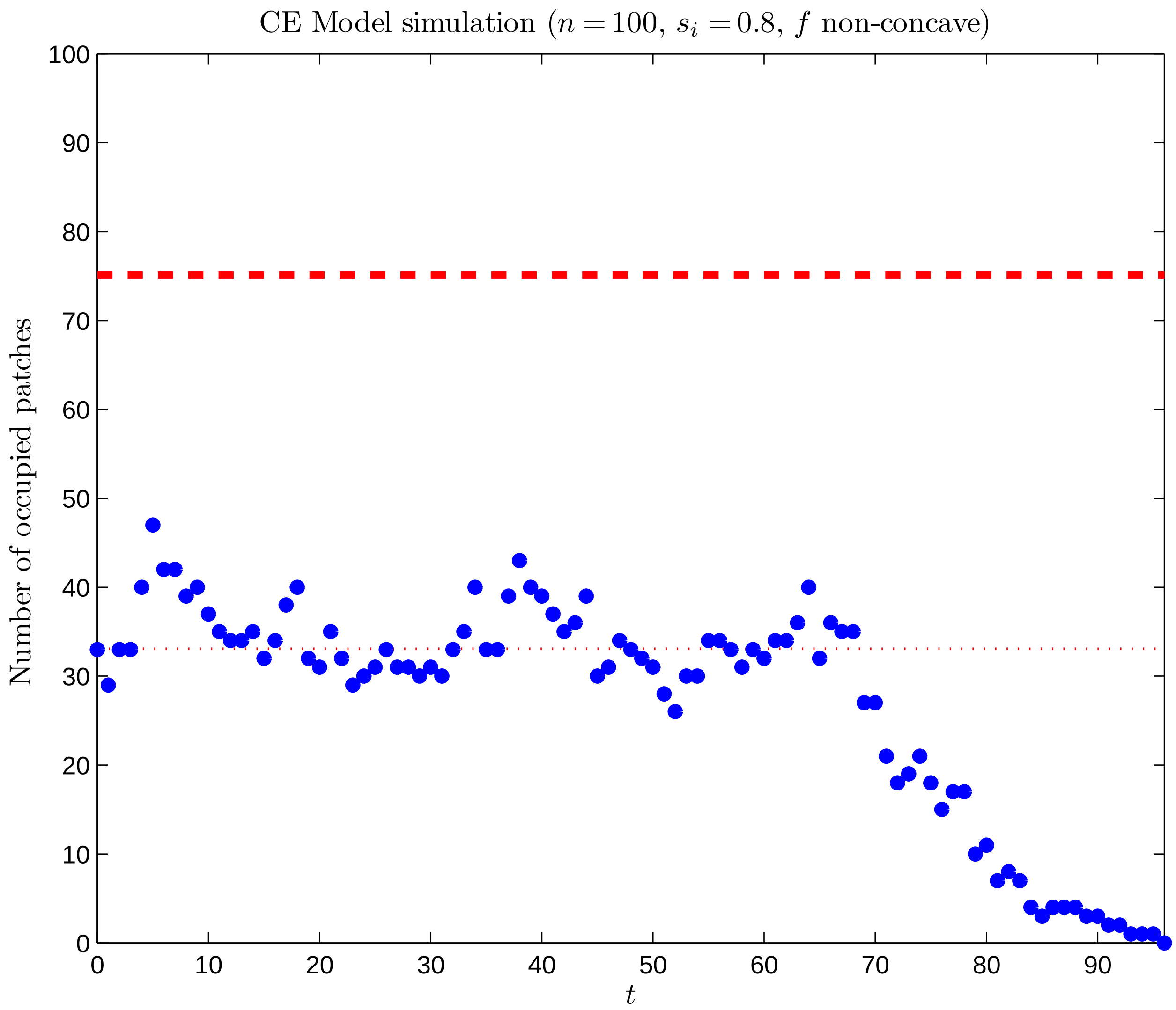

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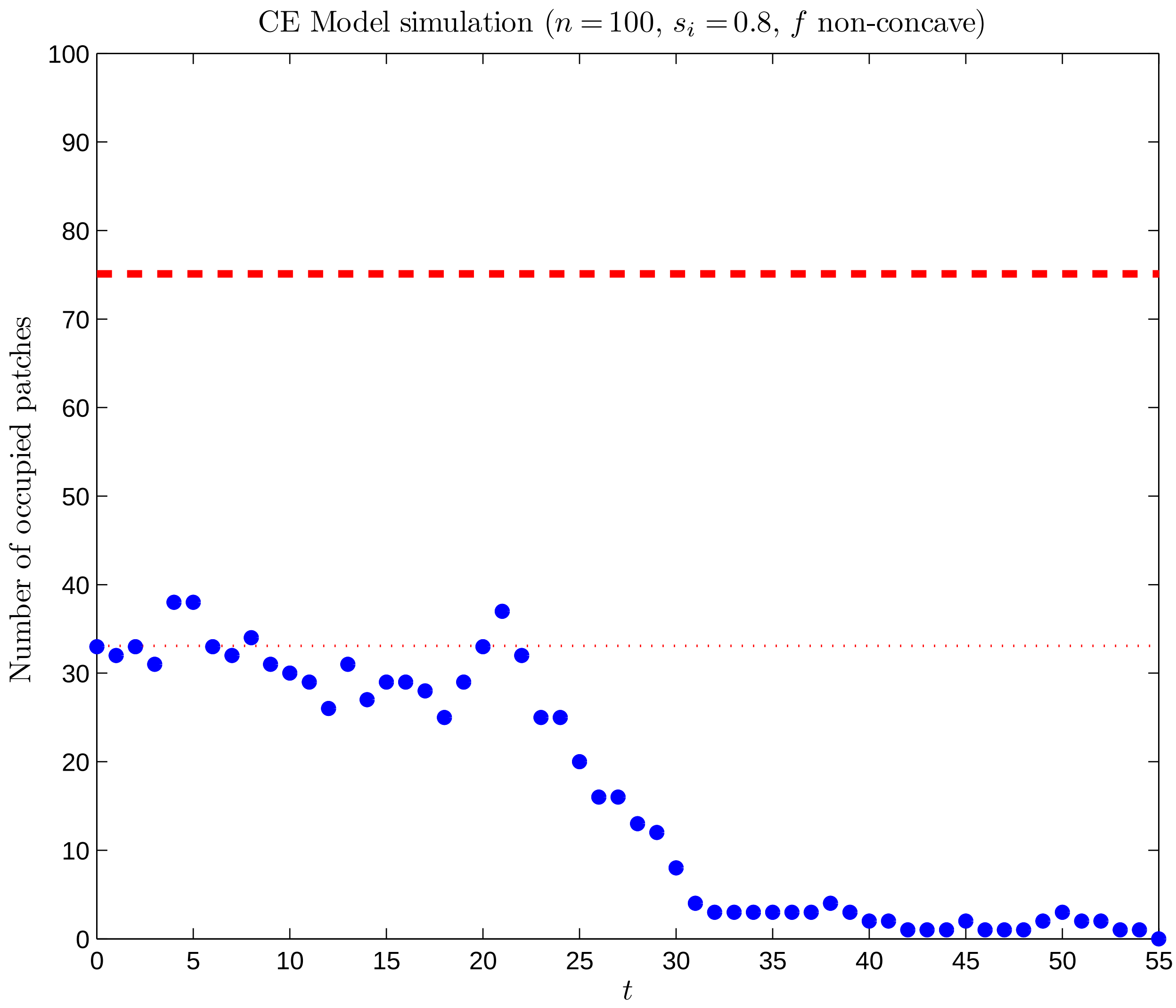

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