Assignment Asterisked Questions

MATH2010

Tutorial Sheet 1 - Week 2

**1.** Find the general solution of the systems

\[ x' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} x, \quad x' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} x. \]

2. Find the general solutions of the following systems

\[ x' = \begin{pmatrix} -1 & -4 & 2 \\ 2 & 5 & -1 \end{pmatrix} x, \quad x' = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \end{pmatrix} x. \]

3. Solve the initial value problems

\[ x' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \]
\[ x' = \begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -11 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 5 \\ 5 \end{pmatrix}. \]

4. Consider the system \( tx' = Ax \), where \( A \) is a constant matrix. Assuming that \( x = \xi t^r \), where \( \xi \) is a constant vector, show that \( \xi \) and \( r \) must satisfy \( (A - rI)\xi = 0 \) in order to obtain nontrivial solution of this system.

**5.** (BONUS) Use the method from (4) to solve the following systems

\[ tx' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} x, \]
\[ tx' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} x. \]

(Here we assume that \( t > 0 \).)

**6.** Find solutions of the following initial value problems

\[ x' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad x' = \begin{pmatrix} 2 \\ -3 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}. \]

7. Find the general solution of the system

\[ x' = \begin{pmatrix} 10 & -10 & -4 \\ -10 & 1 & -14 \end{pmatrix} x. \]

8. Show that all solutions of the system

\[ x' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} x \]

approach 0 as \( t \to \infty \) if and only if \( a + d < 0 \) and \( ad - bc > 0 \).
9 Find the solution of the initial value problem
\[ x' = \begin{pmatrix} -3 & -1 & 2 \\ 0 & -4 & 2 \\ 0 & 1 & -5 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix}. \]

10 Check that \( r = 2 \) is a triple root of the characteristic equation for the system
\[ x' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix} x \]
and find three linearly independent solutions of this system.