$$G'' - n^2 G = 0 \Rightarrow G(y) = A e^{ny} + B e^{-ny}$$

$G(y)$ bdd as $y \to \infty \Rightarrow A = 0$

$$G_n = B_n e^{-ny}$$

$$u_n(x, y) = F_n(x)\, G_n(y)$$
$$= \cos nx\, e^{-ny} \quad , n = 0, 1, 2, \cdots$$

$$u(x, y) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx\, e^{-ny}$$

Bdy condn $u(x, 0) = x, \quad 0 \le x \le \pi$

$$= A_0 + \sum_{n=1}^{\infty} A_n \cos nx$$

$$\Rightarrow A_0 = \frac{1}{\pi} \int_0^{\pi} x\, dx = \pi/2$$

$$A_n = \frac{2}{\pi} \int_0^{\pi} x \cos nx\, dx = \begin{cases} 0, & n \text{ even} \\ -4/(2k-1)^2 \pi, & n \text{ odd}, n = 2k-1 \end{cases}$$

$$u(x, y) = \pi/2 - \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos(2k-1)x}{(2k-1)^2} e^{-(2k-1)\pi}$$

3. $w(x, t) = \frac{x}{2c\sqrt{\pi}} \int_0^t f(t-\tau)\, \tau^{-3/2} e^{-x^2/4c^2\tau}\, d\tau$

$u(t)$ unit step function, $u(t-\tau) = 1$ on $[0, t]$

$$= \frac{x}{2c\sqrt{\pi}} \int_0^t u(t-\tau)\, \tau^{-3/2} e^{-x^2/4c^2\tau}\, d\tau$$

$$= \frac{x}{2c\sqrt{\pi}} \int_0^t \tau^{-3/2} e^{-x^2/4c^2\tau}\, d\tau$$

Put $\sigma = x/2c\sqrt{\tau}, \quad d\sigma = -\frac{1}{4} \frac{x}{2c} \tau^{-3/2} d\tau$

$\tau = t, \ \sigma = x/2c\sqrt{t}, \ \tau \to 0+, \ \sigma \to \infty$

$$w(x, t) = \frac{-2}{\sqrt{\pi}} \int_{\infty}^{x/2c\sqrt{t}} e^{-\sigma^2} d\sigma$$

$$= \frac{2}{\sqrt{\pi}} \int_{x/2c\sqrt{t}}^{\infty} e^{-\sigma^2} d\sigma$$

$$= \frac{2}{\sqrt{\pi}} \left(\int_0^{\infty} - \int_0^{x/2c\sqrt{t}} \right)$$

$$= 1 - \operatorname{erf}\left(x/2c\sqrt{t} \right).$$