## New Zealand Probability Workshop (Mon-Tue) and Australia and New Zealand Applied Probability Workshop (Wed-Fri)

January 23-27, 2012

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Local organisers Mark Holmes, Jeff Hunter, Ilze Ziedins

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All talks take place in room Eng3402 of the Engineering building (Building 403 off Symonds St, see campus map).

## Schedule

Mon. (23 Jan)	Tues.	Wed.	Thur.	Fri. (27 Jan)
09:05 opening remarks		09:10 opening remarks		
09:15 van der Hofstad	9:15 Camia	09:20 Nazarathy	09:20  Ross	09:20 Hunter
10:00 Goodman	10:00 Sakai	10:00 Palmowski	10:00 Black	10:00 Salomon
$10:50 \ tea$	10:50 tea	10:40 tea	10:40 tea	10:40 tea
11:10 Newman	11:10 Sun	11:00 Xia	11:00 O'Reilly	11:00 Costa
11:55 Stein	11:55 Greenwood	11:40 Hautphenne	11:40 Decrouez	11:40 Lu
12:45 lunch	12:45 lunch	12:20 lunch	12:20 lunch	12:20 lunch
14:00 Jones	14:00 Salisbury	followed by excursion	14:00 Daley	14:00 Pearce
14:45 Cotar	14:45 Addario-Berry	to Waiheke Island	14:40 Eshragh	14:40 Bean
15:30 tea	15:30 tea		15:20 Taylor	15:20 tea
16:00 mini-excursion			16:00 tea	
to Devonport			18:00 bus pickup	
			19:00 dinner	

## Titles and Abstracts in Alphabetical Order

## • Louigi Addario-Berry (McGill U.)

## The scaling limit of the minimum spanning tree of the complete graph

Let  $K_n$  be the complete graph on n vertices, and assign iid uniform [0, 1] weights to its edges. Write  $T_n$  for the (almost surely unique) minimum weight spanning tree of  $K_n$  with these weights. The random tree  $T_n$ comes equipped with an intrinsic distance (graph distance) and a mass measure (each vertex has mass one). We show that that after a suitable rescaling of distances and masses,  $T_n$  converges in distribution to a limiting random measured real tree T, in the Gromov–Hausdorff–Prokhorov sense, and establish some basic properties of the limiting tree T.

Joint work with Nicolas Broutin, Christina Goldschmidt, Gregory Miermont

#### • Nigel Bean (The University of Adelaide)

#### QBDs, Rational Arrival Processes and Richard Tweedie

In this talk I will introduce you to Rational Arrival Processes, which are the natural process extension of the family of Matrix Exponential distributions. The family of Matrix Exponential distributions are exactly the family of distributions with rational Laplace-Stieltjes transform, and are a natural generalization of the family of Phase-Type distributions. We use such processes in a queue and so generalize the standard quasi-birth-and-death (QBD) process into a richer environment. Such a process is then an example of a Markov chain on an uncountable state-space with an *operator*-geometric stationary distribution, as studied by Richard Tweedie in 1982. We apply Richard's results and then exploit a peculiar linearity property to reduce the resulting calculations to simple matrix calculations. We further show that any relevant algorithm from the matrix-analytic methods literature can be applied to this richer class of models. This approach provides general results, replacing the laborious derivation of specific results that we had performed earlier.

### • Andrew Black (The University of Adelaide)

## Modelling household disease dynamics and control

We present a modelling framework that allows us to incorporate accurate household structure in a stochastic epidemic model while still being computationally feasible to investigate. We assume that within-household dynamics are governed by a continuous-time Markov chain, and consider the initial stages of disease emergence, during which an assumption of between-household mixing resulting in novel households being infected is justified. This allows us to calculate a number of population level quantities, such as the early growth rate and offspring distribution, for a heterogeneous distribution of household sizes. This methodology also allows us to easily asses the impact of different intervention strategies such as the use of antivirals. We will illustrate the methodology by using census data from several countries and evaluating the impact of delays in antiviral prophylaxis.

## • Federico Camia (Vrije U. Amsterdam) The Massive Brownian Loop Soup

The Brownian Loop Soup is, roughly speaking, a Poisson process of Brownian loops in the plane. It is conjectured to describe the scaling limit geometry of various two-dimensional models of statistical mechanics at their critical points, such as the critical Ising model. Critical models are known to give rise to "massless" (scale-invariant) field theories in the scaling limit. Is there a variant of the Brownian Loop Soup that describes the geometry of near-critical scaling limits, corresponding to "massive" (non-scale-invariant) field theories? I will identify a possible candidate, explain why it is a natural choice, and show how it provides a first positive answer to the question above in the context of the Gaussian Free Field.

## • Renato Costa (Western Institute of Technology) Option Pricing under a Nonlinear and Nonnormal GARCH

We investigate the pricing of options in a class of discrete-time Flexible Coefficient Generalized Autoregressive Conditional Heteroskedastic (FC-GARCH) models with nonnormal innovations. A conditional Esscher transform was used to select a price kernel for valuation in the incomplete market. This choice of the price kernel can be justified by an economic equilibrium argument based on maximizing the expected power utility. We provide a numerical study on the pricing and risk management results when the GARCH and the FCGARCH innovations have a normal distribution or a shifted-Gamma distribution and identify some key features of the pricing results. Joint work with Alvaro Veiga and Tak Kuen Siu.

## • Codina Cotar (Fields Institute, Toronto)

## Density functional theory and optimal transport with Coulomb cost

In this talk I explain a promising and previously unnoticed link between electronic structure of molecules and optimal transportation, and first results. The 'exact' mathematical model for electronic structure, the manyelectron Schroedinger equation, becomes computationally unfeasible for more than a dozen or so electrons. For larger systems, the standard model underlying a huge literature in computational physics/chemistry/materials science is density functional theory (DFT). In DFT, one only computes the single-particle density instead of the full many-particle wave function. In order to obtain a closed equation, one needs a closure assumption which expresses the pair density in terms of the single-particle density  $\rho$ . We show that in the semiclassical limit, there holds an exact closure relation, namely the pair density is the solution to a optimal transport problem with Coulomb cost. We prove that this problem has a unique solution given by an optimal map; moreover we derive an explicit formula for the optimal map in the case when  $\rho$  is radially symmetric (note: atomic ground state densities are radially symmetric for many atoms such as He, Li, N, Ne, Na, Mg, Cu). In my talk I focus on how to deal with its main mathematical novelties (cost decreases with distance; cost has a singularity on the diagonal). I also discus the derivation of the Coulombic OT problem from the many-electron Schroedinger equation for the case with N electrons.

Joint work with Gero Friesecke (TU Munich) and Claudia Klueppelberg (TU Munich)

## D.J. Daley (The University of Melbourne) BRAVO effect in M/M/k/K queueing systems

In its stationary state, the departure process  $N_{dep}$  of a M/M/k/K queueing system with large buffer capacity K is approximately a Poisson process at rate min( $\rho$ , 1) where  $\rho$  is the arrival rate (subject to suitable choice of scale). For the case k = 1 the asymptotic variance behaviour is then

$$\operatorname{var} N_{\operatorname{dep}}(0, t] \sim \min(\rho, 1) \operatorname{E} \left( N_{\operatorname{dep}}(0, t] \right) = \min(\rho, 1) t \quad (t \to \infty)$$

except when  $\rho = 1$ , in which case the constant is replaced by 2/3; Nazarathy and Weiss (2008) demonstrated this (to them) surprising fact, and dubbed it the BRAVO effect ("Balancing [i.e.  $\rho = 1$ ] Reduces Asymptotic Variance of Output"). We have now shown that for general k, a BRAVO effect persists but the factor 2/3 is replaced by one that depends on how k grows (or not) with K. Indeed, more is known: there is nontrivial asymptotic behaviour for  $\rho = 1 - \beta/\sqrt{k}$  for  $k, K \to \infty$  subject to  $K = \sqrt{k}/\alpha$  for positive  $\alpha$  and finite  $\beta$ . Thus, while there is a 'continuous' transition in stochastic behaviour as  $\rho$  increases through the critical value  $\rho = 1$ , higher order moments need not be continuous.

Reference: Nazarathy, Y. and Weiss, G. (2008). The asymptotic variance rate of finite capacity birth-death queues. *Queueing Systems* **59**, 135–156.

# Geoffrey Decrouez (The University of Melbourne) On the Ergodicity and the Stationary distribution of a Stochastic Neuron Network

We present a stochastic network model for real-life neuron networks that takes into account Hebbian learning. The system can be represented by a Markov process whose state space is an infite hierarchy of finitedimensional simplices. nder broad assumptions, the process is shown to be ergodic and have a continuously differentiable density w.r.t. the sum of Lebesgue measures of the simplices. We also demonstrate that the stationary distribution of the network can be approximated by a finite-dimensional one corresponding to a similar Markov process on a truncated version of the state space and that the convergence rate is super-exponential rate. (Joint work with Kostya Borovkov and Matthieu Gilson).

# Ali Eshragh (The University of Adelaide) Optimal Experimental Design for a Growing Population

Our goal is to estimate the rate of growth, lambd), of a population governed by a simple stochastic model. We may choose (n) time points at which to count the number of individuals present, but due to detection difficulties, or constraints on resources, we are able only to observe each individual with fixed probability (p). We discuss the optimal times at which to make our n observations in order to approximately maximize the accuracy of our estimate of ambda. For computational and analytical reasons which will be discussed, we specifically focus on the cases n=1 and n=2, presenting both theoretical and numerical findings.

## • Jesse Goodman (Leiden U.)

## Long paths for first passage percolation on the complete graph.

First passage percolation on the complete graph – also known as the stochastic mean-field model of distance – describes the flows and optimal routing strategies for a network with variable link costs. To each edge of the complete graph, associate an i.i.d. positive edge cost  $X_e$ . The cost of a path is the sum of its edge costs, and the optimal path between two vertices is the path of lowest cost.

When the edge weights are exponential, optimal paths contain of order  $\log n$  edges. Using branching process and coupling techniques, we show how distributions with a heavy tail at 0 produce optimal paths that are much longer, and show how to identify the growth rate explicitly in terms of the tail of the distribution. This allows us to exhibit a smooth transition between path lengths of order  $\log n$  (as in super-critical random graphs) and path lengths of order  $n^{1/3}$  (as in the critical Erdös-Rényi random graph).

## • Priscilla Greenwood (U.B.C.)

## Sustained oscillations for density dependent Markov processes

Simulations of models of epidemics, biochemical systems, and other bio-systems show that when deterministic models yield damped oscillations, stochastic counterparts show sustained oscillations at an amplitude well above the expected noise level. A characterization of damped oscillations in terms of the local linear structure of the associated dynamics is well known, but in general there remains the problem of identifying the stochastic process which is observed in stochastic simulations. Here we show that in a general limiting sense the stochastic path describes a circular motion modulated by a slowly varying Ornstein-Uhlenbeck process. Numerical examples are shown for the Volterra predator-prey model, Sel'kov's model for glycolysis, and a damped linear oscillator. (Joint work with Peter H. Baxendale).

#### • Sophie Hautphenne (The University of Melbourne)

**Extinction probability of branching processes with infinitely many types** We consider multitype branching processes with infinitely many types. We emphasize the differences with the finite-type case in some asymptotic properties and in the extinction criteria. We propose converging sequences to the extinction probability vector and give them a probabilistic interpretation. Joint work with Guy Latouche and Giang Nguyen.

• Remco van der Hofstad (Eurandom/T.U. Eindhoven) Hypercube percolation

Consider bond percolation on the hypercube  $\{0,1\}^n$  at the critical edge probability  $p_c$  defined such that the expected cluster size equals  $2^{n/3}$ , where  $2^{n/3}$  acts as the cube root of the number of vertices of the *n*dimensional hypercube. Percolation on the hypercube was proposed by Erdos and Spencer (1979), and has proved to be substantially harder than percolation on the complete graph due to the non-trivial geometry of the hypercube. In this talk, I will describe the phase transition for percolation on the hypercube, and show that it shares many features with that on the complete graph.

In previous work, we have identified the subcritical and critical regimes of percolation on the hypercube. In particular, we know that for  $p = p_c(1 + O(2^{-n/3}))$ , the largest connected component is of size roughly  $2^{2n/3}$  and that this quantity is non-concentrated. So far, we were missing an analysis of the behavior of the largest connected component just above the critical value. In this work, we identify the supercritical behavior of percolation on the hypercube by showing that, for any sequence  $\epsilon_n$  tending to zero, but  $\epsilon_n$  being much larger than  $2^{-n/3}$ , percolation on the hypercube with edge probability  $p = p_c(1 + \epsilon_n)$  has, with high probability, a unique giant component of size  $(2 + o(1))\epsilon_n 2^n$ . A main tool is the use of non-backtracking random walk, which we use to show that long percolation paths have endpoints that are almost uniform on the hypercube.

This is joint work with Asaf Nachmias, building on previous work with Markus Heydenreich, Gordon Slade, Christian Borgs, Jennifer Chayes and Joel Spencer.

• Jeffrey J. Hunter (Auckland University of Technology)

## The Role of Kemeny's Constant in Properties of Markov Chains

In a finite m-state irreducible Markov chain with stationary probabilities pi and mean first passage times  $m_{ij}$  (mean recurrence time when i = j) it was first shown, by Kemeny and Snell, that  $\sum_{j=1}^{m} \pi_j m_{ij}$  is a constant, K, not depending on i. This constant has since become known as Kemenys constant. We consider a variety of techniques for finding expressions for K, derive some bounds for K, and explore various applications and interpretations of these results. Interpretations include the expected number of links that a surfer on the World Wide Web located on a random page needs to follow before reaching a desired location, as well as the expected time to mixing in a Markov chain. Various applications have been considered including some perturbation results, mixing on directed graphs and its relation to the Kirchhoff index of regular graphs.

• Vaughan Jones (U. Auckland, U.C. Berkeley, Vanderbilt U.) Random matrices and von Neumann algebras.

I will survey the recent constructions due to Guionnet, Shlyakhtenko, Curran and myself of von Neumann algebras associated with planar algebras, together with associated random matrix models with a non-integral number of independent random matrices.

## • Zudi Lu (The University of Adelaide)

## Asymptotic distribution and uniform convergences for local linear fitting under stochastic processes of generalised dependence

Local linear fitting is a popular nonparametric method in nonlinear statistical and econometric modelling; see, for example, Fan and Gijbels (1996), Fan and Yao (2003) and Li and Racine (2007). Under various mixing stochastic processes (in particular alpha-mixing, i.e., strong mixing, which includes many other mixings such as phi-mixing, beta-mixing, as special cases), this technique of local linear fitting has been well studied in the literature by many researchers, c.f., Liebscher (1996), Masry (1996), Bosq (1998), Fan and Yao (2003), Hansen (2008), Kristensen (2009), among others. However, from a practical point of view, the mixing (e.g., alphamixing) processes suffer from many undesirable features. For example, for a lot of popular processes in econometrics such as an ARMA process mixed with ARCH or GARCH errors, it is still difficult to show whether they are alpha-mixing or not except in some very special cases. Even for a very simple linear AR(1) model with innovation being independent symmetric Bernoulli random variables taking on values of -1 and 1, the stationary solution to the model is not alpha-mixing (Andrew 1984).

In this talk, I will first review some of the extensions of the stochastic processes from the mixing ones, in particular a class of generalised stable processes from mixings, or called near epoch dependence, which covers a variety of interesting stochastic processes in time series econometric modelling. Then I will report some recent developments on the local linear fitting technique under this kind of generalised dependent processes. I will particularly introduce some results that my co-authors and I have recently established in this regard, including the pointwise asymptotic distributions for the probability density estimation and local linear estimator of a nonparametric regression function as well as the uniform strong and weak consistencies with convergence rates for the local linear fitting under the condition of near epoch dependence. These results are of wide potential interest in time series semiparametric modelling.

#### • Yoni Nazarathy (Swinburne University of Technology) Scaling limits of cyclically varying birth-death processes

Fluid limits of stochastic queueing systems have received considerable attention in recent years. The general idea is to scale space, time and/or system parameters as to obtain a simpler, yet accurate description of the system. A basic example is the single server queue with time speeded up and space scaled down at the same rate. A second well known example is the Markovian infinite server queue with the arrival rate speeded up and space scaled down at the same rate. Such scalings and their network generalizations are often useful for obtaining stability conditions and approximating optimal control policies. In this talk we consider birth-death processes with general transition rates and obtain an asymptotic scaling result, generalizing the Markovian single server and infinite server cases. We apply our results to the steady-state analysis of queueing systems with cyclic or time varying behaviour. Examples are systems governed by deterministic cycles, queues with hysteresis control and queues with Markov-modulated arrival or service rates. The unifying property of such systems, is that if they are properly scaled, the resulting trajectories follow a cyclic or piece-wise deterministic behavior which is determined by the asymptotic scaling. This yields simple a approximation for the stationary distribution which is shown to be asymptotically exact. Joint work with Matthieu Jonckheere.

## • Chuck Newman (NYU) Short-Range Spin Glasses: an Introduction

Spin glasses are a prototype disordered system whose successful analysis could fill a gap in our understanding of condensed matter physics. They present a mathematical problem of both depth and complexity. Parisi's replica symmetry breaking (RSB) solution of the mean-field spin glass exhibits a new kind of broken symmetry with many novel features. However, more realistic short-range models remain unsolved, and a controversy exists as to whether RSB is valid for them. But work of recent years has clarified and restricted the types of ordering short-range spin glasses can exhibit.

In this talk I will review spin glasses and their history and why they are of interest to both physicists and mathematicians. I will briefly discuss old and newer results about short-range spin glasses at low and zero temperature. Our focus is on low-temperature equilibrium properties of short-range spin glasses, including: numbers of pure and ground states in various dimensions; the question of whether a phase transition and broken spin-flip symmetry occur; the use of metastates to analyze the organization of the (presumed) lowtemperature phase; and whether mean-field spin glasses can shed light on short-range models in moderate dimensions.

This talk will be mostly heuristic. In an accompanying talk, Dan Stein will discuss a rigorous result on spin glass ground states obtained by us and our collaborators Louis-Pierre Arguin and Michael Damron.

## • Malgorzata O'Reilly (The University of Tasmania)

## Infinitesimal Generators of Markovian Stochastic Fluid Flow Models

Recently there has been considerable interest in Markovian stochastic fluid flow models. A number of authors have used different methods to calculate quantities of interest. We discuss methods which allow for the direct analysis of the stochastic fluid flow models, without the need for transformation to other equivalent models. The foundation of the analysis is the derivation of the infinitesimal generator of the model. The advantage of having established the infinitesimal generator is that the expressions for various performance measures can be written in terms of that generator.

First, we discuss our methods in the context of the traditional one-dimensional stochastic fluid flow models [1]. Next, we consider the stochastic two-dimensional fluid flow model [2] which consists of two stochastic fluid flows, one of which is unconstrained, driven by the same underlying Markov chain.

Finally, we consider the stochastic fluid-fluid model [3], which is a stochastic fluid model driven by an uncountable-state process, which is a stochastic fluid model itself. This work is the first direct analysis of a stochastic fluid model that is Markovian on a continuous state space.

#### References:

- Bean, N. G., O'Reilly, M. M. and Taylor, P. G. (2005). Hitting probabilities and hitting times for stochastic fluid flows. Stoch. Proc. Appl.115, 1530-1556.
- [2] Bean, N. G. and O'Reilly M. M. (2011). Stochastic 2-dimensional fluid model. Submitted for publication.
- [3] Bean, N. G. and O'Reilly M. M. (2012). A Stochastic Fluid Model driven by an uncountable-state process, which is a Stochastic Fluid Model itself: Stochastic Fluid-Fluid model. Submitted for publication

#### • Zbigniew Palmowski (University of Wroclaw)

## Forward-backward extrema of Lévy processes and fluid queues

For a Lévy process X and fixed S > t (possibly  $S = +\infty$ ) the future up-down process is defined by:

$$U_{t,S}^* = \sup_{t \le u < t+S} (X_u - X_t).$$

The fluctuations of  $U_{t,S}^*$  are described by the running supremum and running infimum:

$$\overline{U}_{T,S}^* = \sup_{0 \le t \le T} U_{t,S}^*, \qquad \underline{U}_{T,S}^* = \inf_{0 \le t \le T} U_{t,S}^*.$$

The random variables  $\overline{U}_{T,S}^*$  and  $\underline{U}_{T,S}^*$  are path-dependent performance measures of fluid queues:  $U_{t,S}^*$  describes the buffer content of fluid queues observed at time t when queue already has been already running S - t units of time before 0 and  $\overline{U}_{T,S}^*$  and  $\underline{U}_{T,S}^*$  are the maximal and minimal such contents for t ranging over [0,T].

In the case that X has strictly negative mean we find the exact asymptotic decay of the tail distributions of above extrema in both the Cramér and heavy-tailed case.

When the jumps of X are of single sign we explicitly identify the one-dimensional distributions in terms of the scale functions. We also analyze some examples.

• Charles Pearce (The University of Adelaide)

## Duration problem with multiple exhanges and the classical secretary problem

The multiple-choice duration problem has, as objective, maximizing the time of possession of relatively best objects. We show that for the *m*-choice problem there exists a sequence  $(s_1, s_2, \dots, s_m)$  of critical numbers such that, when there are k choices still to be made, the optimal strategy selects a relatively best object if it appears at or after time  $s_k$ . We exhibit an equivalence with the classical secretary problem.

## • Joshua Ross (The University of Adelaide)

#### Invasion of an infectious disease in a finite, homogeneous population

A novel strain of an infectious disease starts life in a single individual. To become established in a population it must be successfully transmitted to at least one other individual in the population. Perhaps the most well-known quantity related to this process is  $R_0$  (the basic reproduction number) which is the expected number of secondary infections arising directly from the initial individual.  $R_0$  is typically evaluated under the assumption of an infinite population size.

Despite the importance of  $R_0$  in infectious disease modelling and management, little attention has been paid to the impact of finite population size, and, as far I know, no one has investigated the probability mass function of secondary infections. We will explore the impact of population size along with the type of infectious period distribution on the distribution and expectation of number of secondary infections.

#### • Akira Sakai (Hokkaido U.)

## Asymptotic behavior of the critical two-point functions for statistical-mechanical models with power-law decaying potentials.

We consider self-avoiding walk, percolation and the Ising model on the *d*-dimensional integer lattice  $\mathbb{Z}^d$  that are defined by power-law decaying pair potentials of the form  $D(x) \approx |x|^{-d-\alpha}$  for some  $\alpha > 0$ . These models are known to exhibit critical behavior as the parameter p, such as the inverse temperature for the Ising model, tends to its critical value  $p_c$ . Let  $d_c$  denote the upper-critical dimension:  $2(\alpha \wedge 2)$  for self-avoiding walk and the Ising model and  $3(\alpha \wedge 2)$  for percolation. I will explain the result of joint work in progress with Lung-Chi Chen that, if  $\alpha \neq 2$ ,  $d > d_c$  and the spread-out parameter L is sufficiently large, then the critical two-point function  $G_{p_c}(x)$  for each model, such as the spin-spin correlation function for the critical Ising model, is asymptotically a multiple of  $|x|^{\alpha \wedge 2-d}$ . This is a key element for the so-called 1-arm exponent to take on its mean-field value: 1/2 for percolation and 1 for the Ising model. The proof is based on application of the lace expansion.

#### • Tom Salisbury (York University)

#### Random Walk in Degenerate Random Environment

I will describe results about random walk in random environments, which degenerate in the sense that standard ellipticity assumptions do not hold. In other words, a walker on the square 2-dimensional lattice uses walk probabilities in which certain nearest neighbour transitions are forbidden. These questions lead naturally into studies of percolation on directed random graphs. This is work with Mark Holmes (Auckland).

#### • Yacov Salomon (The University of Melbourne)

#### Unimodal density estimation using cross entropy minimisation

In a range of applications it is often required to estimate the probability density associated with some random phenomena under study, or uncertain parameters. In some cases, the density is known to be unimodal. Other information regarding the density, such as some of its moments or percentiles, may also be known. If the explicit parametric form of the density cannot be determined from physical considerations alone then parametric methods may prove unsatisfactory. Since only summary statistics, rather than raw data, is available, nonparametric kernel estimation is not applicable. However, if a standard Cross Entropy approach is used, the resulting density may not be unimodal. Using a characterisation of unimodal random variables due to Shepp following the work of Khinchin, we demonstrate a method for incorporating unimodality in cross entropy density estimation. We illustrate our method by applying it in the context of expert elicitation.

#### • Dan Stein (NYU)

## Ground States of the Two-Dimensional Spin Glass

This is joint work with Louis-Pierre Arguin, Michael Damron and Chuck Newman (Commun. Math. Phys. 300 (2010) 641-657). It is an open problem to determine the number of infinite-volume ground states in the Edwards-Anderson (nearest neighbor) spin glass model on  $\mathbb{Z}^d$  for  $d \ge 2$  (with, say, mean zero Gaussian couplings). This is a limiting case of the problem of determining the number of extremal Gibbs states at low temperature. In both cases, there are competing conjectures for  $d \ge 3$ , but no complete results even for d = 2. I report on results which go some way toward proving that (with zero external field, so that ground states come in pairs, related by a global spin flip) there is only a single ground state pair (GSP). Our result is weaker in two ways: First, it applies not to the full plane  $\mathbb{Z}^2$ , but to a half-plane. Second, rather than showing that a.s. (with respect to the quenched random coupling realization J) there is a single GSP, we show that there is a natural joint distribution on J and GSP's such that for a.e. J, the conditional distribution on GSP's given J is supported on only a single GSP. The methods used combine percolation-like geometric arguments

with translation invariance (in one of the two coordinate directions of the half-plane) and uses as a main tool the "excitation metastate" which is a probability measure on GSP's and on how they change as one or more individual couplings vary.

## • Rongfeng Sun (N.U. Singapore)

#### Symmetric Rearrangements Around Infinity with Applications to Levy Processes

We prove a new rearrangement inequality for multiple integrals, which partly generalizes a result of Friedberg and Luttinger (1976) and can be interpreted as involving symmetric rearrangements of domains around infinity. As applications, we prove two comparison results for general Levy processes and their symmetric rearrangements. The first application concerns the survival probability of a point particle in a Poisson field of moving traps following independent Levy motions. We show that the survival probability can only increase if the point particle does not move, and the traps and the Levy motions are symmetrically rearranged. This essentially generalizes an isoperimetric inequality of Peres and Sousi (2011) for the Wiener sausage. In the second application, we show that the q-capacity of a Borel measurable set for a Levy process can only increase if the set and the Levy process are symmetrically rearranged. This result generalizes an inequality obtained by Watanabe (1983) for symmetric Levy processes. Joint work with Alex Drewitz (ETH) and Perla Sousi (Cambridge).

#### • Peter Taylor (The University of Melbourne)

## Queueing Theory and 'Ideal Hospital Occupancy'

I'm getting more and more interested in the contribution that the applied probability community can make to decision-making at the political and administrative levels. The relationship between these areas is not easy.

I shall illustrate some points using the concept of 'ideal hospital occupancy'. The paper that has had the most influence on the dimensioning of hospitals, at least in the UK, was written in 1999 by three health economists Bagust, Place and Posnett. They used an Excel simulation to make the point there is trade-off between availability and utilisation, something that is taught in undergraduate stochastic modelling courses. From a mathematical point of view, their paper has many problems, and yet it had a greater effect on decision-making in the UK heath system than any of the huge number of queueing theory papers that have been written before and since that make essentially the same point.

In this talk, I shall discuss some recent contributions by myself and others on the subject of 'ideal hospital occupancy'. I shall also pose the question of how best the applied probability community can contribute to the political debate in all sorts of areas where there is a need to make decisions under conditions of uncertainty.

• Aihua Xia (The University of Melbourne)

## On asymptotics of locally dependent point processes

We investigate a family of approximating processes that can capture the asymptotic behaviour of locally dependent point processes. We prove two theorems to accommodate respectively the positively and negatively related dependent structures. Three examples are given to illustrate that our approximating processes are doing a better job than compound Poisson processes when the mean number of random events increases. This talk is based on a joint work with Fuxi Zhang.