MATH2010
ANALYSIS OF ORDINARY DIFFERENTIAL EQUATIONS
(Unit Courses)

Time: ONE HOUR for working
TEN minutes for perusal before examination begins

CREDIT WILL ONLY BE GIVEN FOR WORK WRITTEN ON THIS
EXAMINATION SCRIPT.
Answer ALL questions.
All questions carry the same number of marks.
Pocket calculators may not be used.
Check that this examination paper has 11 printed pages!

The Laplace transform table may be removed for convenience in working.

FAMILY NAME (PRINT): ______________________________

GIVEN NAMES (PRINT): ______________________________

STUDENT NUMBER: ______________________________

SIGNATURE: ______________________________

<table>
<thead>
<tr>
<th>EXAMINER’S USE ONLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>QUESTION</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>TOTAL</td>
</tr>
</tbody>
</table>
1. (a) Solve the initial value problem
\[ x' = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} x + \begin{bmatrix} 2e^{2t} \\ 3e^{2t} \end{bmatrix}, \quad x(0) = \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix} \]
(b) Consider the input-output system

\[ x'(t) = Ax(t) + bu(t) \]
\[ y(t) = cx(t) \]

where \( x(t) \) is the state of the system, \( u(t) \) is scalar input and \( y(t) \) is scalar output. Find the transfer function of the system. When \( u(t) = e^{-t} \), find the output as a function of \( t \).
2. (a) Find the Laplace transforms of the functions
   
   (i) \( f(t) = 4t(u(t) - u(t - 2)) \),

   (ii) \( f(t) = t \sin t \).
2. (b) Find the function whose Laplace transform is
\[
\frac{3(s + 1)}{s^2 + 6s + 10},
\]
2. (c) Use the Laplace transform technique to solve the initial value problem

\[ x'' - 5x' + 6x = r(t), \quad x(0) = 1, \quad x'(0) = -2, \]

where \( r(t) = 4e^t \) if \( 0 < t < 2 \) and \( r(t) = 0 \) if \( t > 2 \).
3. Locate the equilibrium points of the system

\[
\begin{align*}
    x'_1 &= x_1 - x_1^2 - x_1x_2 \\
    x'_2 &= \frac{1}{2}x_2 - \frac{1}{4}x_2^2 - \frac{3}{4}x_1x_2.
\end{align*}
\]

Determine their type by linearization (do not sketch the trajectories).