\[ x^2 y'' + y' + \frac{1}{4} y = 0 \]
\[ z = \sqrt{x} \]
\[ y' = \frac{dy}{dx} = \frac{dy}{dz} \frac{dz}{dx} = \frac{dy}{dz} \frac{1}{\frac{dz}{dx}} = \frac{y'}{\sqrt{z}} \]
\[ y'' = \frac{d}{dz} \left( \frac{y'}{\sqrt{z}} \right) = \frac{1}{\sqrt{z}} \left( \frac{dy}{dz} \frac{1}{2z} \right) \frac{1}{2z} = \left( \frac{y''}{2z} - \frac{y'}{2z^2} \sqrt{z} \right) \frac{1}{2z} \]

Subst. in the equation, noting \( z^2 = x \):
\[ z^2 \left( \frac{y''}{2z} - \frac{y'}{2z^2} \sqrt{z} \right) + \frac{y'}{2z} + \frac{1}{4} y = 0 \]
\[ \frac{y''}{4} + \frac{y'}{4z} + \frac{y}{4} = 0 \quad \text{Multiply by} \quad 4z^2 \]
\[ 3z^2 y'' + 3z y' + (3z^2 - 9) y = 0 \quad v = 0 \]

From eq. (12), p 220
\[ J_0 (x) = \sum_{m=0}^\infty \frac{(-1)^m x^{2m}}{2^m (m!)^2} \]

No second solution is given by this method, so we cannot find a general solution without finding a linearly independent second solution. This is not covered in the course, but you can find it (if interested) not examinable) in §4.6, p 228.