MATH4406 (Control Theory)
HW6 (Unit 6): LQR + MPC

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This HW deals primarily with the (highly cited) paper [1]. As opposed to the paper of HW2 which was an expository article, the paper [1] is a research paper. You are not expected to have the full background for all of the details in the paper.

1. Read in detail through the abstract, introduction and section 2.
   - Make a clean list of terms that you do not know and search for their definition (e.g. perhaps detectability). Summarize this list as your answer to this question.
   - Make a bullet point summary of the key points summarized in section 2 – this section summarizes key MPC results. Each bullet point should contain at most 3 lines (i.e. should be an abstract of a property/result/definition related to MPC).
2. Derive equation (7) and the corresponding matrices.
3. Below equation (7) the authors write: *For this reason, the application of MPC has been limited to "slow" and/or "small" processes.* Today (2012) this is debatable as there are many publications about computation of "big" QPs associated with MPC in milliseconds. Search the web for one such paper and cite it as your answer.
4. Move onto Corollary 1 and Corollary 2. It is enough for you to look at the corollary statements and not explicitly needed to look at the proofs (for this exercise). Look now at slide 23 "The closed loop system is non-linear" of the lecture slides of Unit 6. Explain how the slide relies on Corollary 2.
5. Look at Algorithm 1. It is a recursive algorithm, how so? What is the stopping condition?
6. Explain the first 10 lines of subsection 4.4 dealing with simple combinatorial upper bounds on the number of regions. Does this upper bound imply that Algorithm 1 terminates in finite time?
7. Skim through Section 6. For each subsection summarize the main point in one to three sentences.

8. The final question deals solely with Example 7.1 of the paper:

   - Explain equations (27) - i.e. re-derive those numbers (sampling a continuous time system to get a discrete time system)!
   - Explain (re-derive) the numbers in H, F, ..., E.

9. (More time consuming question - perhaps): Assume x lies in Region 1 (in the big displayed equation for u on page 15). Pick such an x and write down the associated u (i.e. simple quick calculation based on the explicit solution given). Now solve the quadratic program associated with the x that you chose and see that you get the same solution as that resulting from the explicit controller. You can solve quadratic programs in MATLAB using quadprog (or other packages also).

References

[1] A. Bemporad, M. Morari, V. Dua, and E.N. Pistikopoulos. The explicit linear quadratic regulator for constrained systems. *Automatica*, 38(1):3–20, 2002.