

MATH4406 (Control Theory)

HW2 (Unit 3): Classic Engineering Control

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This HW deals explicitly with an expository paper:

“Nonminimum-phase zeros - much to do about nothing - classical control - revisited part II”, IEEE Control Systems Magazine, 27(3), 45–57, 2007.

A link to the paper is on the course web-page.

Read through the paper, skipping the MIMO and discrete time sections on pages 54–56. In the reading you should also skip over statements that use concepts that you have not studied, such as “controllable/observable/detectable” (covered in Unit 4 but not required yet) or “nonholonomic constraints” (will not be covered in the course).

Answer / perform the following (some are quick/easy, some are less):

1. Provide a definition for a “non-minimum phase” LTI, SISO, Rational system. Explain the origin of the term: “non-minimum phase”.
2. Derive the transfer function $G(s) = C(sI - A)^{-1}B$ appearing in page 46.
3. Explain what is happening in Figure 1.
4. Reproduce Figure 2 using either Mathematica/MATLAB or any other software package of your choice.
5. Now repeat for a modified $G(s)$ where the real part of the zeros “moves” into the LHP. By this it is meant that you will create a plot of several step responses – or several systems, where for each system you bump the zeros one more “notch” to the left. Make sure your graph is readable (i.e. select your “notches” so that the graph tells the “story” nicely).
6. Re-write the proof of the proposition on page 51. In your re-writing, explicitly spell out terms such as “relative degree” and “Hurwitz”.
7. On page 49, there is a statement about an asymptotically stable transfer function $G(s)$, indicating that it exhibits overshoot if $G(s) - G(0)$ has at least one positive zero. Formalize the proof of this statement (it is outlined in the text briefly).
8. Is the above a characterization of all systems that have overshoot? If yes - prove it. If not, give a counter-example.
9. A servo-mechanism is essentially synonymous with a regulator (look it up on Wikipedia). Derive the transfer functions, $S(s), T(s)$ on pages 50, 51.
10. Formalize (prove) the statement about zero-crossings on page 51.
11. Explain/discuss/expand the “Bicycle Countersteering Revisited” box on page 53. There is no need to be precise here, but rather to illustrate that you are literate on the basic terms. LQG is the subject of Unit 10 (you have seen it briefly in the introduction Unit 1).

Hand in time: Start of class 6.1, August 27, 2012.