Modelling Patient Flow in an Emergency Department

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Motivation

- *Eastern Health closes more beds*
- *Bed closures to cause 'ambulance delays'*
- *Hospital targets cost lives*
- *Girl 'waited for hours for emergency care'*
- *ACT meets elective surgery targets but fails on emergency waits*

Motivation

Public hospitals will be required by 2015 to ensure that 90% of all patients spend no more than 4 hours in the emergency department.

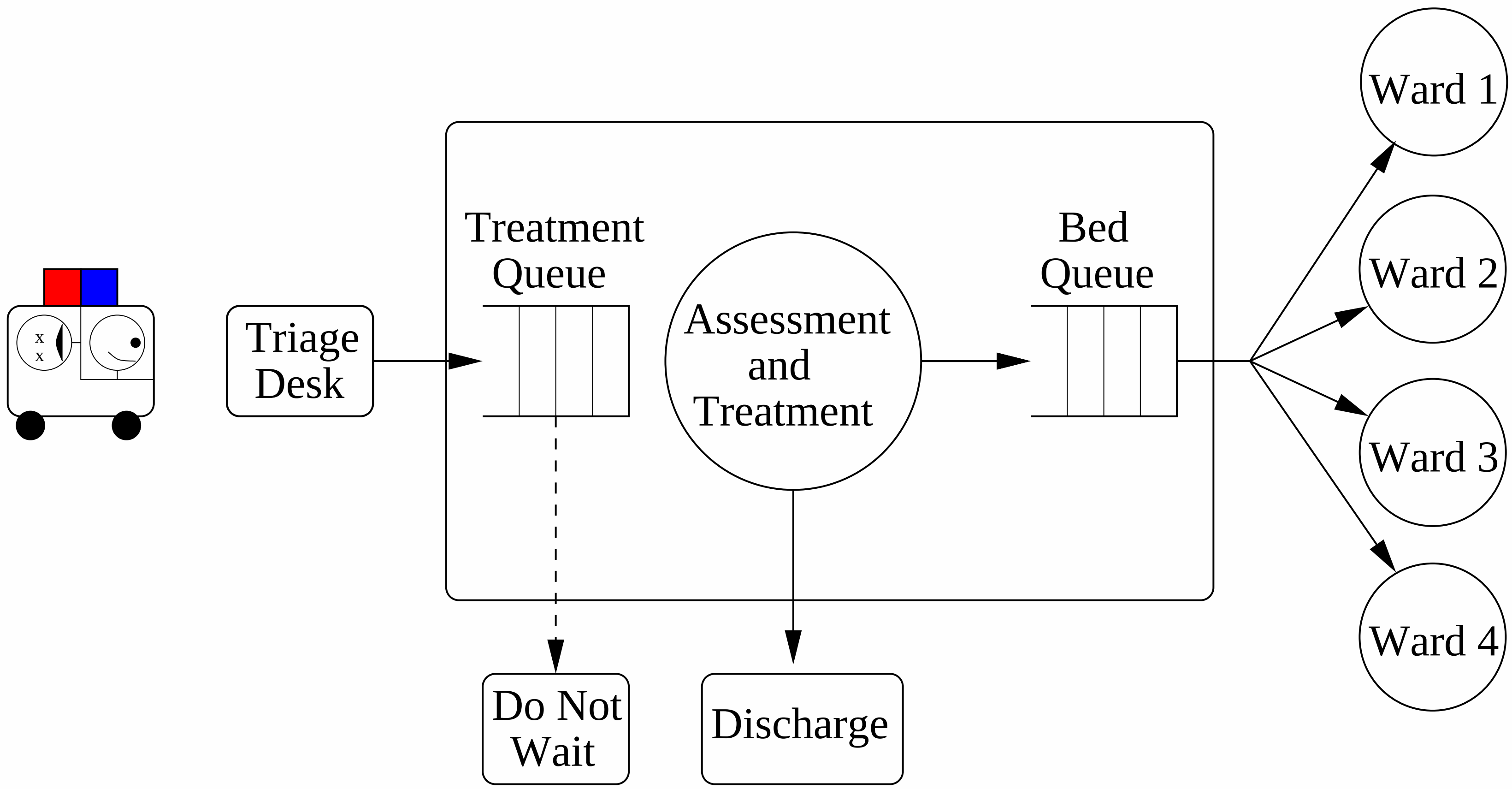
Currently (2012)

- 54% – major metropolitan hospitals
- 63% – major regional hospitals
- 67% – large metropolitan hospitals
- 78% – large regional hospitals

The Australian Triage Scale

ATS Category	Description	Treatment Acuity	% Adherence
1	Resuscitate	Immediate	100%
2	Emergency	10 minutes	80%
3	Urgent	30 minutes	75%
4	Semi-urgent	60 minutes	70%
5	Non-urgent	120 minutes	70%

The Emergency Department



Modelling the Bed Queue

For each six hour block (00-06, 06-12, 12-18, 18-24), and each day of the week, we assumed that the

- arrival rate to the bed queue is constant, ie. λ, and
- departure rate from the bed queue depends on the number of patients n waiting in the bed queue, ie. $\mu(n)$.

Data available from 1 January 2001 to 18 April 2005 (200,000 entries).

Parameters estimated using *spline regression*.

Modelling the Bed Queue

We model the bed queue (service centre) with a *continuous-time Markov chain*.

- C beds (servers)
- State space $S = \{0, 1, 2, \ldots, C\}$
- λ – Arrival rate
- n – current number of beds occupied (number of customers being served)
- $\mu(n)$ – Departure rate
- No queueing (ie. if the bed queue becomes full the emergency department goes on *bypass*.)

Transition rates

The transition rates for the Markov chain are

$$\alpha_{nn+1} = \begin{cases} \lambda, & 0 \leq n \leq C-1 \\ 0, & n = C \end{cases}$$

$$\alpha_{nn-1} = \begin{cases} 0, & n = 0 \\ \mu(n), & 1 \leq n \leq C \end{cases}$$

Transition Probabilities

The Markov chain stays in state n for a random time T_n and then moves to either state $n+1$ or $n-1$ with probabilities

$$P(n \to n+1) = \frac{\lambda}{\lambda + \mu(n)}, \qquad 0 \le n \le C-1$$

$$P(n \to n-1) = \frac{\mu(n)}{\lambda + \mu(n)}, \qquad 1 \le n \le C$$

Time Until an Arrival or Departure

In state n let

- T_n^+ be the time until the next arrival, and
- T_n^- be the time until the next departure.

Note that $T_n^+ \sim \exp(\lambda)$ and $T_n^- \sim \exp(\mu(n))$.

We have that

$$T_n = \begin{cases} T_0^+, & n = 0 \\ \min(T_n^+, T_n^-), & 1 \le n \le C-1 \\ T_C^-, & n = C \end{cases}$$

Density Function

The density function of T_n is

$$f_n(t) = \begin{cases} \lambda e^{-\lambda t}, & n = 0 \\ (\lambda + \mu(n)) e^{-(\lambda+\mu(n))t}, & 1 \le n \le C-1 \\ \mu(C) e^{-\mu(C)t}, & n = C \end{cases}$$

Probability of Reaching Capacity

- $p_n(t)$ – probability of moving from n to C patients in the time interval $[0, t]$
- $p_n(t|x)$ – probability of moving from n to C in $[0, t]$ *given* that the first transition from n occurs at time x

$$p_n(t|x) = \begin{cases} 0, & n < C, x > t \\ \begin{array}{c} \frac{\lambda}{\lambda + \mu(n)} p_{n+1}(t-x) \\ + \frac{\mu(n)}{\lambda + \mu(n)} p_{n-1}(t-x) \end{array}, & n < C, x \leq t \\ 1, & n = C \end{cases}$$

Probability of Reaching Capacity

For $0 \leq n \leq C$

$$p_n(t) \ = \int_0^\infty p_n(t|x) f_n(x) dx.$$

Thus,

$$p_0(t) = \int_0^t p_1(t-x)\lambda e^{-\lambda x} dx$$

$$p_n(t) = \int_0^t \left[\lambda p_{n+1}(t-x) + \mu(n) p_{n-1}(t-x)\right] e^{(\lambda+\mu(n))t} dx$$

$$p_C(t) = 1$$

Laplace Transforms

Taking Laplace transforms gives

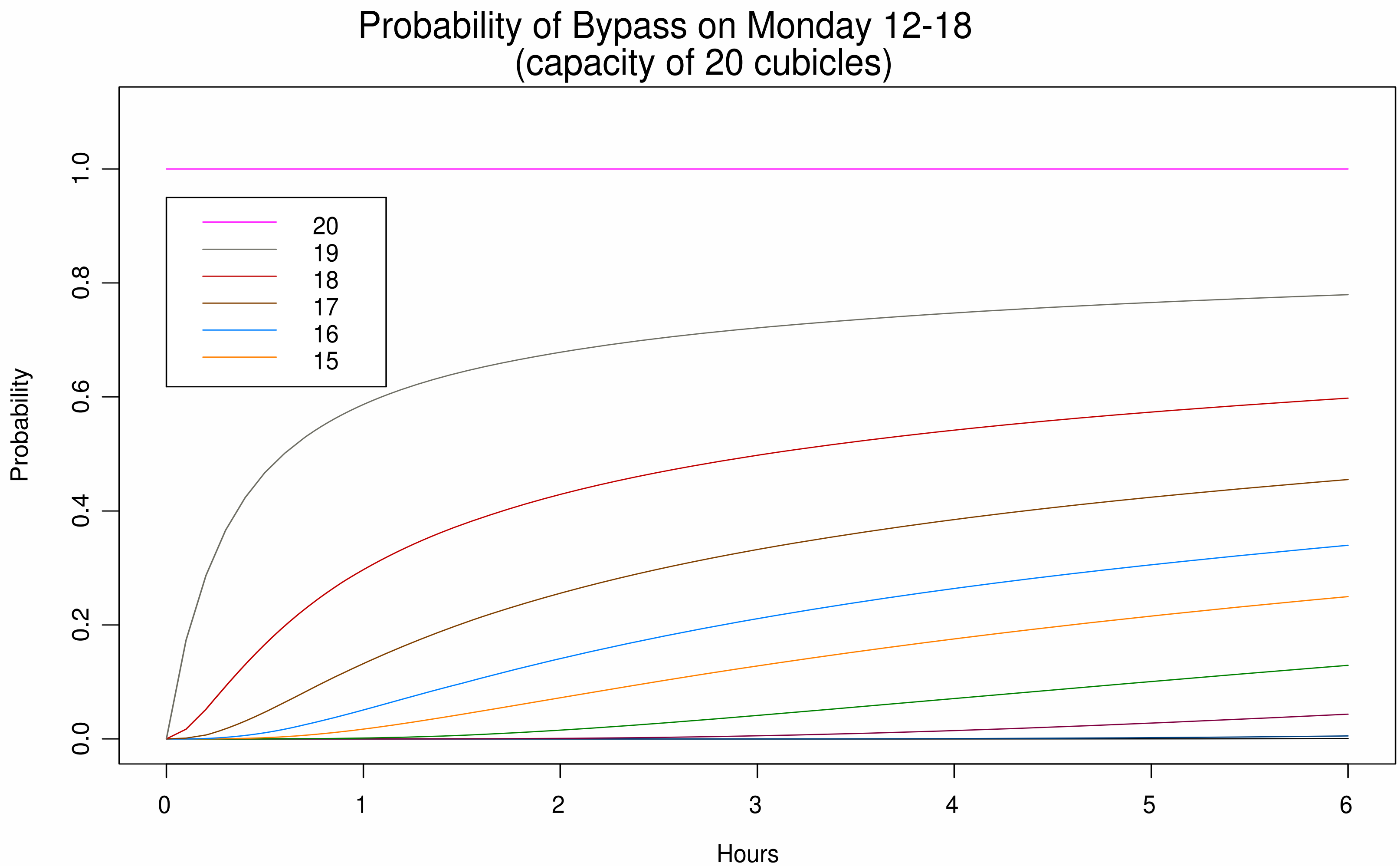
$$\widehat{P}_0(s) = \frac{\lambda}{s+\lambda}\widehat{P}_1(s)$$

$$\widehat{P}_n(s) = \frac{\lambda}{s+\lambda+\mu(n)}\widehat{P}_{n+1}(s) + \frac{\mu(n)}{s+\lambda+\mu(n)}\widehat{P}_{n-1}(s)$$

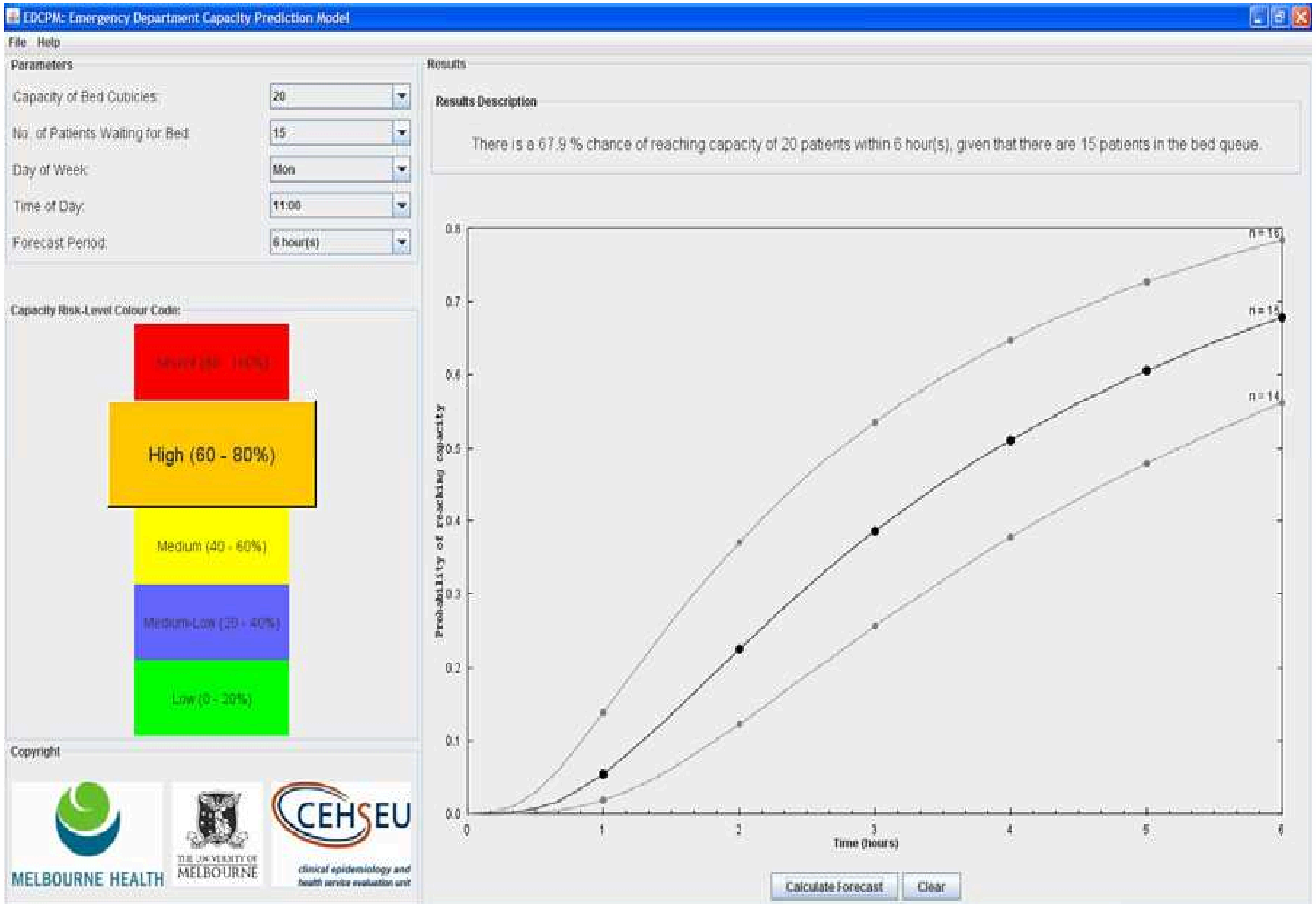
$$\widehat{P}_C(s) = \frac{1}{s}$$

To find $p_n(t)$ invert $\widehat{P}_n(s)$ numerically – Euler method, Abate and Whitt, 1995.

Probability of Ambulance Bypass



The ED Capacity Prediction Tool



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Model Validation

- To validate the model we used data observed from 19 April 2005 to 26 September 2006 (48,000 entries).
- Choose a threshold C.
- At the beginning of each 6 hour block calculate the probability of reaching capacity C using the model.
- Group the blocks into 10 bins of equal size N_i according to the probabilities.
- For each bin i, calculate the mean probability $\bar{p}_i$, and the variance of the probabilities V_i.

Model Validation

- For $i = 1, 2, \ldots, 10$, record the number of times capacity C is reached, O_i.
- Calculate the expected number of times capacity C is reached, $E_i = N_i \bar{p}_i$.
- The goodness-of-fit statistic is

$$G^2 = \sum_{i=1}^{10} \frac{(O_i - E_i)^2}{\text{var}(O_i)}$$

where $G^2 \sim \chi^2_{10}$ (Hosmer and Lemeshow, 1980) and

$$\text{var}(O_i) \approx N_i \bar{p}_i (1 - \bar{p}_i) - N_i V_i.$$

Model Validation

Bin	N_i	O_i	E_i	$\frac{(O_i - E_i)^2}{\text{var}(O_i)}$
1	182	0	0.01	0.008
2	182	0	0.10	0.105
3	182	1	0.38	1.04
4	182	2	1.03	0.91
5	182	4	2.36	1.16
6	182	8	4.92	1.98
7	182	13	9.77	1.13
8	182	25	19.21	1.95
9	182	55	42.45	4.84
10	182	110	106.70	0.25

p-value $= 0.16$

Future Work

- Apply and validate the model with more recent data.
- Analyse the data to see if the mandated government targets were met.
- Model patient flow through the entire emergency department.
- Determine strategies to improve the running of the emergency department based on stochastic models.

Headlines You'll Never See

“More hospital beds to open at Lyell McEwin”

ABC News Website – 5 July, 2013