1. The sample space $S$ of an experiment consists of the non-negative integers up to 10. Let $A$, $B$ and $C$ be the events defined by

$$A = \{0, 1, 2, 4, 7, 8, 9\}, \quad B = \{0, 3, 4, 5, 9, 10\} \quad \text{and} \quad C = \{0, 2, 4, 6, 8, 10\}.$$ 

List the outcomes corresponding to each of the following events: (a) $A \cup B$, (b) $B \cup C$ and (c) $A \cap B^c$. [3]

Verify that the distributive law holds for the events $A$, $B$ and $C$ listed above: that is, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. [1]

2. A survey is classified as successful if usable data is obtained from it. Let $A_i$ denote the event that at most $i$ repetitions of the survey are required for success. Use set notation to identify the following events:

(a) exactly one repetition is required for success, [1]
(b) at least two repetitions are required for success, [1]
(c) at least $i$ (where $i \geq 2$) repetitions are required for success, and [1]
(d) exactly $i$ (where $i \geq 2$) repetitions are required for success. [1]

Is it true that $A_1 \cup A_2 = A_2$? [1]

What then is $A_1 \cap A_2 \cap A_3$? [1]

3. A positive integer (natural number) is chosen. Use a Venn diagram to illustrate the relationship between the following events: $E$, the event that the chosen integer is even, $P$, the event that the chosen integer is prime$^1$, and $D$, the event that the chosen integer is divisible by 3. [3]

Your Venn diagram should delineate seven regions corresponding to disjoint events whose union is the sample space $S$ (which in the present case is the natural numbers). Show that they are non-empty by indicating the smallest number that lies in each region; for example, 6 is both divisible by 3 and even, and so it lies in the region represented by $E \cap D$, and 6 is clearly the smallest such number. [7]

Total [20]

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$^1$A prime number is an integer $p > 1$ that has no positive integer divisors other than 1 and $p$ itself.
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Extra Tutorial Problems

The more challenging problems (marked with an asterisk) are just for fun

1. The following problems are taken from the textbook:


   Chapter 1: Problems 1.1 to 1.11
   Chapter 2: Problems 2.1 to 2.20

2. *Consider a conventional knock-out tournament, such as the Australian Open Tennis Championship. There are $2^n$ competitors and $n$ rounds. By labelling the players $1, 2, \ldots, 2^n$, give a precise description of the sample space $S$ of all outcomes. If we were only interested in who wins the tournament, then we would set $S = \{1, 2, \ldots, 2^n\}$. But, no, try to describe all outcomes of all rounds (not just the final). Verify that $|S| = 2^{2^n} - 1$.*

3. *In a group of 100 people, 58 are men, 60 are married and 32 are working. There are 14 working men, 21 married people who are working, 25 married men and 8 working men who are married. If a person is chosen at random from the group, what is the probability that the person is an unmarried working woman.*

4. *Six cups and saucers come in pairs: two red cups and saucers, two white cups and saucers, and, two cups and saucers (tastefully emblazoned) with stars and stripes. If the cups are randomly placed on the saucers, what is the probability that no cup is upon a saucer of the same pattern.*