4.5.6 $\quad xy'' + y' + \frac{1}{4}y = 0, \quad z = \sqrt{x}$

$y' = dy/dx = dy/dz \, dz/dx = dy/dz \, \frac{1}{2z} = \mathring{y}/2z, \quad \mathring{y} = dy/dz$

$y'' = \frac{d}{dz}(y')\, dz/dx = \frac{d}{dz}\left(\frac{dy}{dz}\frac{1}{2z}\right)\frac{1}{2z} = \left(\ddot{y}/2z - \mathring{y}/2z^2\right)/2z$

Subst. in the equation, noting $z^2 = x$:

$z^2 \cdot \frac{1}{2z}\left(\ddot{y}/2z - \mathring{y}/2z^2\right) + \mathring{y}/2z + \frac{1}{4}y = 0$

$\ddot{y}/4 + \mathring{y}/4z + y/4 = 0$ $\quad$ Multiply by $4z^2$

$z^2\ddot{y} + z\mathring{y} + (z^2 - 0^2)y = 0 \qquad \nu = 0.$

From eq (12), p 220

$$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m}(m!)^2}$$

No second solution is given by this method, so we cannot find a general solution without finding a linearly indept. second solution. This is not covered in the course, but you can find it (if interested; not examinable) in §4.6, p228.