Chapter 7: Optimal Linear-Quadratic Control
Preface

This booklet contains lecture notes and exercises for a 2016 AMSI Summer School Course: “Linear Control Theory and Structured Markov Chains” taught at RMIT in Melbourne by Yoni Nazarathy. The notes are based on a subset of a draft book about a similar subject by Sophie Hautphenne, Erjen Lefeber, Yoni Nazarathy and Peter Taylor. The course includes 28 lecture hours spread over 3.5 weeks. The course includes assignments, short in-class quizzes and a take-home exam. These assement items are to appear in the notes as well.

The associated book is designed to teach readers, elements of linear control theory and structured Markov chains. These two fields rarely receive a unified treatment as is given here. It is assumed that the readers have a minimal knowledge of calculus, linear algebra and probability, yet most of the needed facts are summarized in the appendix, with the exception of basic calculus. Nevertheless, the level of mathematical maturity assumed is that of a person who has covered 2-4 years of applied mathematics, computer science and/or analytic engineering courses.

Linear control theory is all about mathematical models of systems that abstract dynamic behavior governed by actuators and sensed by sensors. By designing state feedback controllers, one is often able to modify the behavior of a system which otherwise would operate in an undesirable manner. The underlying mathematical models are inherently deterministic, as is suited for many real life systems governed by elementary physical laws. The general constructs are system models, feedback control, observers and optimal control under quadratic costs. The basic theory covered in this book has reached relative maturity nearly half a century ago: the 1960’s, following some of the contributions by Kalman and others. The working mathematics needed to master basic linear control theory is centered around linear algebra and basic integral transforms. The theory relies heavily on eigenvalues, eigenvectors and others aspects related to the spectral decomposition of matrices.

Markov chains are naturally related to linear dynamical systems and hence linear control theory, since the state transition probabilities of Markov chains evolve as a linear dynamical system. In addition the use of spectral decompositions of matrices, the matrix exponential and other related features also resembles linear dynamical systems. The field of structured Markov chains, also referred to as Matrix Analytic Methods, goes back to the mid 1970’s, yet has gained popularity in the teletraffic, operations research
and applied probability community only in the past two decades. It is unarguably a more esoteric branch of applied mathematics in comparison to linear control theory and it is currently not applied as abundantly as the former field.

A few books at a similar level to this one focus on dynamical systems and show that the probabilistic evolution of Markov chains over finite state spaces behaves as linear dynamical systems. This appears most notably in [?]. Yet, structured Markov chains are more specialized and possess more miracles. In certain cases, one is able to analyze the behavior of Markov chains on infinite state spaces, by using their structure. E.g. underlying matrices may be of block diagonal form. This field of research often focuses on finding effective algorithms for solutions of the underlying performance analysis problems. In this book we simply illustrate the basic ideas and methods of the field. It should be noted that structured Markov chains (as Markov chains in general) often make heavy use of non-negative matrix theory (e.g. the celebrated Perron-Frobenius Theorem). This aspect of linear algebra does not play a role in the classic linear control theory that we present here, yet appears in the more specialized study of control of non-negative systems.

Besides the mathematical relation between linear control theory and structured Markov chains, there is also a much more practical relation which we stress in this book. Both fields, together with their underlying methods, are geared for improving the way we understand and operate dynamical systems. Such systems may be physical, chemical, biological, electronic or human. With its styled models, the field of linear control theory allows us to find good ways to actually control such systems, on-line. With its ability to capture truly random behavior, the field of structured Markov chains allows us to both describe some significant behaviors governed by randomness, as well as to efficiently quantify (solve) their behaviors. But control does not really play a role.

With the exception of a few places around the world (e.g. the Mechanical Engineering Department at Eindhoven University of Technology), these two fields are rarely taught simultaneously. Our goal is to facilitate such action through this book. Such a unified treatment will allow applied mathematicians and systems engineers to understand the underlying concepts of both fields in parallel, building on the connections between the two.

Below is a detailed outline of the structure of the book. Our choice of material to cover was such as to demonstrate most of the basic features of both linear control theory and structured Markov chains, in a treatment that is as unified as possible.

Outline of the contents:

The notes contains a few chapters and some appendices. The chapters are best read sequentially. Notation is introduced sequentially. The chapters contain embedded short exercises. These are meant to help the reader as she progresses through the book, yet at the same time may serve as mini-theorems. That is, these exercises are both deductive
and informative. They often contain statements that are useful in their own right. The end of each chapter contains a few additional exercises. Some of these exercises are often more demanding, either requiring computer computation or deeper thought. We do not refer to computer commands related to the methods and algorithms in the book explicitly. Nevertheless, in several selected places, we have illustrated example MATLAB code that can be used.

For the 2016 AMSI summer school, we have indicated besides each chapter the in-class duration that this chapter will receive in hours.

**Chapter 1 (2h)** is an elementary introduction to systems modeling and processes. In this chapter we introduce the types of mathematical objects that are analyzed, give a feel for some applications, and describe the various use-cases in which such an analysis can be carried out. By a use-case we mean an activity carried out by a person analyzing such processes. Such use cases include “performance evaluation”, “controller design”, “optimization” as well as more refined tasks such as stability analysis, pole placement or evaluation of hitting time distributions.

**Chapter 2 (7h)** deals with two elementary concepts: Linear Time Invariant (LTI) Systems and Probability Distributions. LTI systems are presented from the viewpoint of an engineering-based “signals and systems” course. A signal is essentially a time function and system is an operator on functional space. Operators that have the linearity and time-invariance property are LTI and are described neatly by either their impulse response, step response, or integral transforms of one of these (the transfer function). It is here that the convolution of two signals plays a key role. Signals can also be used to describe probability distributions. A probability distribution is essentially an integrable non-negative signal. Basic relations between signals, systems and probability distributions are introduced. In passing we also describe an input–output form of stability: BIBO stability, standing for “bounded input results in bounded output”. We also present feedback configurations of LTI systems, showing the usefulness of the frequency domain (s-plane) representation of such systems.

**Chapter 3 (9h)** moves onto dynamical models. It is here that the notion of state is introduced. The chapter begins by introducing linear (deterministic) dynamical systems. These are basically solutions to systems of linear differential equations where the free variable represents time. Solutions are characterized by matrix powers in discrete time and matrix exponentials in continuous time. Evaluation of matrix powers and matrix exponentials is a subject of its right as it has to do with the spectral properties of matrices, this is surveyed as well. The chapter then moves onto systems with discrete countable (finite or infinite) state spaces evolving stochastically: Markov chains. The basics of discrete time and continuous time Markov chains are surveyed. In doing this a
few example systems are presented. We then move onto presenting input–state–output systems, which we refer to as \((A, B, C, D)\) systems. These again are deterministic objects. This notation is often used in control theory and we adopt it throughout the book. The matrices \(A\) and \(B\) describe the effect on input on state. The matrices \(C\) and \(D\) are used to describe the effect on state and input on the output. After describing \((A, B, C, D)\) systems we move onto distributions that are commonly called Matrix Exponential distributions. These can be shown to be directly related to \((A, B, C, D)\) systems. We then move onto the special case of phase type (PH) distributions that are matrix exponential distributions that have a probabilistic interpretation related to absorbing Markov chains. In presenting PH distributions we also show parameterized special cases.

**Chapter 4 (0h)** is not taught as part of the course. This chapter dives into the heart of Matrix Analytic Modeling and analysis, describing quasi birth and deaths processes, Markovian arrival processes and Markovian Binary trees, together with the algorithms for such models. The chapter begins by describing QBDs both in discrete and continuous time. Then moves onto Matrix Geometric Solutions for the stationary distribution showing the importance of the matrices \(G\) and \(R\). The chapter then shows elementary algorithms to solve for \(G\) and \(R\) focusing on the probabilistic interpretation of iterations of the algorithms. State of the art methods are summarized but are not described in detail. Markovian Arrival Point Processes and their various sub-classes are also surveyed. As examples, the chapter considers the M/PH/1 queue, PH/M/1 queue as well as the PH/PH/1 generalization. The idea is to illustrate the power of algorithmic analysis of stochastic systems.

**Chapter 5 (4h)** focuses on \((A, B, C, D)\) systems as used in control theory. Two main concepts are introduced and analyzed: state feedback control and observers. These are cast in the theoretical framework of basic linear control theory, showing the notions of controllability and observability. The chapter begins by introducing two physical examples of \((A, B, C, D)\) systems. The chapter also introduces canonical forms of \((A, B, C, D)\) systems.

**Chapter 6 (3h)** deals with stability of both deterministic and stochastic systems. Notions and conditions for stability were alluded to in previous chapters, yet this chapter gives a comprehensive treatment. At first stability conditions for general deterministic dynamical systems are presented. The concept of a Lyapunov function is introduced. This is the applied to linear systems and after that stability of arbitrary systems by means of linearization is introduced. Following this, examples of setting stabilizing feedback control rules are given. We then move onto stability of stochastic systems (essentially positive recurrence). The concept of a Foster-Lyapunov function is given for showing positive recurrence of Markov chains. We then apply it to quasi-birth-death processes.
proving some of the stability conditions given in Chapter 4 hold. Further stability conditions of QBD's are also given. The chapter also contains the Routh-Hourwitz and Jury criterions.

**Chapter 7 (3h)** is about optimal linear quadratic control. At first Bellman’s dynamic programming principle is introduced in generality, and then it is formulated for systems with linear dynamics and quadratic costs of state and control efforts. The linear quadratic regulator (LQR) is introduced together with its state feedback control mechanism, obtained by solving Ricati equations. Relations to stability are overviewed. The chapter then moves onto Model-predictive control and constrained LQR.

**Chapter 8 (0h) is not taught as part of the course.** This chapter deals with fluid buffers. The chapter involves both results from applied probability (and MAM), as well as a few optimal control examples for deterministic fluid systems controlled by a switching server. The chapter begins with an account of the classic fluid model of Anick, Mitra and Sondhi. It then moves onto additional models including deterministic switching models.

**Chapter 9 (0h) is not taught as part of the course.** This chapter introduces methods for dealing with deterministic models with additive noise. As opposed to Markov chain models, such models behave according to deterministic laws, e.g. \((A, B, C, D)\) systems, but are subject to (relatively small) stochastic disturbances as well as measurement errors that are stochastic. After introducing basic concepts of estimation, the chapter introduces the celebrated Kalman filter. There is also brief mention of linear quadratic Gaussian control (LQG).

The notes also contains an extensive appendix which the students are required to cover by themselves as demand arises. The appendix contains proofs of results in cases where we believe that understanding the proof is instructive to understanding the general development in the text. In other cases, proofs are omitted.

**Appendix A** touches on a variety of basics: Sets, Counting, Number Systems (including complex numbers), Polynomials and basic operations on vectors and matrices.

**Appendix B** covers the basic results of linear algebra, dealing with vector spaces, linear transformations and their associated spaces, linear independence, bases, determinants and basics of characteristic polynomials, eigenvalues and eigenvectors including the Jordan Canonical Form.

**Appendix C** covers additional needed results of linear algebra.
Appendix D contains probabilistic background.

Appendix E contains further Markov chain results, complementing the results presented in the book.

Appendix F deals with integral transforms, convolutions and generalized functions. At first convolutions are presented, motivated by the need to know the distribution of the sum of two independent random variables. Then generalized functions (e.g. the delta function) are introduced in an informal manner, related to convolutions. We then present the Laplace transform (one sided) and the Laplace-Stiltjes Transform. Also dealing with the region of convergence (ROC). In here we also present an elementary treatment of partial fraction expansions, a method often used for inverting rational Laplace transforms. The special case of the Fourier transform is briefly surveyed, together with a discussion of the characteristic function of a probability distribution and the moment generating function. We then briefly outline results of the z-transform and of probability generating functions.

Besides thanking Sophie, Erjen and Peter, my co-authors for the book on which these notes are based, I would also like to thank (on their behalf) to several colleagues and students for valuable input that helped improve the book. Mark Fackrell and Nigel Bean’s analysis of Matrix Exponential Distributions has motivated us to treat the subjects of this book in a unified treatment. Guy Latouche was helpful with comments dealing with MAM. Giang Nguyen taught jointly with Sophie Hautphenene a course in Vietnam covering some of the subjects. A Master’s student from Eindhoven, Kay Peeters, visiting Brisbane and Melbourne for 3 months and prepared a variety of numerical examples and illustrations, on which some of the current illustrations are based. Also thanks to Azam Asanjarani and to Darcy Bermingham. The backbone of the book originated while the authors were teaching an AMSI summer school course, in Melbourne during January 2013. Comments from a few students such as Jessica Yue Ze Chan were helpful.

I hope you find these notes useful,
Yoni.
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Chapter 7

Optimal Linear-Quadratic Control (3h)

This chapter gives a flavor of optimal control theory with specialization into optimal control of linear systems. It ends with an introduction of Model Predictive Control, a sub-optimal control method that has become popular in both theory in practice in the past 20 years.

7.1 Bellman’s Optimality Principle

Consider a general nonlinear time-invariant control system in discrete time:

\[ x(\ell + 1) = f(x(\ell), u(\ell)) \quad x(0) = x_0. \]

We consider the problem of controlling this system optimally, that is, we want to find inputs \( u(0), u(1), \ldots, u(N-1) \) such that the following objective is minimized:

\[ J = g_N(x(N)) + \sum_{k=0}^{N-1} g(x(k), u(k)). \]

Richard Bellman (1920-1984) made a rather obvious observation, which became known as the Principle of Optimality:

**Principle of Optimality [Bellman]** Let \( u^* = \{u(0), u(1), \ldots, u(N-1)\} \) be an optimal policy for minimizing (7.2) subject to the dynamics (7.1), and assume that when using \( u^* \), a given state \( x(i) \) occurs at time \( i \). Consider now the subproblem where we are at \( x(i) \) at time \( i \) and wish to minimize the “cost-to-go” from time \( i \) to time \( N \):

\[ g_N(x(N)) + \sum_{k=i}^{N-1} g(x(k), u(k)). \]

Then the truncated policy \( \{u(i), u(i+1), \ldots, u(N-1)\} \) is optimal for this subproblem.
The intuitive idea behind this principle is very simple. If the truncated policy were not optimal, then we could reduce the cost for the original problem by switching to the better alternative once we reach \( x(i) \).

Consider for instance the problem of driving by car from Melbourne to Perth. The fastest route passes through Adelaide. Now the principle of optimality states the obvious fact that the Adelaide-Perth part of the route is also the fastest route from Adelaide to Perth. However, this simple observation has far-reaching consequences. Since in order to solve the optimal control problem (7.1), (7.2), we can first determine the optimal control for the subproblem starting at \( i = N - 1 \). Next, we can determine the optimal control for the subproblem starting at \( i = N - 2 \), etc., and proceed backwards until we determine the optimal control for the original problem.

This approach for solving the optimization problem (7.2), subject to (7.1), is also known as Dynamic Programming (DP).

**Exercise 7.1.1.** Consider the grid, shown in the figure below. Find the shortest path from \( A \) to \( B \) (moving only east or north) using Dynamic Programming.

**Exercise 7.1.2.** Consider again the grid of Exercise 7.1.1. This time, find the longest path from \( A \) to \( B \) (moving only east or north) using Dynamic Programming.

From the above exercises the DP algorithm should have become clear:

**Theorem 7.1.3.** For every initial state \( x_0 \), the optimal cost \( J^*(x_0) \) resulting from minimizing (7.2) subject to (7.1) is equal to \( J_0(x_0) \), given by the last step of the following algorithm, which proceeds backwards in the time from period \( N - 1 \) to period 0:

\[
J_N(x(N)) = g_N(x(N))
\]
\[ J_k(x(k)) = \min_{u(k)} g(x(k), u(k)) + J_{k+1}(f(x(k), u(k))) \quad k = 0, 1, \ldots, N - 1 \quad (7.3) \]

Furthermore, if \( u^*(x(k)) \) minimizes the right hand side of (7.3) for each \( x(k) \) and \( k \), the policy \( u^* = \{u^*(x(0)), u^*(x(1)), \ldots, u^*(x(N-1))\} \) is optimal.

The function \( J_k(x(k)) \) can be interpreted as the costs associated with the subproblem of starting in \( x(k) \) at time \( k \), and therefore is often referred to as the cost-to-go function at time \( k \).

### 7.2 The Linear Quadratic Regulator

In the previous section we considered optimal control in a more general setting. Now, we restrict ourselves to linear systems:

\[ x(\ell + 1) = Ax(\ell) + Bu(\ell) \quad x(0) = x_0 \]

and quadratic costs

\[ J = x(N)'Q_f x + \sum_{k=0}^{N-1} x(k)'Qx(k) + u(k)'Ru(k). \]

Engineers typically take \( Q \) and \( R \) to be diagonal matrices, where \( Q_{ii} = 1/u_{i,\text{max}} \) and \( R_{ii} = 1/x_{i,\text{max}} \). Here \( u_{i,\text{max}} \) and \( x_{i,\text{max}} \) denote the maximally allowed value for \( u_i \) and \( x_i \) respectively. Not that this choice for \( Q \) and \( R \) guarantees that these constraints will not be violated, but it is a rule of thumb, commonly used in practice. How to explicitly deal with constraints is subject of the next section.

For solving the optimal control problem we need to make the following assumptions:

- The pair \((A, B)\) is controllable.
- \( Q' = Q \geq 0 \), i.e. positive semi-definite.
- \( R' = R > 0 \), i.e. positive definite.
- The pair \((A, \bar{C})\) is observable, where \( \bar{C} \) solves \( Q = \bar{C}'\bar{C} \)

These conditions guarantee that in the remainder of this section inverses exist, limits exist, and that the resulting controller stabilizes the system at \( x = 0 \).

We can solve this optimal control problem by means of dynamic programming. As a first step, we obtain

\[ J_N(x(N)) = x(N)'Q_f x(N). \]
Writing (7.3) for \(k = N - 1\) results in

\[
J_k(x(k)) = \min_{u(N-1)} x(N-1)'Qx(N-1) + u(N-1)'Ru(N-1) + J_N(Ax(N-1) + Bu(N-1))
\]

\[
= \min_{u(N-1)} x(N-1)'Qx(N-1) + u(N-1)'Ru(N-1) + (Ax(N-1) + Bu(N-1))'Q_f(Ax(N-1) + Bu(N-1))
\]

\[
= \min_{u(N-1)} u(N-1)'((R + B'Q_f B)u(N-1) + 2x(N-1)'A'Q_f B + x'(Q + A'Q_f A)x)
\]

Now we differentiate the right hand term w.r.t. \(u(N-1)\), set the derivative to zero, and obtain

\[
(R + B'Q_f B)u(N-1) = -B'Q_f A x(N-1)
\]

and therefore

\[
u^*(N-1) = -(R + B'Q_f B)^{-1}B'Q_f A x(N-1)
\]

By substituting this back into the equation for \(J_k(x(k))\) we obtain:

\[
J_k(x(k)) = x(N-1)'A'Q_f B(R + B'Q_f B)^{-1}B'Q_f A x(N-1) - 2x(N-1)'A'Q_f B(R + B'Q_f B)^{-1}B'Q_f A x(N-1) + x'(Q + A'Q_f A)x
\]

\[
= x(N-1)' \left( A'Q_f A - (A'Q_f B)(R + B'Q_f B)^{-1}(B'Q_f A) + Q \right) x(N-1)
\]

Proceeding similarly for the other steps we obtain

\[
u^*(k) = -(R + B'P(k)B)^{-1}B'P(k) A x(k)
\]

where \(P(k)\) is given by the backward recursion

\[
\]

So for the finite time horizon optimal control problem, the resulting optimal controller is a linear (time-varying) feedback controller.

Now if we let \(N \to \infty\), that is, we consider the infinite horizon optimal control problem, then it can be shown that \(P(k)\) converges to a fixed matrix \(P\) which is the unique positive definite solution of the discrete time algebraic Riccati equation:

\[
P = A'PA - (A'PB)(R + B'PB)^{-1}(B'PA) + Q
\]

The corresponding optimal input now also becomes a static state feedback

\[
u^*(\ell) = -(R + B'PB)^{-1}B'PA x(\ell).
\]
Exercise 7.2.1. Solve the infinite horizon optimal control problem in discrete time for the scalar linear system

\[ x(\ell + 1) = x(\ell) + 2u(\ell) \quad x(0) = x_0 \]

and cost function

\[ J = \sum_{k=0}^{\infty} 2x(k)^2 + 6u(k)^2. \]

That is, determine the optimal steady state feedback \( u(\ell) = -k_f x(\ell) \), as well as the cost-to-go \( p x(\ell)^2 \).

Remark 7.2.2. Similar results can be derived for continuous time. So consider the system

\[ \dot{x}(t) = Ax(t) + Bu(t) \quad x(0) = x_0 \]

and cost function

\[ J = \int_0^T x(s)'Qx(s) + u(s)'R u(s) \, ds \]

with the same assumptions on \( A, B, Q \) and \( R \). Then the optimal input is given by the linear time-varying state feedback

\[ u^*(t) = -R^{-1}B'P(t)x(t) \]

where \( P(t) \) is given by the following differential equation (in backward time):

\[ \dot{P}(t) = A'P(t) + P(t)A - P(t)BR^{-1}B'P(t) + Q \quad P(T) = Q_f. \]

Again, if we let \( T \to \infty \), we get a steady state feedback controller

\[ u^*(t) = \underbrace{-R^{-1}B'P}_{k_f} x(t) \]

where \( P \) is the unique positive definite solution to the continuous time algebraic Riccati equation:

\[ A'P + PA - PBR^{-1}B'P + Q = 0. \]

7.3 Riccati Equations

What the exercises at the end of the previous section show, is that in general MPC does not yield a stabilizing controller. However, by taking the prediction horizon \( p \) sufficiently large, and by adding terminal constraints, a stabilizing controller can be obtained.
In this section we show that by adding a terminal constraint, a specific choice for the terminal costs, and carefully selecting the prediction horizon (given the current state), the resulting MPC controller solves the infinite horizon LQR-problem with constraints. This guarantees that the resulting controller stabilizes the system. Furthermore, we obtain the optimal controller for the infinite horizon LQR-problem with constraints, where we only have to solve a finite optimization problem.

To be precise, we consider the system

$$\textbf{x}(\ell + 1) = A\textbf{x}(\ell) + B\textbf{u}(\ell) \quad \textbf{x}(0) = \textbf{x}_0,$$

and want to minimize the objective

$$J = \sum_{k=0}^{\infty} \textbf{x}(k)'Q\textbf{x}(k) + \textbf{u}(k)'R\textbf{u}(k)$$

subject to the constraints

$$E\textbf{x} + F\textbf{u} \leq \textbf{g}.$$

In addition to the assumptions we made in section 7.2, we require that the elements of the vector $\textbf{g}$ satisfy $g_i > 0$. Furthermore we assume that the initial condition $\textbf{x}_0$ is in the set of states for which the optimal control problem is feasible. In case the matrix $A$ is such that the system $\textbf{x}(\ell + 1) = A\textbf{x}(\ell)$ is marginally stable, we can take for this set $\mathbb{R}^n$.

In the remainder of this section we outline how to solve this problem. Since the problem is feasible, i.e., a solution exists which results in finite costs, we also know, due to the fact that $g_i > 0$, that after a finite amount of time the constraints (7.5) will not be active anymore. From Bellmans Optimality Principle we know that from then on, the solution to the unconstrained infinite horizon optimal control problem as presented in section 7.2 is followed.

So the first step is to solve the Discrete Time Algebraic Riccati equation

$$P = A'PA - (A'PB)(R + B'PB)^{-1}(B'PA) + Q.$$ 

From section 7.2 we know that the associated optimal input is given by $\textbf{u}(\ell) = -K_f\textbf{x}(\ell)$ where $K_f = R^{-1}B'P$. Furthermore, the cost to go is given by $\textbf{x}(\ell)'Px(\ell)$.

The second step is to determine the maximally output admissible set. That is, the largest set $Z$ of $\textbf{x}$ satisfying $(E - FK_f)\textbf{x} \leq \textbf{g}$ such that $(A - BK_f)\textbf{x}$ is also contained in that set.

As a third step, we consider the MPC problem to minimize

$$J = \textbf{x}(\ell + p)'P\textbf{x}(\ell + p) + \sum_{k=0}^{p-1} \textbf{x}(\ell + k|\ell)'Q\textbf{x}(\ell + k|\ell) + \textbf{u}(\ell + k|\ell)'R\textbf{u}(\ell + k|\ell).$$

subject to the dynamics (7.4), the constraints (7.5), and the terminal constraints $\textbf{x}(\ell + p) \in Z$. Here we should take $p$ large enough such that this MPC problem is feasible.
7.4 Model-based Predictive Control (omitted)

This section is omitted from this version.

Bibliographic Remarks

Exercises