3. Boundary conditions $u(0,t)=0$, $u_x(L,t)=0$.

Going back to basics, $F'' - kF = 0$, $\dot{G} - kc^2 G = 0$ and determine k from the bdy condns.

Since $u(x,t) = F(x)G(t)$, get $F(0)=0$, $F'(L)=0$.

$k = \mu^2 > 0$, $F(x) = Ae^{\mu x} + Be^{-\mu x}$

$$F(0)=0 \Rightarrow A+B=0 \Rightarrow F(x) = A(e^{\mu x} - e^{-\mu x})$$

$$F'(L)=0 \Rightarrow A\mu(e^{\mu L} + e^{-\mu L}) = 0 \Rightarrow A = 0$$

Zero solution, impossible.

$k=0$, $F(x) = Ax + B$, $\left.\begin{array}{l} F(0)=0 \Rightarrow B=0 \\ F'(L)=0 \Rightarrow A=0 \end{array}\right\}$ impossible again

$k = -p^2 < 0$, $F(x) = A\cos px + B\sin px$

$$F(0)=0 \Rightarrow A=0 \Rightarrow F(x) = B\sin px$$

$$F'(L)=0 \Rightarrow Bp\cos pL = 0$$

$$\Rightarrow \cos pL = 0 \Rightarrow pL = \pi/2, 3\pi/2, 5\pi/2, \dots$$

So $p_n = (2n-1)\pi/2L$, $n=1,2,\dots$.

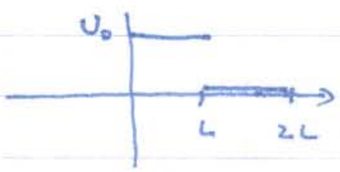
$$F_{2n-1}(x) = B_{2n-1}\sin(2n-1)\pi x/2L$$

$$G_{2n-1}(t) = e^{-\lambda_{2n-1}t}, \quad \lambda_{2n-1} = c^2(2n-1)\pi/2L$$

$$u(x,t) = \sum_{n=1}^{\infty} B_{2n-1}e^{-\lambda_{2n-1}t}\sin(2n-1)\pi x/2L$$

$$u(x,0) = U_0 = \sum_{n=1}^{\infty} B_{2n-1}\sin(2n-1)\pi x/2L$$

$$B_{2n-1} = \frac{2}{L}\int_0^L U_0 \sin(2n-1)\pi x/_{2L}\, dx$$



$$= -\frac{4U_0}{\pi(2n-1)}\Big[\cos(2n-1)\pi x/2L\Big]_0^L$$

$$= \frac{4U_0}{\pi(2n-1)}\Big[1 - \underbrace{\cos(2n-1)\pi/2}_{=0,\ n=1,2,\dots}\Big]$$

$$= 4U_0/\pi(2n-1).$$

$$u(x,t) = \frac{4U_0}{\pi}\sum_{n=1}^{\infty}\sin\frac{(2n-1)\pi x}{2L}\,\frac{e^{-c^2(2n-1)\pi t/2L}}{2n-1}$$