1. Suppose that the probability a battery lasts at least 40 hours is 0.95, while the probability it lasts at least 65 hours is 0.7. What is the probability that it lasts at least another 25 hours, given that it has already lasted 40 hours? [5]

2. One of two components is selected at random and tested. Component 1 is faulty with probability $1/5$ and Component 2 is faulty with probability $1/10$. Determine the following probabilities: (a) the probability that the selected component is faulty, and (b) the probability that Component 2 was the one selected given that the selected component is faulty. [9]

3. Items on a production line are inspected one at a time and the probability that a given item is found to be defective is $p = 0.05$, independently of the other items. Suppose that 4 items are inspected. What is the probability of at least one defective item? What is the probability that exactly half are defective? [6]

Total [20]
Probability Models for Engineering & Science

Extra Tutorial Problems

The more challenging problems (marked with an asterisk) are just for fun

1. The following problems are taken from the textbook:


   Chapter 2: Problems 2.21 to 2.108

2. *A biased coin is tossed repeatedly. Each time there is a probability $p$ of heads. Let $p_n$ be the probability that an even number of heads has occurred after $n$ tosses, 0 being an even number. By conditioning on the result of the first toss, derive a difference equation relating elements of the sequence \{${p_n}$\} and use this to obtain an explicit expression for $p_n$, $n \geq 1$.

3. *A bowl contains twenty cherries, exactly fifteen of which have had their stones removed. A greedy pig eats five quickly, without commenting on the presence or absence of stones. Subsequently, a cherry is picked at random from the remaining fifteen.

   (a) What is the probability that this cherry contains a stone?

   (b) Given that this cherry contains a stone, what is the probability that the pig consumed at least one stone?

4. *A bus has six request stops to visit. At each of these, the bus may or may not be hailed and it stops only when it is hailed. Suppose that, at any given stop, the bus is hailed with probability $p$, independently of its behaviour at the other stops. Determine the probability that the bus passes at least three stops in succession without stopping.

5. *A random number $N$ of dice is thrown and $A_i$ is the event that this number is $i$, with $P(A_i) = (1/2)^i$, $i \geq 1$. If the sum of the scores on the dice is $S$, find the probability that:

   (a) $N = 2$ given that $S = 4$.

   (b) $S = 4$ given that $N$ is even.

   (c) $N = 2$ given that $S = 4$ and the first die showed 1.

   (d) The largest number shown on any die is $r$. 