Assignment Asterisked Questions

MATH2010

Tutorial Sheet 1 - Week 2

$**$1. Find the general solution of the systems

$$x' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} x, \quad x' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} x$$

2. Find the general solutions of the following systems

$$x' = \begin{pmatrix} -1 & -4 & 2 \\ 2 & 5 & -1 \\ 2 & 2 & 2 \end{pmatrix} x, \quad x' = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix} x.$$

3. Solve the initial value problems

$$x' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix},$$

$$x' = \begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -11 \\ 4 & -4 & -22 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}.$$

4. Consider the system $tx' = Ax$, where A is a constant matrix. Assuming that $x = \xi t^r$, where ξ is a constant vector, show that ξ and r must satisfy $(A - rI)\xi = 0$ in order to obtain nontrivial solution of this system.

$**$5. (**BONUS**) Use the method from (4) to solve the following systems

$$tx' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} x,$$

$$tx' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} x.$$

(Here we assume that $t > 0$.)

$**$6. Find solutions of the following initial value problems

$$x' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, x' = \begin{pmatrix} 2 & \frac{3}{2} \\ -\frac{3}{2} & -1 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

7. Find the general solution of the system

$$x' = \begin{pmatrix} 10 & -10 & -4 \\ -10 & 1 & -14 \\ -4 & -14 & -2 \end{pmatrix} x.$$

8. Show that all solutions of the system

$$x' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} x$$

approach 0 as $t \to \infty$ if and only if $a + d < 0$ and $ad - bc > 0$.

**9 Find the solution of the initial value problem

$$x' = \begin{pmatrix} -3 & -1 & 2 \\ 0 & -4 & 2 \\ 0 & 1 & -5 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix}.$$

10 Check that $r = 2$ is a triple root of the characteristic equation for the system

$$x' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix} x$$

and find three linearly independent solutions of this system.