Question 1

(a) Find the solution of the initial value problem
\[ y'' + 3y' + 2y = 2x^2 + 6x + 7 + e^x, \]
\[ y(0) = y'(0) = 0. \]

(b) Find the general solution of the differential equation
\[ y'' - 6y' + 9y = 18 \sin x + 2e^{3x}. \]

Question 2

(a) Find the Laplace transforms of the functions
\[
(i) \quad f(x) = x^3 e^{-2x} + x^2 e^{3x}, \\
(ii) \quad f(t) = t \cos t + t \sin t.
\]

(b) Find the functions whose Laplace transforms are
\[
(i) \quad \frac{3s}{s^2 + 4s + 13}, \\
(ii) \quad \frac{1}{s^2(s^2 + 1)}.
\]

(c) Use the Laplace Transform technique to solve the initial value problem
\[ y'' + y = t^2, \quad y(0) = 6, \quad y'(0) = 0. \]

Question 3

Let \( r(t) = \begin{cases} 
  t + \pi/2 & \text{if } -\pi < t \leq 0, \\
  -t + \pi/2 & \text{if } 0 < t \leq \pi,
\end{cases} \)
and \( r(t) = r(t + 2\pi) \) for all \( t \). Find the Fourier series of \( r(t) \). From the Fourier series find the steady state solution of
\[ y'' + 0.02y' + 25y = r(t). \]

Question 4
(a) Given that \( y_1(x) = x^{-1} \cos x \) is a solution of the differential equation

\[
xy'' + 2y' + xy = 0,
\]

find a second linearly independent solution. Find the general solution of

\[
xy'' + 2y' + xy = x.
\]

(b) Find the Laplace transform of the function

\[
f(t) = t, \quad 0 < t < 2\pi, \quad f(t) = f(t + 2\pi), \quad t > 0.
\]

**Question 5**

(a) Let \( y = (y_1 \ y_2)^T \). Find the general solution of the system

\[
y' = \begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix} y
\]

Identify the nature of the equilibrium point at the origin and give a rough sketch of some trajectories.

(b) Convert the differential equation

\[
y'' + y' + y - y^4 = 0
\]

to a system of first order differential equations. Find the critical points of the system and determine their type by linearization. *(Do not sketch the trajectories.)*