Name: ____________________________

Let $X_1(t)$ and $X_2(t)$ be two CTMCs operating independently with respective state spaces $S_1 = \{1, 2\}$ and $S_2 = \{3, 4\}$ and generator matrices,

$$Q_1 = \begin{bmatrix} -1 & 1 \\ \mu_1 & -\mu_1 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} -1 & 1 \\ \mu_2 & -\mu_2 \end{bmatrix}.$$ 

Assume further that $P(X_1(0) = 1) = 1$ and $P(X_2(0) = 3) = 1$. Denote,

$$\tau_1 = \inf\{t \geq 0 : X_1(t) = 2\} \quad \text{and} \quad \tau_2 = \inf\{t \geq \tau_1 : X_2(t) = 3\} - \tau_1.$$

1) Argue why $\tau_1$ is an exponentially distributed random variable.

2) What is $E[\tau_1]$?

3) Is the distribution of $\tau_1$ phase-type?

4) Denote $\eta(t) = P(X_2(t) = 3)$ and $\alpha = P(\tau_2 = 0)$. Represent $\alpha$ in terms of $\eta(t)$.

5) Is the distribution of $\tau_2$ phase-type? If yes, give parameters of it in terms of $\alpha$ and $Q_2$. 