

Chapter 2: The Photosphere

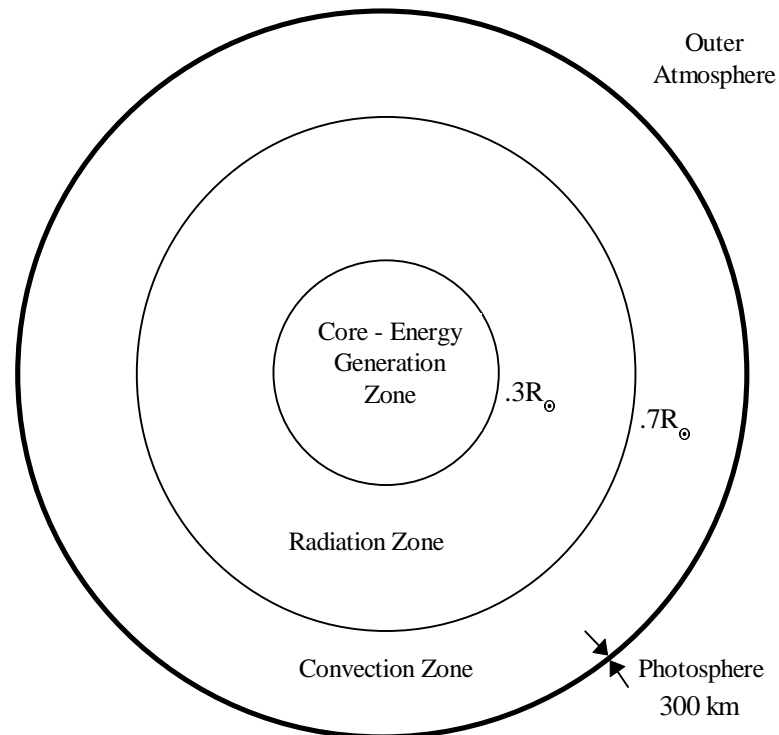


Fig 2-1: The Sun - Overall Structure

2.1: The Large Scale Structure of the Photosphere

The photosphere, the visible surface of the sun, is the uppermost opaque level in the sun. Light from deeper regions will not escape and higher material, being almost transparent, will emit relatively little light. Thus, the photosphere is the transitional region between deeper opaque regions of the sun and overlying relatively transparent material. This leads to the important features of the photosphere; in the photosphere the opacity drops from high to low, and the temperatures fall (although not as fast as the opacity) with increasing height.

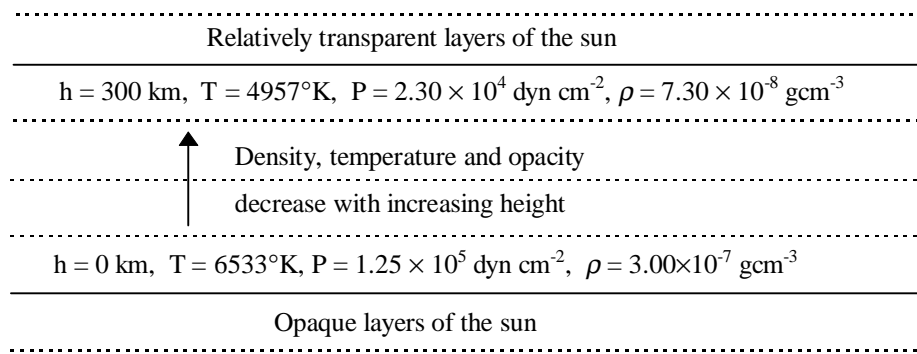


Fig 2-2: Large Scale Structure of the Photosphere

The temperature falls with increasing height until the temperature minimum in the lower chromosphere is reached, after which the temperature rises with increasing height.

The structure of the photosphere is discussed in more detail in sections 2.1.3 and 2.1.4. Figure 2-3 shows the variation of various properties of the photosphere with height.

2.1.1: The Solar Interior

Thermonuclear fusion reactions in the core of the sun provide the energy that reaches us from the sun. These reactions only occur in the innermost region of the sun, where the temperature and density are very high (out to about $0.3R_{\odot}$) and this energy must then be transported to the surface of the sun before it can escape into space. This energy can be transported by radiation or convection; whether or not convective flow occurs depends on the convective stability of the medium, which can be determined by the **Schwarzschild criterion**¹ for the occurrence of convection:

$$\left| \left(\frac{dT}{dr} \right)_{\text{adiabatic}} \right| < \left| \left(\frac{dT}{dr} \right)_{\text{radiative}} \right| \quad (2-1)$$

¹After Karl Schwarzschild, who proposed it in 1906.

Thus, if the adiabatic temperature gradient is less than the gradient in the absence of convection, convection will occur.

In a dense opaque medium, convection, if it occurs, is very efficient and the actual temperature gradient of the medium will be very close to the adiabatic gradient. In the sun, radiative transport dominates to a distance of about $0.7R_{\odot}$ from the centre, and then convective instability sets in, and convective transport dominates from $0.7R_{\odot}$ to the surface of the sun.

The photosphere itself, with its lower opacities and superadiabatic temperature gradient, is stable against convection. The solar granulation (see chapter 6) is believed to arise as a result of convection in this zone, as convective motions will tend to overshoot into stable regions; the flow in the convection zone can cause variations in the lower regions of the photosphere. An upward flow will not simply stop as soon as it comes to a stable region; its momentum will cause it to proceed into the stable region, and then it will fall back down after coming to a stop. Large convective velocities can be maintained in this way for some distance into the stable region, as, due to the rapid decrease of density with height, only a fraction of the mass in the flow needs to continue upwards in order to maintain the same volume flow.

2.1.2: The Outer Atmosphere of the Sun

The outer solar atmosphere, consisting of the chromosphere and the corona, is relatively transparent and of very low density. It is responsible for some features of the solar spectrum, such as ultraviolet emission lines, but has very little effect on Fraunhofer lines.

2.1.3: The Photosphere

The photosphere exhibits many complex motions and other inhomogeneities. Despite this, the large scale structure is dominated by the variation of properties such as pressure and density with height. Thus, a reasonable approximation of the photosphere can be obtained by considering the photosphere to be composed of relatively uniform horizontal strata; this is known as the plane-parallel approximation. The dependence of the physical properties of the photosphere on height must then be considered.

The change in pressure with height for an atmosphere in hydrostatic equilibrium will be

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2} \rho \quad (2-2)$$

where r is the distance from the centre of the sun, $M(r)$ is the mass enclosed by this radius, and ρ is the density at this radius. The assumption of hydrostatic equilibrium is a strong assumption, especially considering that the photosphere is in motion. The motions in the photosphere are not rapid enough to cause a strong departure from hydrostatic equilibrium², so the assumption of hydrostatic equilibrium will give a reasonable approximation. In the photosphere, $M(r)$ is essentially constant as the contribution to the mass of the sun due to the low density photosphere is small, so we can write this in terms of a local gravitational acceleration, g , as

$$\frac{dP}{dr} = -g\rho. \quad (2-3)$$

The gravitational acceleration g in the photosphere is about 274 ms^{-2} .

The pressure and density are also related by the ideal gas law

$$\begin{aligned} P &= NkT \\ &= \frac{\rho kT}{\bar{\mu}} \end{aligned} \quad (2-4)$$

where k is Boltzmann's constant and $\bar{\mu}$ is the mean mass of the particles contributing to the pressure, where

²The pressure variations driving the flow are small compared to the pressure differences involved with the stratification. As the surface gravity of the sun is high, the atmosphere is highly stratified, and the resultant pressure differences are large.

$$\bar{\mu} = \frac{\sum_{\substack{\text{all particle} \\ \text{types } i}} m_i N_i}{N}. \quad (2-5)$$

The temperature gradient is determined by the total energy flow and the resistance of the medium to the energy flow. Thus, it will depend on the energy flow mechanism. For radiative transport of energy,

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa \rho}{T^3} \frac{L}{4\pi r^2} \quad (2-6)$$

where κ is the flux mean opacity, or mass absorption coefficient averaged over all wavelengths (and suitably weighted to account for the wavelength distribution of the flux), and L is the luminosity, or radiative energy flux, at the radius in question. For convective energy transport, the temperature gradient will be close to the adiabatic temperature gradient

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \frac{dP}{dr}, \quad (2-7)$$

where the ratio of the specific heats, γ , for a monatomic ideal gas is given by $\gamma = \frac{5}{3}$.

Radiative transport is the dominant mechanism in the photosphere, with about 6% of the energy being transported by convection at the base of the photosphere, and virtually none at a height of 60 km.³ This decrease in the fraction of energy transported by convection is a necessary result of the rapid decrease in density with increasing height; the velocities of a convective flow would have to rise enormously in order to maintain the same mass flow needed to maintain the same convective energy flux. The radiative transport is not hampered at all, the decrease in opacity resulting from the decrease in density only makes it even easier for the energy to radiate outwards.

As the temperature changes in the photosphere are small compared to the changes in density and pressure, the pressure and density fall at an approximately exponential rate as the height increases:

$$P \approx P_0 e^{-\frac{z}{H(T)}} \quad (2-8)$$

³See pg 44 in Durrant, C. J. "The Atmosphere of the Sun" Hilger (1988). These values are determined from an analysis of the temperature and velocity variations in the granulation.

$$\rho \approx \rho_0 e^{-\frac{z}{H(T)}} \quad (2-9)$$

where $H(T)$ is the scale height for pressure and density. This scale height is given by

$$H(T) = \frac{N_0 k T}{g}, \quad (2-10)$$

where N_0 is Avogadro's number. For a temperature of 6300°K, the scale height is 150 km.⁴

The main feature of the photosphere is the extremely rapid drop in pressure and density with height. The temperature also falls with increasing height in the photosphere,⁵ although much more slowly than the pressure and density. As the opacity of the photosphere is proportional to the number of absorbers, the opacity must also drop rapidly.

2.1.4: The Model Photosphere

As conditions in the photosphere cannot be directly measured, a model atmosphere can be constructed so that these conditions are satisfied, and the spectrum produced matches the observed spectrum. Modifications made necessary by inhomogeneities will be considered later.⁶ The photospheric model used in this work is shown in table 2-1 below.

⁴See pg 10 in Durrant, C. J. "The Atmosphere of the Sun" Hilger (1988).

⁵The upper chromosphere has a temperature of about 100 000°K, and corona is even hotter, with a temperature in the millions of degrees. Lines from Fe XVII and other highly ionised elements have been identified in the coronal spectrum.

⁶See chapter 7.

Table 2-1: The Holweger-Müller Model Atmosphere⁷

Height ⁸ (km)	Optical Depth (τ_{5000})	Temper- ature (°K)	Pressure (dynes cm ⁻²)	Electron Pressure (dynes cm ⁻²)	Density (g cm ⁻³)	Opacity (κ_{5000})
550	5.0×10^{-5}	4306	5.20×10^2	5.14×10^{-2}	1.90×10^{-9}	0.0033
507	1.0×10^{-4}	4368	8.54×10^2	8.31×10^{-2}	3.07×10^{-9}	0.0048
441	3.2×10^{-4}	4475	1.75×10^3	1.68×10^{-1}	6.13×10^{-9}	0.0084
404	6.3×10^{-4}	4530	2.61×10^3	2.48×10^{-1}	9.04×10^{-9}	0.012
366	0.0013	4592	3.86×10^3	3.64×10^{-1}	1.32×10^{-8}	0.016
304	0.0040	4682	7.35×10^3	6.76×10^{-1}	2.47×10^{-8}	0.027
254	0.010	4782	1.23×10^4	1.12	4.03×10^{-8}	0.040
202	0.025	4917	2.04×10^4	1.92	6.52×10^{-8}	0.061
176	0.040	5005	2.63×10^4	2.54	8.26×10^{-8}	0.075
149	0.063	5113	3.39×10^4	3.42	1.04×10^{-7}	0.092
121	0.10	5236	4.37×10^4	4.68	1.31×10^{-7}	0.11
94	0.16	5357	5.61×10^4	6.43	1.64×10^{-7}	0.14
66	0.25	5527	7.16×10^4	9.38	2.03×10^{-7}	0.19
29	0.50	5963	9.88×10^4	22.7	2.60×10^{-7}	0.34
0	1.0	6533	1.25×10^5	73.3	3.00×10^{-7}	0.80
-34	3.2	7672	1.59×10^5	551	3.24×10^{-7}	3.7
-75	16	8700	2.00×10^5	2.37×10^3	3.57×10^{-7}	12

⁷Holweger, H. and Müller, E. A. "The Photospheric Barium Spectrum: Solar Abundance and Collision Broadening of Ba II Lines by Hydrogen", *Solar Physics* **39**, pg 19-30 (1974). Extra points have been cubic spline interpolated by J. E. Ross. The optical properties (such as the optical depth and the opacity) of a model atmosphere are, obviously, very important, and will be considered later. See table C-4 for complete details of the Holweger-Müller model atmosphere including all depth points used.

⁸The height scale is not arbitrary. The base of the photosphere (height = 0 km) is chosen to be at standard optical depth of one (i.e. $\tau_{5000\text{Å}} = 1$).

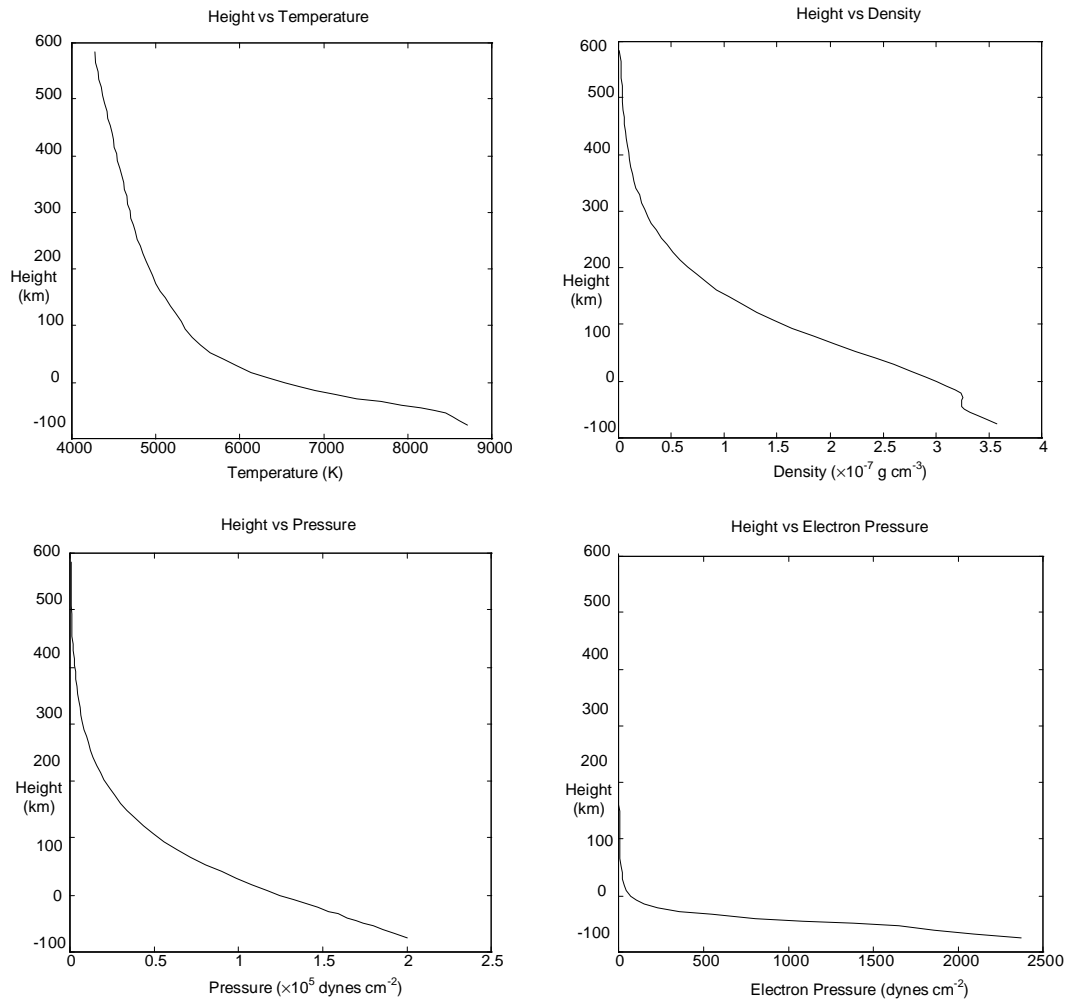


Figure 2-3: The Holweger-Müller Model Atmosphere

The variation of physical properties with height can be readily seen for this model. The temperature increases with decreasing height and the pressure increases exponentially. When the temperatures become high enough, the electron pressure increases rapidly as various atomic species begin to ionise more. The electronic contribution to the total pressure becomes significant, and the density does not rise as rapidly at this depth, as the pressure increase is provided by the increased ionisation.

Other model atmospheres differ in detail, but have the same general structure as the one shown here. The regions responsible for the production of the solar spectrum are very similar between model atmospheres; the regions from which very little radiation emerges are where most of the differences are.

2.2: Chemical Composition of the Photosphere

The photosphere, like the rest of the sun, is mostly composed of hydrogen. Helium is also common, and other elements are less abundant. The abundances of many elements are only poorly known, but the abundances of the most important elements are known to a reasonable degree of accuracy.⁹

The abundance of elements is usually given as

$$\text{abundance}_{\text{element}} = \log_{10} \frac{N_{\text{element}}}{N_H} + 12. \quad (2-11)$$

The factor of 12 is added to the logarithmic abundance ratio to make the abundances of most elements greater than zero. Figure 2-4 shows the chemical composition of the photosphere. (See table C-3 for abundances used in this work.)

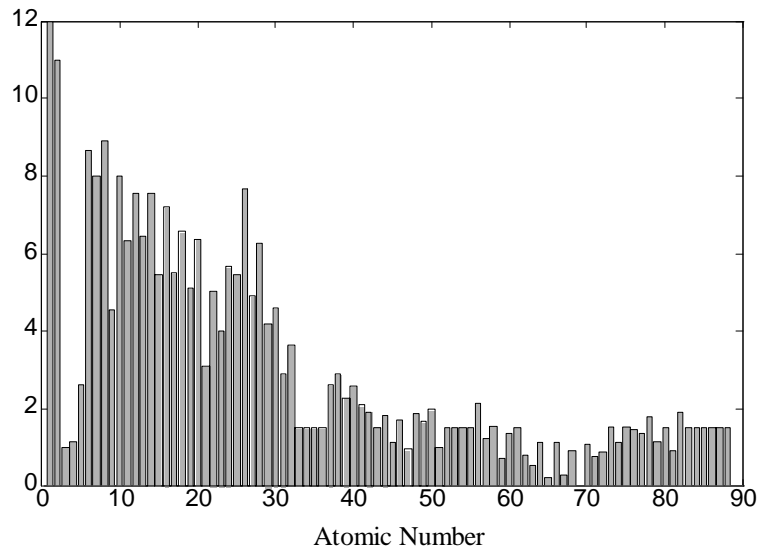


Figure 2-4: Solar Abundance of Elements

The chemical composition of the photosphere seems to be the same as the average composition of the entire solar system, so meteoric abundance measurements can be used to improve the accuracy of solar determinations, or solar measurements can be used in cases outside the sun.

⁹See Ross, J. and Aller, L. "The Chemical Composition of the Sun" *Science* **191**, pg 1223-1229 (1976), Grevesse, N. "Accurate Atomic Data and Solar Photospheric Spectroscopy" *Physica Scripta* **T8**, pg 49-58 (1984), and Anders, E. and Grevesse, N. "Abundances of the Elements: Meteoric and Solar" *Geochimica et Cosmochimica Acta* **53**, pg 197-214 (1989).

2.3: Microscopic Properties and Behaviour

2.3.1: Thermodynamic Equilibrium

If the energy in a system is equally distributed among the available states, the ratio of the occupation numbers of any two states is determined by the temperature T and is given by

$$\frac{N_1}{N_2} = \frac{g_1}{g_2} \frac{e^{-E_1/kT}}{e^{-E_2/kT}} \quad (2-12)$$

where k is Boltzmann's constant, g_1 and g_2 are the statistical weights of the two states (the effective number of sub-states making up each state)¹⁰, and E_1 and E_2 are the energies of the two states. This dependence of occupation of states upon the temperature only is characteristic of systems in thermodynamic equilibrium. A system is in true thermodynamic equilibrium only if it does not exchange energy with its surroundings, but if the energy flows in and out of the system are balanced, and the occupation of states depends only on the temperature, the system can be regarded as being in thermodynamic equilibrium.

The states of the system include the motion states of the particles, the excitation and ionisation states of atoms, and the energy of photons in the radiation field. The equi-partition of energy among the photon energy states gives the radiation field in thermodynamic equilibrium:

$$I_\lambda = B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{-hc/\lambda kT} - 1} \quad (2-13)$$

which is the well-known Planck radiation function for the black-body radiation field. This equilibrium radiation field is obtained through interaction with the particles in the system (due to the absence of photon-photon interaction).

¹⁰The statistical weight is given in terms of the atomic quantum number j by $g = 2j + 1$.

2.3.2: Local Thermodynamic Equilibrium - the LTE

Approximation

The photosphere, however, cannot be regarded as being in true thermodynamic equilibrium. Although thermodynamic equilibrium prevails in the solar interior, in the photosphere, due to the lower opacities and the higher temperature gradient, the radiation field at any point contains contributions from regions of different temperatures, and will not be equal to the black-body field. Also, if any atomic states strongly interact with the radiation field, their populations will be affected by the radiation field and will not be solely determined by the local temperature.

If the particles interact with each other much more strongly than with the radiation field, their state populations will still be given by the Boltzmann equation, (eqn 2-12) even if the radiation field is not given by the Planck function. A system with these characteristics is said to be in **LTE**, or **local thermodynamic equilibrium**. The temperature of a system in LTE can be defined as the temperature of the particles. The radiation field can be quite different from the Planck function, being in general anisotropic and non-Planckian.

2.3.3: The LTE Equation of State

The population of different energy levels or states for a system in LTE (or in thermodynamic equilibrium) can be found from the Boltzmann equation (eqn 2-12). Since the occupation of a state i is proportional to $g_i e^{-E_i/kT}$, the sum of the occupations of all states can be used to normalise this to find the probability of occupation of a state for one particle, giving:

$$\frac{N_i}{N} = \frac{g_i e^{-E_i/kT}}{\sum_{\text{all } j} g_j e^{-E_j/kT}} \quad (2-14)$$

where N is the total number of particles. The normalisation factor, $\sum_{\text{all } j} g_j e^{-E_j/kT}$ is called the **partition function**.

$$U(T) = \sum_{\text{all } j} g_j e^{-E_j/kT} \quad (2-15)$$

The number of particles in a particular state is then given in terms of the total number of particles by

$$\frac{N_i}{N} = \frac{g_i e^{-E_i/kT}}{U(T)} \quad (2-16)$$

If we consider only the population of atoms in a given ionisation state, then this gives the population of atoms in the energy level i if N is the total population of atoms in the ionisation state, and the partition function is calculated over all energy states available to atoms **in this ionisation state**.

If we consider the ratio between the populations of atoms in the ground state in two successive ionisation states, equation (2-12) gives

$$\frac{N_{0,i+1}}{N_{0,i}} = \frac{g}{g_{0,i}} e^{-(\chi_{i+1} + \text{KE}_{\text{electron}})/kT} \quad (2-17)$$

if we consider the energy of the ground state of the lower ionisation state to be zero.

The multiplicity of the $i+1$ state is given by

$$g = g_{0,i+1} g_{\text{electron}} \quad (2-18)$$

The number of states available to the electron is

$$g_{\text{electron}} = \frac{8\sqrt{2}\pi m_e^{\frac{3}{2}} \text{KE}_{\text{electron}}^{\frac{1}{2}}}{N_e h^3} d\text{KE}_{\text{electron}} \quad (2-19)$$

where N_e is the electron number density, m_e is the mass of an electron, and h is Planck's constant, so we can then integrate equation (2-17) over all electron kinetic energies to give

$$\frac{N_{0,i+1}}{N_{0,i}} = \left(\frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} \frac{2g_{0,i+1}}{N_e g_{0,i}} e^{-\chi_{i+1}/kT} \quad (2-20)$$

or in terms of the total populations of atoms in the two ionisation states,

$$\frac{N_{0,i+1} N_e}{N_{0,i}} = \left(\frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} \frac{2U_{i+1}(T)}{U_i(T)} e^{-\chi_{i+1}/kT} \quad (2-21)$$

which is known as Saha's equation.

This can then be used repeatedly to find the fraction of the total population of any atom in any ionisation state:

$$\begin{aligned}
\frac{N_i}{N} &= \frac{N_i}{N_0 + N_1 + N_2 + N_3 + \dots} \\
&= \frac{\frac{N_i}{N_0}}{1 + \frac{N_1}{N_0} + \frac{N_2}{N_0} + \frac{N_3}{N_0} + \dots} \\
&= \frac{\frac{N_i}{N_{i-1}} \frac{N_{i-1}}{N_{i-2}} \dots \frac{N_2}{N_1} \frac{N_1}{N_0}}{1 + \frac{N_1}{N_0} + \frac{N_2}{N_1} \frac{N_1}{N_0} + \frac{N_3}{N_2} \frac{N_2}{N_1} \frac{N_1}{N_0} + \dots}
\end{aligned} \tag{2-22}$$

Thus, the fraction of the population in any ionisation state can be found from ratios between populations of successive states, which can be calculated using Saha's equation (equation (2-21)). Usually, it will be sufficient to only consider the more likely ionisation states when using equation (2-22), calculating only the first few terms in the sum in the denominator. This is a strong technique, as we need only know the appropriate partition functions and the local temperature. This ease of calculating populations is what makes the LTE approximation so attractive.

Using these techniques to calculate ionisation fractions and populations for Iron, it can be seen that the Fe I population is strongly dependent on height in the photosphere. The population is affected by both the temperature and the electron concentration, which in turn depends on the ionisation levels of other elements. (See figure 2-5 below.)

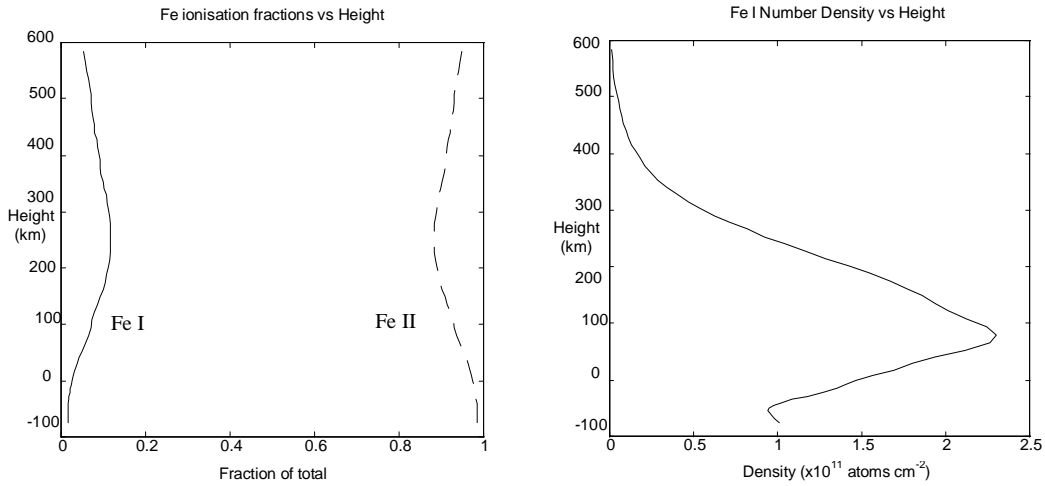


Figure 2-5: Variation of Iron ionisation states and populations with height

2.3.4: Non-LTE Conditions

If the LTE approximation is not valid, matters become more complicated. In the extreme case we would not be able to make use of any of the thermodynamic equilibrium results, and we could not, in fact, even sensibly define a temperature for the system. As collisions dominate the transfer of energy between states, the photosphere is almost in LTE, and only those states which interact strongly with the radiation field will have populations differing significantly from the LTE values. The populations of any weakly interacting atomic excitation states will still be at their LTE values, the ionisation equilibrium will still be given by equation (2-22) (at least for most atoms) and the velocity distribution of particles will still be Maxwellian.¹¹

The population of a non-LTE state can be determined from the rate of all transitions to or from this state:

$$\begin{aligned}
 & \sum_{j>i} A_{ji} N_j + \sum_{j \neq i} B_{ji} I_{\lambda_{ji}} N_j + \sum R_{\text{collisions}_{ji}} N_j \\
 & = \left(\sum_{j<i} A_{ij} + \sum_{j \neq i} B_{ij} I_{\lambda_{ij}} + \sum R_{\text{collisions}_{ij}} \right) N_i
 \end{aligned} \tag{2-23}$$

¹¹See section 3.5 for the effect of a Maxwellian velocity distribution on spectral line profiles.

where A_{ij} is the Einstein spontaneous emission coefficient and B_{ij} is the Einstein induced transition rate for the i - j transition. If these radiative transition rates are small, the state will be in LTE (as the collision excitation and de-excitation rates act to produce a Boltzmann distribution of states if the velocities are Maxwellian), but if they are large, the population will differ from the LTE population. (If all the radiation terms balance, an LTE population can result by accident.)

To calculate the population, we need to know the various reaction rates, and the radiation field as well as the temperature. The deviation from the LTE case can be expressed in terms of a non-LTE departure coefficient b_i , where

$$b_i = \frac{N_i}{(N_i)_{\text{in LTE}}}. \quad (2-24)$$

Departure coefficients depend on the height in the photosphere and on the energy level involved. They can be determined by fitting calculated spectral lines to observed spectral lines and then incorporated into the model atmosphere, but in general, it is desirable to assume LTE whenever possible, and to restrict ourselves to transitions between levels in LTE.

LTE is most likely to be a good approximation if the absorption cross-sections and emission rates for transitions to and from the level are low, and collision excitation and de-excitation rates are high. These conditions are likely to be satisfied for a given spectral line if transitions involving the upper and lower levels are not excessively strong, and if the line is formed deep in the photosphere (where the density and collision rates are higher). In the outer atmosphere of the sun, where extremely low densities result in much lower collision rates, populations can be far removed from LTE populations.

Cases where departures from LTE were significant were avoided in this work. In practice, non-LTE cases usually involve very strong lines, which are more likely to be blended than weaker lines (due to the greater widths). Of the unblended lines used in this work, only the potassium resonance line at 7699\AA shows serious departure from LTE.¹²

¹²Non-LTE calculations can be performed, but are more involved than LTE calculations. See section 5.2.1 for a brief discussion of non-LTE methods.

