Assignment Asterisked Questions

MATH2010

Tutorial Sheet 3 - Week 4

**1. Find the inverse Laplace transform of the following functions

$$\frac{2s+1}{s^2-2s+2}, \qquad \frac{8s^2-4s+2}{s(s^2+1)}, \qquad \frac{6}{(s-1)^4}.$$

2. Let $F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$. Assuming that f is continuous on $[0, \infty)$ and that $|f(t)| \leq Ke^{at}$ for $t \geq 0$ and some constants $a > 0$ and $K > 0$, it is possible to show that it is legitimate to differentiate under the integral sign with respect to the parameter s when $s > a$. Check that

$$F'(s) = \mathcal{L}\{-tf(t)\}, \quad F^{(n)}(s) = \mathcal{L}\{(-1)^n f(t)\}.$$

Hence evaluate the Laplace transforms of

$$t^n e^{at}\,(n \geq 1, \text{ integer }) \text{ and } t^2 \sin bt.$$

**3. Use the Laplace transform to solve the initial value problem

$$y'' + \omega^2 y = \cos 2t,\ \omega^2 \neq 4,\ y(0) = 1,\ y'(0) = 0.$$

4. Use the Laplace transform to solve the initial value problem

$$y'''' - 4y''' + 6y'' - 4y' + y = 0,\ y(0) = 0,\ y'(0) = 1,\ y''(0) = 0,\ y'''(0) = 1.$$

5. In each of problems (i) through (v) use the differentiation formula $-F'(s) = \mathcal{L}\{tf(t)\}$ to find the Laplace transform of the function.

(i) $2t \cos 2t$.

**(ii) $t \sin \omega t$.

(iii) $t \sinh 3t$.

**(iv) $t^2 e^t$.

(v) $t^2 \cos t$.