1. Choose one of the four application examples appearing in Section 1.3 (Inverted Pendulum, Chemical Plant, Manufacturing Line, Communication Router). For this example do the following:

(a) Describe the application in your own words while stating the importance of having a mathematical model for this application. Use a figure if necessary. Your description should be half a page to two pages long.

(b) Suggest the flavor of the type of mathematical model (or models) that you would use to analyze, optimize and control this example. Justify your choice.

(c) Refer to the uses cases appearing in Section 1.2. Suggest how each of these applies to the application example and to the model.

(d) Consider the performance analysis measures described under the use case “computation and Analysis” in Section 1.2. How does each of these use cases apply to the application example and model that you selected?

2. Consider the function \( f(t) = e^{at} + e^{bt} \) with \( a, b, t \in \mathbb{R} \).

   (a) Find the Laplace transform of \( f(\cdot) \).

   (b) Find the Laplace transform of \( g_1(t) := \frac{d}{dt} f(t) \)

   (c) Find the Laplace transform of \( g_2(t) := \int_0^t f(\tau) d\tau \)

3. Prove that the Laplace transform of the convolution of two functions is the product of the Laplace transforms of the individual functions.

4. Consider Theorem 2.10 about BIBO stability. Prove this theorem for discrete time complex valued signals.
5. Carry out exercise 2.16 from Section 2.4.

6. Consider the differential equation:

\[ \dot{y}(t) + ay(t) = u(t), \quad y(0) = 0. \]

Treat the differential equation as a system, \( y(\cdot) = \mathcal{O}(u(\cdot)) \).

(a) Is it an LTI system?

(b) If so, find the system’s transfer function.

(c) Assume the system is a plant controlled in feedback as described in Section 2.5, with \( g_1(s) = 1 \) and \( g_2(s) = K \) for some constant \( K \). Plot (using software) the step response of the resulting closed loop system for \( a = 1 \) and for various values of \( K \) (you choose the values).

7. Consider a sequence of \( n \) systems in tandem where the output of one system is input to the next. Assume each of the systems has the impulse response \( h(t) = e^{-t}1(t) \). As input to the first system take \( u(t) = h(t) \).

(a) What is the output from this sequence of systems? I.e. find \( y(t) \), such that,

\[ y(\cdot) = \mathcal{O}(\mathcal{O}(\cdots \mathcal{O}(u(t)) \cdots)), \]

such that the composition is repeated \( n \) times.

(b) Relate your result to Exercise 2.21 in Section 2.6.

(c) Assume \( n \) grows large, what can you say about the output of the sequence of systems? (Hint: Consider the Central Limit Theorem).