11. $u(r,\theta,t) = F(r,\theta)G(t)$, $u_{tt} = F\ddot{G}$, $u_r = F_r G$, $u_{rr} = F_{rr}G$, $u_{\theta\theta} = F_{\theta\theta}G$

$$F\ddot{G} = c^2(F_{rr} + F_r/r + F_{\theta\theta}/r^2)$$

$\ddot{G}/c^2G = (F_{rr} + F_r/r + F_{\theta\theta}/r^2)/F = -k^2$, since LHS only a function of t, RHS of r,θ, so both a constant μ. If $\mu \geq 0$, $G(t) \to \infty$ (infinite displacement) so, as always, $\mu = -k^2$. This gives $\ddot{G} + (ck)G = 0$, $F_{rr} + F_r/r + F_{\theta\theta}/r^2 + k^2F = 0$

12. $F(r,\theta) = W(r)Q(\theta)$, $F_r = W'Q$, $F_{rr} = W''Q$, $F_{\theta\theta} = WQ''$

$(W'' + W'/r + k^2W)Q + WQ''/r^2 = 0$

$(r^2W'' + rW' + k^2r^2W)/W = -Q''/Q = \text{const}$, say l.

If $l \leq 0$, by periodicity of polars $Q(0) = Q(2\pi) \Rightarrow Q(\theta) = 0$

So must have something a bit different $l = +n^2$!!

$$Q'' + n^2 Q = 0$$

$$r^2W'' + rW' + (kr^2 - n^2)W = 0$$

13. θ in polar coordinates has period 2π, So

$Q(\theta) = a_n \cos n\theta + b_n \sin n\theta$ has $Q(0) = Q(2\pi)$

and n must be a positive integer, $n = 0, 1, 2, \cdots$

Or, if you like $Q_n(\theta) = \cos n\theta$, $Q_n^*(\theta) = \sin n\theta$ are linearly indept. solutions. In the equation for W, put $s = rk$, $W' = \frac{dW}{ds}k$, $W'' = \frac{d^2W}{ds^2}k^2$, so the eqn becomes

$$\left(\frac{s}{k}\right)^2 \frac{d^2W}{ds^2}k^2 + \frac{s}{k}\frac{dW}{ds}k + (s^2 - n^2)W = 0$$

$$s^2 d^2W/ds^2 + s\,dW/ds + (s^2 - n^2)W = 0$$

Bessel's equation solutions $J_n(s)$, $Y_n(s)$. But $Y_n(s) \to \infty$ as $s = rk \to 0$ (centre of membrane), so can only have $J_n(s) = J_n(rk)$, $n = 0, 1, 2, \cdots$

14. $u_{kn} = F_n(r,\theta)G_k(t) = \cos n\theta\, J_n(kr)(A\cos ckt + B\sin ckt)$

$u_{kn}^* = F_n^*(r,\theta)G_k(t) = \sin n\theta\, J_n(kr)(A\cos ckt + B\sin ckt)$

$u_{kn}(R,\theta,t) = u_{kn}^*(R,\theta,t) = 0 \Rightarrow J_n(kR) = 0$. If α_{mn} is the m-th zero of $J_n(r)$, then $kR = k_{mn}R = \alpha_{mn}$.