1. Consider the discrete time \((A, B)\) system,

\[ x(\ell + 1) = x(\ell) + 2u(\ell). \]

(a) Find a state feedback controller, \(g(\cdot)\), \(u(t) = g(x(t))\), that minimizes

\[ \sum_{\ell=0}^{\infty} 2x(\ell)^2 + 6u(\ell)^2. \]

(b) Represent the resulting linear dynamical system as \(x(\ell + 1) = \tilde{a}x(\ell)\) and determine if it is asymptotically stable or not.

2. Consider the \((A, B)\) system,

\[ \dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \]

with state-feedback controller,

\[ u(t) = [\alpha \quad 1] x(t). \]

Assume we want to show that the closed loop system (with 0 input is stable) using the Lyapounov function \(V(x) = x'x\). For which values of \(\alpha\) is this possible?
3. Items arrive to a service facility according to a Poisson process with rate $\lambda$. The items queue up and are served one after the other. There are two servers that are designed to simultaneously give service to the same item (to the item at the head of the queue). When both servers are working, the item at the head of the queue is served at rate $2\mu$. As opposed to that, when only one of the servers is working, the item being served is served at rate $\mu$. One of the servers works without interruptions non-stop (unless the queue is empty). The other server experiences “on” and “off” periods. The transitions between “on” and “off” periods follow a CTMC with a $2 \times 2$ generator matrix $1'1 - 2I$. The transitions are not affected by the number of items in the queue.

(a) Describe this system as a quasi birth and death process (QBD).
(b) What is the stability condition of the system in terms of $\lambda$ and $\mu$?
(c) Find the stationary distribution of the system in terms of $\lambda$ and $\mu$. If you are not able to find an explicit expression (in terms of $\lambda$ and $\mu$), do so numerically to receive partial points.
(d) Find an expression for the mean queue level. Again, if you are not able to find an explicit expression, do so numerically (to receive partial points).

Good Luck!