AMSI Summer School 2016
Linear Control Theory & Structured Markov Chains
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Solution for Quiz #7

Consider the \((A, B, C, D)\) SISO system,

\[
\dot{x}(t) = Ax(t) + bu(t) \\
y(t) = c'x(t),
\]

with

\[
A = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad c' = \begin{bmatrix} 1 & 1 \end{bmatrix}.
\]

1) What is the controllability matrix of this system?

**Answer:**

\[
\text{con}(A, b) = [b \quad A b] = \begin{bmatrix} 1 & 1 + \alpha \\ 1 & 1 \end{bmatrix}.
\]

2) For what values of \(\alpha\) is the system controllable?

**Answer:** For \(\alpha \neq 0\) since in this case the controllability matrix is full rank.

3) What is the observability matrix of this system?

**Answer:**

\[
\text{obs}(A, c') = \begin{bmatrix} c' \\ c' A \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & \alpha + 1 \end{bmatrix}.
\]

4) For what values of \(\alpha\) is the system observable?

**Answer:** For \(\alpha \neq 0\) since in this case the observability matrix is full rank.