2010/2011   nb "Soln 3" is to §11.12, Q9, p646 & not in this assignment.

MT ~~2531~~ ASSIGNMENT 10/5

1. $u_{xx} + u_{yy} = 0$, $u(x,y) = F(x)G(y)$

Bdy conditions: $u(x,2) = u(0,y) = u(2,y) = 0$

$u(x,0) = \sin \pi x/2$

$F''/F = -G''/G = k \Rightarrow k = -p^2$ for bdy condns to work

$F(x) = A\cos px + B\sin px$

$F(0) = 0 \Rightarrow A = 0$

$F(2) = 0 \Rightarrow \sin 2p = 0 \Rightarrow 2p = n\pi, \quad n = 1, 2, \cdots$

$\Rightarrow p_n = n\pi/2$

$F_n x = \sin p_n x.$

$G'' - p^2 G = 0 \Rightarrow G_n(y) = A_n e^{p_n y} + B_n e^{-p_n y}$

Bdy condn $u(x,2) = 0 \Rightarrow G_n(2) = 0 \Rightarrow A_n e^{n\pi} + B_n = 0$

$\Rightarrow G_n(y) = 2A_n e^{n\pi/2} \sinh \frac{n\pi}{2}(y-1)$

$$u_n(x,y) = e^{n\pi/2} \sin \frac{n\pi x}{2} \sinh n\pi(y-1)/2$$

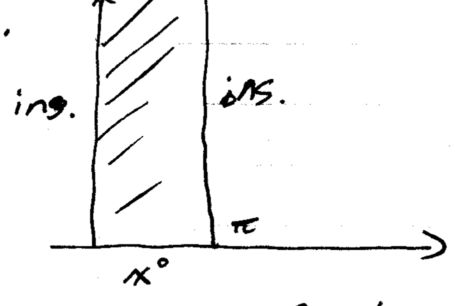
$$u(x,y) = \sum_1 C_n e^{n\pi/2} \sin \frac{n\pi x}{2} \sinh \frac{n\pi(y-1)}{2}$$

Bdy condn $u(x,0) = \sin \pi x/2$

$$\Rightarrow \begin{cases} C_1 = -e^{-\pi/2}/\sinh \pi/2, \\ C_n = 0, \quad n = 2, 3, \cdots \end{cases}$$

$$\Rightarrow u(x,y) = \frac{-1}{e^{\pi/2}\sinh \pi/2} \sin \pi x/2 \sinh \pi(y-1)/2$$

2.



$u_{xx} + u_{yy} = 0$ gives

$F''/F = -G''/G = k$

Bdy condns
$$\begin{cases} u_x(0,y) = u_x(\pi,y) = 0 \\ u(x,y) \text{ bdd as } y \to \infty \\ u(x,0) = x \end{cases}$$

$k = -p^2$, $F(x) = A\cos px + B\sin px$

$F'(x) = p(-A\sin px + B\cos px)$

Bdy condns
$$\begin{cases} F'(0) = 0 \Rightarrow B = 0 \\ F'(\pi) = 0 \Rightarrow \sin p\pi = 0, \quad p\pi = n\pi, \quad n = 0, 1, 2, \cdots \end{cases}$$

$\Rightarrow p = n, \quad n = 0, 1, 2, \cdots$

$F_n(x) = \cos nx, \quad n = 0, 1, 2, \cdots$