4. $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left(\int_{-\pi}^{-\pi/2} 1\,dx + \int_{-\pi/2}^{0}(-1)\,dx\right) = 0$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{2\pi}\left(\int_{-\pi}^{-\pi/2}\cos nx\,dx - \int_{-\pi/2}^{0}\cos nx\,dx\right)$

$= \frac{1}{2\pi n}\left(\left[\sin nx\right]_{-\pi}^{-\pi/2} - \left[\sin nx\right]_{-\pi/2}^{0}\right)$ $\quad \sin n\pi = 0$, $\sin(-\theta) = -\sin\theta$

$= \frac{1}{2\pi n}\left(-2\sin n\pi/2\right) = \frac{-1}{\pi n}\sin n\pi/2$

$a_1 = -\frac{2}{\pi}$, $a_2 = 0$, $a_3 = \frac{2}{3\pi}$, $a_4 = 0$, $a_5 = \frac{-2}{5\pi}$, ---

$a_{2k} = 0$, $k = 1, 2, \ldots$

$a_{2k-1} = 2(-1)^k/k\pi$, $k = 1, 2, \ldots$

$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{-\pi/2}\sin nx\,dx - \int_{-\pi/2}^{0}\sin nx\,dx\right)$

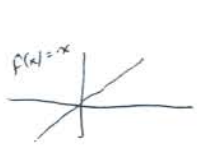
$= \frac{-1}{\pi n}\left(\left[\cos nx\right]_{-\pi}^{-\pi/2} - \left[\cos nx\right]_{-\pi/2}^{0}\right)$ $\quad \cos n\pi = (-1)^n$, $\cos(-\theta) = \cos\theta$

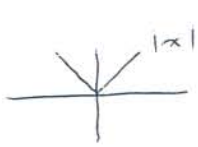
$= \frac{-1}{\pi n}\left(2\cos n\pi/2 - \cos n\pi - 1\right)$

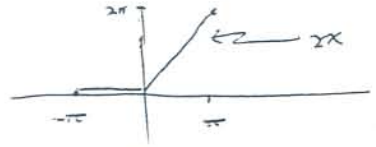
$b_1 = 0$, $b_2 = \frac{2}{\pi}$, $b_3 = 0$, $b_4 = 0$, $b_5 = 0$, $b_6 = \frac{2}{3\pi}$, ---

Series: $-\frac{2}{\pi}\left(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - \ldots\right)$

$+ \frac{2}{\pi}\left(\sin 2x + \frac{1}{3}\sin 6x + \ldots\right)$







$a_0 = \frac{1}{2\pi}\int_0^{\pi} 2x\,dx = \frac{1}{2\pi}\pi^2 = \pi/2$

$\int x\cos nx = \frac{1}{n}x\sin nx - \frac{1}{n}\int \sin nx\,dx = \frac{1}{n}x\sin nx + \frac{1}{n^2}\cos nx$

$\int x\sin nx = -\frac{1}{n}x\cos nx + \frac{1}{n}\int\cos nx\,dx = -\frac{1}{n}x\cos nx - \frac{1}{n^2}\sin nx$

$a_n = \frac{1}{\pi}\int_0^{\pi} 2x\cos nx\,dx = \frac{2}{\pi}\left(0 + \frac{1}{n^2}\left(\cos n\pi - 1\right)\right)$

$= \frac{2}{\pi n^2}\left(\cos n\pi - 1\right) = \frac{2}{\pi n^2}\left((-)^n - 1\right)$

$a_1 = -4/\pi$, $a_2 = 0$, $a_3 = -4/\pi 3^2$, $a_4 = 0$, $a_5 = -4/\pi 5^2$, ---

$b_n = \frac{1}{\pi}\int_0^{\pi} 2x\sin nx\,dx = \frac{2}{\pi}\left(-\frac{\pi}{n}\cos n\pi\right) = -\frac{2}{n}\cos n\pi$

$b_1 = 2$, $b_2 = -\frac{2}{2}$, $b_3 = 2/3$, $b_4 = -2/4$, ---

Series: $\pi/2 - \frac{4}{\pi}\left(\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \ldots\right)$

$+ 2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + \ldots\right)$