Online Supplementary Materials

1. Probability distributions

Table 1. Probability distributions.

Notation	Description
$\mathsf{U}(a,b)$	uniform distribution with density $f(x;a,b)=\frac{1}{b-a}$ for $x\in[a,b]$
$\mathsf{N}(\mu,\sigma^2)$	normal distribution with density $f(x;\mu,\sigma^2)=(2\pi\sigma^2)^{-\frac{1}{2}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
$\mathsf{Gamma}(\alpha,\beta)$	Gamma distribution with density $f(x;\alpha,\beta)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}\mathrm{e}^{-\frac{x}{\beta}}$
$\mathsf{IG}(\alpha,\beta)$	Inverse-Gamma distribution with density $f(x;\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{-(\alpha+1)}\mathrm{e}^{-\frac{\beta}{x}}$
$\mathsf{t}(\nu,\mu,\sigma^2)$	Student's t distribution with density $f(x;\nu,\mu,\sigma^2)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\;\Gamma\left(\frac{\nu}{2}\right)\sigma}\left(1+\frac{1}{v}\left(\frac{x-\mu}{\sigma}\right)^2\right)^{-\frac{\nu+1}{2}}$
$\mathsf{MVN}(\boldsymbol{\mu},\Sigma)$	Multivariate normal distribution with density $f(\mathbf{x};\boldsymbol{\mu},\Sigma)=\lvert 2\pi\Sigma\rvert^{-\frac{1}{2}}\,\mathrm{e}^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\top}\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})}$
$\mathsf{IW}(A,a)$	Inverse-Wishart distribution with density $f(\mathbf{x};A,a)=\frac{\lvert\mathbf{x}\rvert^{-\frac{(a+p+1)}{2}}\exp\left\{-\frac{1}{2}\mathrm{trace}\left(A\mathbf{x}^{-1}\right)\right\}}{2^{\frac{ap}{2}}\lvert A\rvert^{-\frac{a}{2}}\Gamma_p\left(\frac{a}{2}\right)}$
	where Γ_p is the multivariate Gamma function

2. Conditional distributions derivation

2.1. Conditional distribution of the factor loading matrix $\boldsymbol{\Lambda}$

The conditional distribution of the factor loading matrix is:

$$\begin{aligned}
p(\boldsymbol{\Lambda}|\mathbf{y},\boldsymbol{\Phi},\boldsymbol{\Psi}_\epsilon,\boldsymbol{\omega},\boldsymbol{\nu},\boldsymbol{Z}) &\propto \left(\prod_{i=1}^{n} p(\mathbf{y}_i|\boldsymbol{\Lambda},\boldsymbol{\Phi},\boldsymbol{\Psi}_\epsilon,\boldsymbol{Z},\boldsymbol{\nu},\boldsymbol{\omega}_i)\right)p(\boldsymbol{\Lambda})\\
&\propto |\boldsymbol{\Psi}_\epsilon|^{-0.5n}\left(\prod_{i=1}^{n}|\boldsymbol{Z}_i|^{-0.5}\right)\exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}(\mathbf{y}_i-\boldsymbol{\Lambda}\boldsymbol{\omega}_i)^{\top}(\boldsymbol{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}(\mathbf{y}_i-\boldsymbol{\Lambda}\boldsymbol{\omega}_i)\right]\right\}\times\\
&\quad\times\prod_{k=1}^{p}\prod_{j=1}^{q}\sigma_{k,j}^{-1}\exp\left\{-\frac{1}{2}\left(\frac{\lambda_{k,j}}{\sigma_{k,j}}\right)^2\right\},
\end{aligned}$$

and therefore for $k=1,\ldots,p$ and $j=1,\ldots,q$,

$$\begin{aligned}
p(\lambda_{k,j}|\mathbf{y},\boldsymbol{\Lambda}_{-\lambda_{k,j}},\boldsymbol{\Phi},\boldsymbol{\Psi}_\epsilon,\boldsymbol{\omega},\boldsymbol{\nu},\boldsymbol{Z}) &\propto \exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}\frac{\left(y_{i,k}-\boldsymbol{\Lambda}_k\boldsymbol{\omega}_i\right)^2}{\psi_k z_{i,k}}\right]\right\}\times\\
&\quad\times\sigma_{k,j}^{-1}\exp\left\{-\frac{1}{2}\left(\frac{\lambda_{k,j}}{\sigma_{k,j}}\right)^2\right\}\\
&\propto \exp\left\{-\frac{1}{2}\left[\frac{\lambda_{k,j}^2}{\sigma_{k,j}^2}+\sum_{i=1}^{n}\frac{\left(y_{i,k}-\boldsymbol{\Lambda}_k\boldsymbol{\omega}_i\right)^2}{\psi_k z_{i,k}}\right]\right\}\\
&\propto \exp\left\{-\frac{1}{2}\left[\frac{\lambda_{k,j}^2}{\sigma_{k,j}^2}+\sum_{i=1}^{n}\frac{\left(y_{i,k}-\sum_{l=1}^{q}\lambda_{kl}\omega_{il}\right)^2}{\psi_k z_{i,k}}\right]\right\}\\
&\propto \exp\left\{-\frac{1}{2}\left[\frac{\lambda_{k,j}^2}{\sigma_{k,j}^2}+\sum_{i=1}^{n}\frac{\left((y_{i,k}-\sum_{l=1}^{q}\lambda_{kl}\omega_{il}\mathbb{1}_{\{l\neq j\}})-\lambda_{k,j}\omega_{i,j}\right)^2}{\psi_k z_{i,k}}\right]\right\}\\
&\propto \exp\left\{-\frac{1}{2}\left[\lambda_{k,j}^2\left(\frac{1}{\sigma_{k,j}^2}+\sum_{i=1}^{n}\frac{\omega_{i,j}^2}{\psi_k z_{i,k}}\right)-2\lambda_{k,j}\sum_{i=1}^{n}\frac{\omega_{i,j}(y_{i,k}-\sum_{l=1}^{q}\lambda_{kl}\omega_{il}\mathbb{1}_{\{l\neq j\}})}{\psi_k z_{i,k}}\right]\right\}.
\end{aligned}$$

The above equation can be recognized as a quadratic form of the normal distribution with mean μ and variance σ^2 such that

$$\frac{1}{\sigma^2} = \left(\frac{1}{\sigma^2_{k,j}} + \sum_{i=1}^{n} \frac{\omega^2_{i,j}}{\psi_k z_{i,k}}\right) \text{ and } \frac{\mu}{\sigma^2} = \sum_{i=1}^{n} \frac{\omega_{i,j}(y_{i,k} - \sum_{l=1}^{q} \lambda_{kl}\omega_{il}\mathbb{1}_{\{l\neq j\}})}{\psi_k z_{i,k}},$$

and thus, $\lambda_{k,j}|\mathbf{y}, \mathbf{\Lambda}_{-\lambda_{k,j}}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}, \boldsymbol{\nu}, \boldsymbol{Z} \sim \mathsf{N}\left(\mu_{\lambda_{k,j}}, \sigma^2_{\lambda_{k,j}}\right)$, where

$$\sigma^2_{\lambda_{k,j}} = \left(\frac{1}{\sigma^2_{k,j}} + \sum_{i=1}^{n} \frac{\omega^2_{i,j}}{\psi_k z_{i,k}}\right)^{-1} \text{ and } \mu_{\lambda_{k,j}} = \sigma^2_{\lambda_{k,j}} \left(\sum_{i=1}^{n} \frac{\omega_{i,j}(y_{i,k} - \sum_{l=1}^{q} \lambda_{kl}\omega_{il}\mathbb{1}_{\{l\neq j\}})}{\psi_k z_{i,k}}\right).$$

2.2. Conditional distribution of the covariance matrix $\mathbf{\Phi}$

The conditional distribution of the general covariance matrix $\mathbf{\Phi}$ is:

$$\begin{aligned}
p(\mathbf{\Phi}|\mathbf{y}, \mathbf{\Lambda}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}, \boldsymbol{\nu}, \boldsymbol{Z}) &\propto \left(\prod_{i=1}^{n} p(\mathbf{y}_i|\mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}_i, \boldsymbol{\nu}, \boldsymbol{Z})\right) \times \\
&\times \prod_{i=1}^{n} p(\boldsymbol{\omega}_i|\mathbf{\Phi})p(\mathbf{\Phi}) \propto \prod_{i=1}^{n} p(\boldsymbol{\omega}_i|\mathbf{\Phi})p(\mathbf{\Phi}) \\
&\propto |\mathbf{\Phi}|^{-0.5n} \exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}\left(\boldsymbol{\omega}_i^\top \mathbf{\Phi}^{-1}\boldsymbol{\omega}_i\right)\right]\right\} \times \frac{|\mathbf{\Phi}|^{-\frac{(a+q+1)}{2}} \exp\left\{-\frac{1}{2}\text{trace}\left(A\mathbf{\Phi}^{-1}\right)\right\}}{2^{\frac{aq}{2}}\,|A|^{-\frac{a}{2}}\,\Gamma_q\left(\frac{a}{2}\right)} \\
&\propto |\mathbf{\Phi}|^{-\frac{(n+a+q+1)}{2}} \exp\left\{-\frac{1}{2}\left[\text{trace}\left(A\mathbf{\Phi}^{-1}\right) + \sum_{i=1}^{n}\left(\boldsymbol{\omega}_i^\top \mathbf{\Phi}^{-1}\boldsymbol{\omega}_i\right)\right]\right\} \\
&\propto |\mathbf{\Phi}|^{-\frac{(n+a+q+1)}{2}} \exp\left\{-\frac{1}{2}\left[\text{trace}\left(A\mathbf{\Phi}^{-1}\right) + \text{trace}\left(\sum_{i=1}^{n}\left(\boldsymbol{\omega}_i^\top \mathbf{\Phi}^{-1}\boldsymbol{\omega}_i\right)\right)\right]\right\} \\
&\propto |\mathbf{\Phi}|^{-\frac{(n+a+q+1)}{2}} \exp\left\{-\frac{1}{2}\left[\text{trace}\left(\left(A + \left(\sum_{i=1}^{n}\boldsymbol{\omega}_i^\top\boldsymbol{\omega}_i\right)\right)\mathbf{\Phi}^{-1}\right)\right]\right\} \\
&\propto |\mathbf{\Phi}|^{-\frac{(n+a+q+1)}{2}} \exp\left\{-\frac{1}{2}\left[\text{trace}\left(\left(A + \boldsymbol{\omega}^\top\boldsymbol{\omega}\right)\mathbf{\Phi}^{-1}\right)\right]\right\},
\end{aligned}$$

and thus: $\mathbf{\Phi}|\mathbf{y}, \mathbf{\Lambda}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}, \boldsymbol{\nu}, \boldsymbol{Z} \sim \mathsf{IW}\left(A + \boldsymbol{\omega}^\top\boldsymbol{\omega}, a + n\right)$.

2.3. Conditional distribution of the covariance matrix $\mathbf{\Psi}_\epsilon$

The conditional distribution of the diagonal covariance matrix is:

$$p(\mathbf{\Psi}_\epsilon|\mathbf{y}, \mathbf{\Lambda}, \mathbf{\Phi}, \boldsymbol{\omega}, \boldsymbol{\nu}, \boldsymbol{Z}) \propto \left(\prod_{i=1}^{n} p(\mathbf{y}_i|\mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}_i, \boldsymbol{\nu}, \boldsymbol{Z})\right) \prod_{k=1}^{p} p(\psi_k),$$

and therefore, for all $k = 1, \ldots, p$, we arrive at:

$$\begin{aligned}
p(\psi_k|\mathbf{y}, \boldsymbol{\theta} \setminus \mathbf{\Psi}_\epsilon) &\propto \prod_{i=1}^{n}\left[\left(\prod_{k=1}^{p}(\psi_k z_{i,k})\right)^{-0.5} \exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}\left(\mathbf{y}_i - \mathbf{\Lambda}\boldsymbol{\omega}_i\right)^\top \times\right.\right.\right. \\
&\times \left.\left.\left.\left(\prod_{k=1}^{p}\psi_k z_{i,k}\right)^{-1}\left(\mathbf{y}_i - \mathbf{\Lambda}\boldsymbol{\omega}_i\right)\right]\right\}\right] \times \prod_{k=1}^{p} \frac{\beta_k^{\alpha_k}}{\Gamma(\alpha_k)}\psi_k^{-(\alpha_k+1)}\mathrm{e}^{-\frac{\beta_k}{\psi_k}} \\
&\propto \psi_k^{-0.5n} \exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}\left(y_{i,k} - \mathbf{\Lambda}_k\boldsymbol{\omega}_i\right)^\top \psi_k^{-1}(z_{i,k})^{-1}\left(y_{i,k} - \mathbf{\Lambda}_k\boldsymbol{\omega}_i\right)\right]\right\} \times \\
&\times \frac{\beta_k^{\alpha_k}}{\Gamma(\alpha_k)}\psi_k^{-(\alpha_k+1)}\mathrm{e}^{-\frac{\beta_k}{\psi_k}} \\
&\psi_k^{-0.5n} \exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}(z_{i,k})^{-1}\frac{\left(y_{i,k} - \mathbf{\Lambda}_k\boldsymbol{\omega}_i\right)^2}{\psi_k}\right]\right\} \psi_k^{-(\alpha_k+1)}\mathrm{e}^{-\frac{\beta_k}{\psi_k}}
\end{aligned}$$

$$\propto \psi_k^{-(0.5n+\alpha_k+1)} \exp\left\{-\frac{\beta_k}{\psi_k} - \frac{1}{2}\left[\sum_{i=1}^{n}(z_{i,k})^{-1}\frac{\left(y_{i,k} - \mathbf{\Lambda}_k\boldsymbol{\omega}_i\right)^2}{\psi_k}\right]\right\}$$
$$= \psi_k^{-(0.5n+\alpha_k+1)} \exp\left\{-\frac{-\left(\beta_k + \frac{1}{2}\sum_{i=1}^{n}(z_{i,k})^{-1}\left(y_{i,k} - \mathbf{\Lambda}_k\boldsymbol{\omega}_i\right)^2\right)}{\psi_k}\right\}.$$

That is, $\psi_k|\mathbf{y}, \mathbf{\Lambda}, \mathbf{\Phi}, \boldsymbol{\omega}, \boldsymbol{\nu}, \boldsymbol{Z} \sim \mathsf{IG}\left(0.5n + \alpha_k, \beta_k + \frac{1}{2}\sum_{i=1}^{n}(z_{i,k})^{-1}\left(y_{i,k} - \mathbf{\Lambda}_k\boldsymbol{\omega}_i\right)^2\right)$, for $k = 1, \dots, p$.

2.4. Conditional distribution of the latent variables $\boldsymbol{\omega}$

The conditional distribution of the latent variable $\boldsymbol{\omega}$ is:

$$p(\boldsymbol{\omega}|\mathbf{y}, \mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{\nu}, \boldsymbol{Z}) \propto \left(\prod_{i=1}^{n} p(\mathbf{y}_i|\mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}_i, \boldsymbol{\nu}, \boldsymbol{Z})\right)\prod_{i=1}^{m} p(\boldsymbol{\omega}_i|\mathbf{\Phi}),$$

and therefore, for all $i = 1, \dots, n$, we arrive at:

$$p(\boldsymbol{\omega}_i|\boldsymbol{\theta} \setminus \boldsymbol{\omega}) \propto p(\mathbf{y}_i|\mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{Z}, \boldsymbol{\nu}, \boldsymbol{\omega}_i)p(\boldsymbol{\omega}_i|\mathbf{\Phi})$$
$$\propto |\mathbf{\Psi}_\epsilon|^{-0.5}\left(\prod_{i=1}^{n}|\boldsymbol{Z}_i|^{-0.5}\right)\exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}\left(\mathbf{y}_i - \mathbf{\Lambda}\boldsymbol{\omega}_i\right)^\top\left(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i\right)^{-1}\left(\mathbf{y}_i - \mathbf{\Lambda}\boldsymbol{\omega}_i\right)\right]\right\}\times$$
$$\times |\mathbf{\Phi}|^{-0.5}\exp\left\{-\frac{1}{2}\boldsymbol{\omega}_i^\top\mathbf{\Phi}^{-1}\boldsymbol{\omega}_i\right\}$$
$$\propto \exp\left\{-\frac{1}{2}\left[\boldsymbol{\omega}_i^\top\mathbf{\Phi}^{-1}\boldsymbol{\omega}_i + \left(\mathbf{y}_i - \mathbf{\Lambda}\boldsymbol{\omega}_i\right)^\top\left(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i\right)^{-1}\left(\mathbf{y}_i - \mathbf{\Lambda}\boldsymbol{\omega}_i\right)\right]\right\}$$
$$\propto \exp\left\{-\frac{1}{2}\left[\boldsymbol{\omega}_i^\top\mathbf{\Phi}^{-1}\boldsymbol{\omega}_i + (\mathbf{\Lambda}\boldsymbol{\omega}_i)^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{\Lambda}\boldsymbol{\omega}_i - 2(\mathbf{\Lambda}\boldsymbol{\omega}_i)^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{y}_i\right]\right\}$$
$$\propto \exp\left\{-\frac{1}{2}\left[\boldsymbol{\omega}_i^\top\mathbf{\Phi}^{-1}\boldsymbol{\omega}_i + \boldsymbol{\omega}_i^\top\left(\mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{\Lambda}\right)\boldsymbol{\omega}_i - 2\boldsymbol{\omega}_i^\top\left(\mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\right)\mathbf{y}_i\right]\right\}$$
$$\propto \exp\left\{-\frac{1}{2}\left[\boldsymbol{\omega}_i^\top\left(\mathbf{\Phi}^{-1} + \mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{\Lambda}\right)\boldsymbol{\omega}_i - 2\boldsymbol{\omega}_i^\top\left(\mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\right)\mathbf{y}_i\right]\right\}.$$

The above expression can be recognized as a quadratic form of the multivariate normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix Σ such that

$$\Sigma^{-1} = \left(\mathbf{\Phi}^{-1} + \mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{\Lambda}\right) \text{ and } \mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{y}_i = \Sigma^{-1}\boldsymbol{\mu}$$
$$\Rightarrow \quad \boldsymbol{\mu} = \left(\mathbf{\Phi}^{-1} + \mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{\Lambda}\right)^{-1}(\mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{y}_i),$$

and therefore, for $\boldsymbol{\mu}_{\boldsymbol{\omega}_i} = \left(\mathbf{\Phi}^{-1} + \mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{\Lambda}\right)^{-1}\left(\mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{y}_i\right)$ and for $\Sigma_{\boldsymbol{\omega}_i} = \left(\mathbf{\Phi}^{-1} + \mathbf{\Lambda}^\top(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i)^{-1}\mathbf{\Lambda}\right)^{-1}$, it holds that $\boldsymbol{\omega}_i|\mathbf{y}, \mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{Z}_i \sim \mathsf{MVN}\left(\boldsymbol{\mu}_{\boldsymbol{\omega}_i}, \Sigma_{\boldsymbol{\omega}_i}\right)$ for all $1 \leq i \leq n$.

2.5. Conditional distribution of the latent variable $\boldsymbol{Z}$

The conditional distribution of the latent variable $\boldsymbol{Z}$ is:

$$p(\boldsymbol{Z}|\mathbf{y}, \mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}, \boldsymbol{\nu}) \propto p(\mathbf{y}|\mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}, \boldsymbol{\nu}, \boldsymbol{Z})p(\boldsymbol{Z}|\boldsymbol{\nu})$$
$$= |\mathbf{\Psi}_\epsilon|^{-0.5n}\left(\prod_{i=1}^{n}|\boldsymbol{Z}_i|^{-0.5}\right)\exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}\left(\mathbf{y}_i - \mathbf{\Lambda}\boldsymbol{\omega}_i\right)^\top\left(\mathbf{\Psi}_\epsilon\boldsymbol{Z}_i\right)^{-1}\left(\mathbf{y}_i - \mathbf{\Lambda}\boldsymbol{\omega}_i\right)\right]\right\}\times$$
$$\times\prod_{i=1}^{n}\prod_{k=1}^{p}p(z_{i,k}|\nu_k) \propto \prod_{i=1}^{n}\prod_{k=1}^{p}\frac{(0.5\nu_k)^{0.5\nu_k}}{\Gamma(0.5\nu_k)}(z_{i,k})^{-(0.5\nu_k+1)}\mathrm{e}^{-\frac{0.5\nu_k}{z_{i,k}}}.$$

Thus, for all $i = 1, \dots, n$ and $k = 1, \dots, p$, we have:

$$p(z_{i,k}|\mathbf{y}, \mathbf{\Lambda}, \mathbf{\Phi}, \mathbf{\Psi}_\epsilon, \boldsymbol{\omega}, \boldsymbol{\nu}, \boldsymbol{Z}_{-z_{i,k}}) \propto (z_{i,k})^{-0.5}\exp\left\{-\frac{1}{2}\left[\sum_{i'=1}^{n}\frac{\left(y_{i',k} - \mathbf{\Lambda}_k\boldsymbol{\omega}_{i'}\right)^2}{\psi_k z_{i',k}}\right]\right\}\times$$

$$
\begin{aligned}
&\times \frac{(0.5\nu_k)^{0.5\nu_k}}{\Gamma(0.5\nu_k)} (z_{i,k})^{-(0.5\nu_k+1)} \mathrm{e}^{-\frac{0.5\nu_k}{z_{i,k}}} \\
&\propto {z_{i,k}}^{-(0.5+0.5\nu_k+1)} \exp\left\{ -\frac{0.5\nu_k}{z_{i,k}} - \frac{1}{2}\left[\psi_k^{-1} \frac{\left(y_{i,k} - \boldsymbol{\Lambda}_k \boldsymbol{\omega}_i\right)^2}{z_{i,k}} \right] \right\} \\
&= {z_{i,k}}^{-(0.5+0.5\nu_k+1)} \exp\left\{ -\frac{\left(0.5\nu_k + \frac{1}{2}\psi_k^{-1}\left(y_{i,k} - \boldsymbol{\Lambda}_k \boldsymbol{\omega}_i\right)^2\right)}{z_{i,k}} \right\}.
\end{aligned}
$$

That is, $z_{i,k}|\mathbf{y}, \boldsymbol{\Phi}, \boldsymbol{\Psi}_\epsilon, \boldsymbol{\omega}, \boldsymbol{\nu} \sim \mathsf{IG}\left(0.5 + 0.5\nu_k, 0.5\nu_k + \frac{\psi_k^{-1}}{2}\left(y_{i,k} - \boldsymbol{\Lambda}_k \boldsymbol{\omega}_i\right)^2\right)$ for all $k = 1, \ldots, p$.

3. Graphical MCMC convergence diagnostics for the Holzinger Swineford 1939 school data

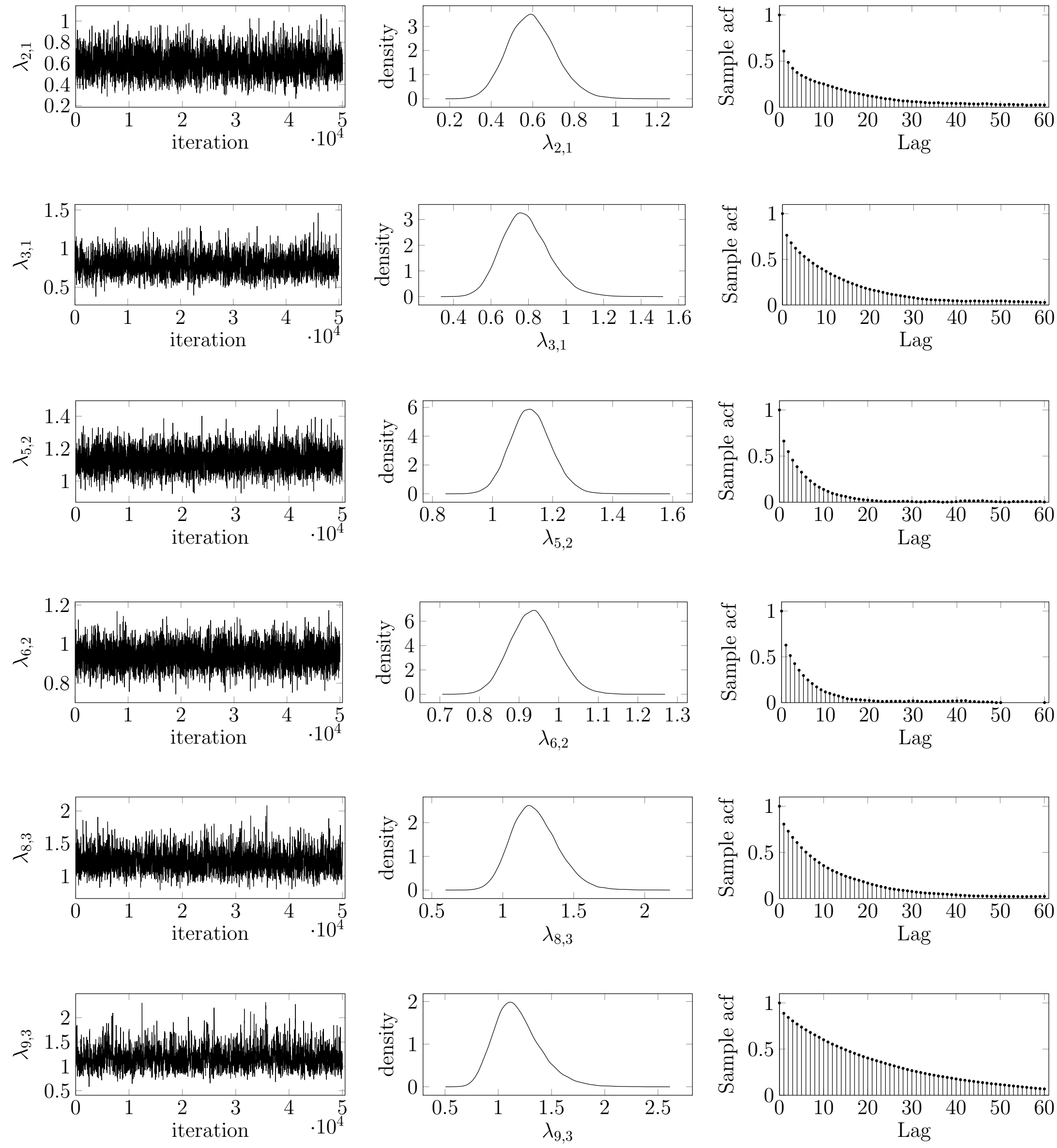

Figure 1. Typical nCFA convergence diagnostics graphical summaries. For each component of the factor loading matrix, three convergence plots (trace plot, kernel density estimator plot, and sample autocorrelation plot) are presented.

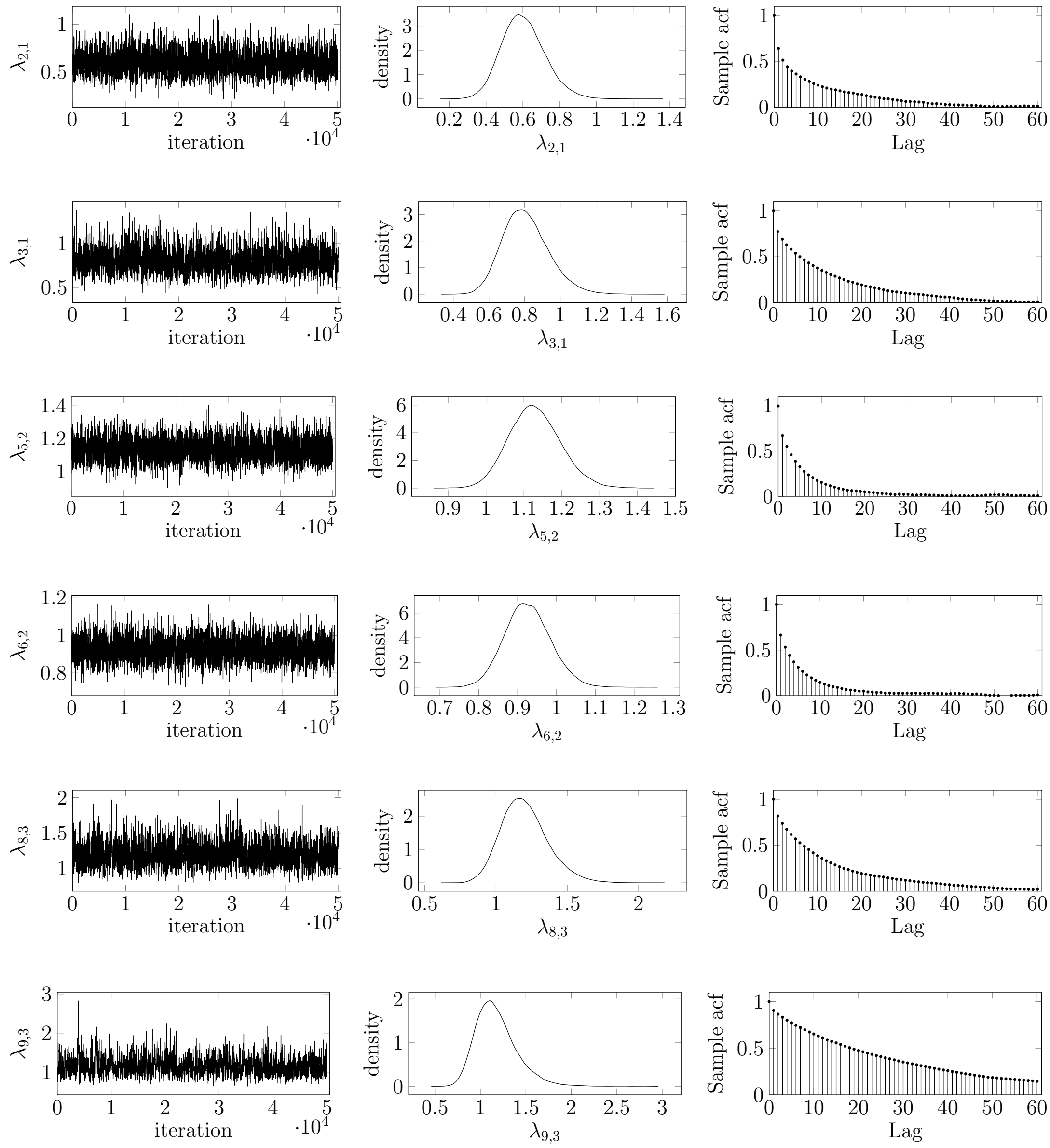

Figure 2. Typical tCFA convergence diagnostics graphical summaries. For each component of the factor loading matrix, three convergence plots (trace plot, kernel density estimator plot, and sample autocorrelation plot) are presented.

4. Typical convergence diagnostic of nCFA and tCFA when applied to the g-and-k measurement error instance of size $n = 250$

Table 2. MCMC summary for the nCFA model for a synthetic dataset of size $n = 250$ with g-and-k measurement errors.

Parameter	true value	mean	sd	2.5%	50%	97.5%	$\widehat{R}$	squared error	bias
ψ_1	0.1	9.285	0.869	7.715	9.237	11.122	1	84.37	9.19
ψ_2	0.2	8.339	1.762	1.892	8.650	10.517	1	66.25	8.14
ψ_3	0.3	8.158	2.617	0.828	8.939	11.255	1	61.75	7.86
ψ_4	0.4	7.363	0.722	6.076	7.336	8.831	1	48.49	6.96
ψ_5	0.5	3.575	2.952	0.314	2.492	9.541	1	9.45	3.07
ψ_6	0.6	12.937	1.216	10.731	12.876	15.483	1	152.20	12.34
ψ_7	0.7	8.161	0.912	6.599	8.166	9.821	1	55.67	7.46
ψ_8	0.8	2.473	2.250	0.277	1.580	8.205	1	2.80	1.67
$\lambda_{2,1}$	0.5	0.760	1.946	-1.928	0.378	6.567	1	0.07	0.26
$\lambda_{3,1}$	0.8	0.985	3.416	-7.382	0.842	8.530	1	0.03	0.19
$\lambda_{5,2}$	0.5	4.845	2.416	0.143	5.101	9.226	1	18.88	4.34
$\lambda_{7,3}$	0.5	0.618	0.656	-0.135	0.507	2.105	1	0.01	0.12
$\lambda_{8,3}$	0.8	4.004	1.676	1.032	3.859	7.684	1	10.27	3.20
$\mathbf{\Phi}_{1,1}$	1	0.244	0.180	0.073	0.193	0.715	1	0.57	-0.76
$\mathbf{\Phi}_{2,1}$	0.5	0.031	0.089	-0.124	0.027	0.223	1	0.22	-0.47
$\mathbf{\Phi}_{3,1}$	0	-0.043	0.100	-0.304	-0.019	0.091	1	0.00	-0.04
$\mathbf{\Phi}_{1,2}$	0.5	0.031	0.089	-0.124	0.027	0.223	1	0.22	-0.47
$\mathbf{\Phi}_{2,2}$	1.5	0.291	0.272	0.091	0.233	0.829	1	1.46	-1.21
$\mathbf{\Phi}_{3,2}$	-0.5	-0.023	0.066	-0.193	-0.011	0.063	1	0.23	0.48
$\mathbf{\Phi}_{1,3}$	0	-0.043	0.100	-0.304	-0.019	0.091	1	0.00	-0.04
$\mathbf{\Phi}_{2,3}$	-0.5	-0.023	0.066	-0.193	-0.011	0.063	1	0.23	0.48
$\mathbf{\Phi}_{3,3}$	2	0.526	0.400	0.121	0.426	1.499	1	2.17	-1.47
Average								23.42	2.79

Table 3. MCMC summary for the tCFA model for a synthetic dataset of size $n = 250$ with g-and-k measurement errors.

Parameter	true value	mean	sd	2.5%	50%	97.5%	$\widehat{R}$	squared error	bias
ψ_1	0.1	1.828	0.380	1.194	1.791	2.678	1	2.987	1.728
ψ_2	0.2	1.345	0.278	0.875	1.319	1.958	1	1.312	1.145
ψ_3	0.3	2.168	0.449	1.418	2.125	3.174	1	3.491	1.868
ψ_4	0.4	2.871	0.643	1.768	2.820	4.265	1	6.104	2.471
ψ_5	0.5	1.699	0.374	1.074	1.662	2.541	1	1.438	1.199
ψ_6	0.6	2.642	0.600	1.641	2.582	3.978	1	4.169	2.042
ψ_7	0.7	1.483	0.312	0.962	1.453	2.183	1	0.613	0.783
ψ_8	0.8	1.908	0.433	1.205	1.858	2.897	1	1.227	1.108
$\lambda_{2,1}$	0.5	0.958	0.181	0.650	0.942	1.366	1	0.210	0.458
$\lambda_{3,1}$	0.8	1.020	0.198	0.675	1.005	1.454	1	0.049	0.220
$\lambda_{5,2}$	0.5	1.218	0.288	0.753	1.182	1.883	1	0.516	0.718
$\lambda_{7,3}$	0.5	0.795	0.140	0.563	0.780	1.110	1	0.087	0.295
$\lambda_{8,3}$	0.8	0.954	0.155	0.693	0.938	1.302	1	0.024	0.154
$\mathbf{\Phi}_{1,1}$	1	1.155	0.311	0.600	1.136	1.819	1	0.024	0.155
$\mathbf{\Phi}_{2,1}$	0.5	0.694	0.203	0.341	0.679	1.128	1	0.038	0.194
$\mathbf{\Phi}_{3,1}$	0	0.822	0.219	0.435	0.807	1.291	1	0.676	0.822
$\mathbf{\Phi}_{1,2}$	0.5	0.694	0.203	0.341	0.679	1.128	1	0.038	0.194
$\mathbf{\Phi}_{2,2}$	1.5	0.829	0.320	0.331	0.789	1.557	1	0.450	-0.671
$\mathbf{\Phi}_{3,2}$	-0.5	0.391	0.195	0.035	0.380	0.805	1	0.793	0.891
$\mathbf{\Phi}_{1,3}$	0	0.822	0.219	0.435	0.807	1.291	1	0.676	0.822
$\mathbf{\Phi}_{2,3}$	-0.5	0.391	0.195	0.035	0.380	0.805	1	0.793	0.891
$\mathbf{\Phi}_{3,3}$	2	2.143	0.552	1.135	2.115	3.304	1	0.020	0.143
ν_1		2.347	0.305	2.012	2.269	3.138	1		
ν_2		2.229	0.207	2.006	2.173	2.770	1		
ν_3		2.365	0.314	2.013	2.284	3.178	1		
ν_4		3.132	0.831	2.082	2.964	5.168	1		
ν_5		2.248	0.231	2.007	2.184	2.856	1		
ν_6		2.398	0.345	2.014	2.309	3.286	1		
ν_7		2.244	0.227	2.007	2.180	2.841	1		
ν_8		2.325	0.305	2.010	2.239	3.135	1		
Average								1.170	0.801

Table 4. Summary of average squared error and average bias for all model parameters under the nCFA and the tCFA models for 100 synthetic datasets of size $n = 250$ with g-and-k measurement errors.

Parameter	squared error (nCFA)	squared error (tCFA)	bias (nCFA)	bias(tCFA)
ψ_1	95.359 ± 33.552	$\mathbf{5.878 \pm 4.982}$	9.621 ± 1.671	$\mathbf{2.289 \pm 0.800}$
ψ_2	63.679 ± 37.333	$\mathbf{3.020 \pm 2.000}$	7.600 ± 2.432	$\mathbf{1.658 \pm 0.520}$
ψ_3	48.454 ± 36.005	$\mathbf{2.901 \pm 1.991}$	6.464 ± 2.582	$\mathbf{1.629 \pm 0.498}$
ψ_4	96.535 ± 37.868	$\mathbf{6.941 \pm 4.583}$	9.642 ± 1.890	$\mathbf{2.506 \pm 0.814}$
ψ_5	28.958 ± 21.698	$\mathbf{2.257 \pm 1.732}$	4.960 ± 2.087	$\mathbf{1.412 \pm 0.514}$
ψ_6	97.473 ± 36.429	$\mathbf{4.145 \pm 3.120}$	9.714 ± 1.763	$\mathbf{1.915 \pm 0.692}$
ψ_7	54.532 ± 38.658	$\mathbf{1.724 \pm 1.703}$	6.909 ± 2.606	$\mathbf{1.189 \pm 0.558}$
ψ_8	33.185 ± 34.736	$\mathbf{1.439 \pm 1.524}$	4.949 ± 2.949	$\mathbf{1.078 \pm 0.525}$
Average	64.772 ± 34.535	$\mathbf{3.538 \pm 2.704}$	7.482 ± 2.248	$\mathbf{1.709 \pm 0.615}$
$\lambda_{2,1}$	3.071 ± 5.094	$\mathbf{0.723 \pm 2.171}$	0.591 ± 1.650	$\mathbf{0.338 \pm 0.780}$
$\lambda_{3,1}$	6.105 ± 7.391	$\mathbf{0.659 \pm 2.585}$	1.401 ± 2.035	$\mathbf{0.143 \pm 0.799}$
$\lambda_{5,2}$	8.785 ± 8.261	$\mathbf{0.441 \pm 0.409}$	2.178 ± 2.010	$\mathbf{0.606 \pm 0.272}$
$\lambda_{7,3}$	3.715 ± 6.540	$\mathbf{0.272 \pm 0.334}$	1.122 ± 1.567	$\mathbf{0.456 \pm 0.254}$
$\lambda_{8,3}$	8.325 ± 7.702	$\mathbf{0.158 \pm 0.256}$	2.322 ± 1.713	$\mathbf{0.310 \pm 0.249}$
Average	6.000 ± 6.998	$\mathbf{0.451 \pm 1.151}$	1.523 ± 1.795	$\mathbf{0.371 \pm 0.471}$
$\mathbf{\Phi}_{1,1}$	$\mathbf{0.054 \pm 0.036}$	0.673 ± 0.587	$\mathbf{-0.196 \pm 0.126}$	0.717 ± 0.398
$\mathbf{\Phi}_{2,1}$	$\mathbf{0.005 \pm 0.017}$	0.718 ± 0.439	$\mathbf{0.037 \pm 0.063}$	0.787 ± 0.314
$\mathbf{\Phi}_{3,1}$	$\mathbf{0.003 \pm 0.009}$	0.354 ± 0.242	$\mathbf{-0.006 \pm 0.052}$	0.544 ± 0.241
$\mathbf{\Phi}_{1,2}$	0.218 ± 0.049	$\mathbf{0.181 \pm 0.191}$	-0.463 ± 0.063	$\mathbf{0.287 \pm 0.314}$
$\mathbf{\Phi}_{2,2}$	1.281 ± 0.270	$\mathbf{0.235 \pm 0.263}$	-1.123 ± 0.143	$\mathbf{-0.310 \pm 0.372}$
$\mathbf{\Phi}_{3,2}$	$\mathbf{0.211 \pm 0.055}$	0.634 ± 0.334	$\mathbf{0.455 \pm 0.068}$	0.770 ± 0.201
$\mathbf{\Phi}_{1,3}$	$\mathbf{0.003 \pm 0.009}$	0.354 ± 0.242	$\mathbf{-0.006 \pm 0.052}$	0.544 ± 0.241
$\mathbf{\Phi}_{2,3}$	$\mathbf{0.211 \pm 0.055}$	0.634 ± 0.334	$\mathbf{0.455 \pm 0.068}$	0.770 ± 0.201
$\mathbf{\Phi}_{3,3}$	2.605 ± 0.614	$\mathbf{0.337 \pm 0.423}$	-1.594 ± 0.252	$\mathbf{-0.226 \pm 0.535}$
Average	0.510 ± 0.124	$\mathbf{0.458 \pm 0.339}$	$\mathbf{-0.271 \pm 0.098}$	0.431 ± 0.313

5. Example usage of the tCFA package

```
library(tCFA)

# we use lavaan syntax for structural equation
# modeling and growth curve models https://lavaan.ugent.be/
cfa_model <- 'visual  =~ x1 + x2 + x3
                textual =~ x4 + x5 + x6
                speed   =~ x7 + x8 + x9'

# sample size
burnsz <- 1000
samplesz <- 5000

# prior parameters
alpha_prior = 1
beta_prior = 1
sigma_prior = 10
max_t_degree_freedom_prior = 100

inputFile = "HolzingerSwineford1939data.csv"
```

```
# mcmc sampling step
tCFA::NormalCFA(inputFile,modelString = cfa_model,
      mcmcOutFilePath = "1N.csv", mcmcLatentOutFilePath = "W1N.csv",
      alpha_prior = alpha_prior,beta_prior = beta_prior,
      sigma_prior = sigma_prior, burnin = burnsz, samples = samplesz,
      seed = 42)

# marginal likelihood estimation
tCFA::NormalCalculateMargLikl(inputFile,modelString = cfa_model,
      mcmcOutFilePath = "1N.csv", mcmcLatentOutFilePath = "W1N.csv",
      alpha_prior = alpha_prior,beta_prior = beta_prior,
      sigma_prior = sigma_prior)

# remove temporary files
file.remove("1N.csv")
file.remove("W1N.csv")

# mcmc sampling step
tCFA::StudentCFA(inputFile,modelString = cfa_model,
      mcmcOutFilePath = "1T.csv", mcmcLatentOutFilePath = "W1T.csv",
      alpha_prior = alpha_prior,beta_prior = beta_prior,
      sigma_prior = sigma_prior, burnin = burnsz, samples = samplesz,
      max_t_degree_freedom_prior = max_t_degree_freedom_prior)

# marginal likelihood estimation
tCFA::StudentCalculateMargLikl(inputFile,modelString = cfa_model,
      mcmcOutFilePath = "1T.csv", mcmcLatentOutFilePath = "W1T.csv",
      alpha_prior = alpha_prior,beta_prior = beta_prior,
      sigma_prior = sigma_prior, nu_max = max_t_degree_freedom_prior)

# remove temporary files
file.remove("1T.csv")
file.remove("W1T.csv")
```