MATH4406 (Control Theory)
HW3 (Unit 4): State Space Linear Systems - 1

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1. Consider the scalar differential equation,
$$\dot{x}(t) = a(t)x(t), \quad x(0) = x_0,$$
with $a(\cdot)$ a continuous function. Use Picard iterations to obtain the solution,
$$x(t) = x_0 \, e^{\int_0^t a(s)ds}.$$

2. Show that for any $A \in \mathbb{R}^{n \times n}$, there exist functions, $\alpha_0(t), \ldots, \alpha_{n-1}(t)$ such that,
$$e^{At} = \sum_{i=0}^{n-1} \alpha_i(t) A^i.$$

3. Consider the continuous time, (A, B, C, D) system parameterized by,
$$A = \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -10 \end{bmatrix}, \quad D = 1.$$
For this system,
   (a) Find the transfer function, $H(s)$.
   (b) Find the impulse response matrix.
   (c) Find the controllability matrix, is the system controllable?
   (d) Find the observability matrix, is the system observable?
   (e) Design a Luenberger observer for the system such that the estimation error, $e(t) = \hat{x}(t) - x(t)$, converges to 0 without oscillations.
   (f) Illustrate the Luenberger observer above, numerically (i.e. supply graphs of $\hat{x}(t)$ vs. $x(t)$ for some initial conditions.

4. Consider problem 3.38 in [AntMich07] (photocopy passed out in class), carry out (a) and (b) as stated in the problem.

5. Consider the "inverted pendulum positioning system" example from the lecture notes (taken from the book [SivKwa72]). Assume full state observation, i.e. $C = I_4$, $D = 0_{4\times 1}$. Design a state feedback controller that will result in a critically stable system (i.e. stable but not asymptotically stable). Plot illustrative trajectories of the evolution of $s(t)$ and $\phi(t)$ for various initial conditions and reference inputs.

6. Prove (from first principles) that a continuous time (A, B, C, D) system is observable, if and only if $\text{obs}(A, C)$ is full-rank.