Control Theory
Homework 7
Due 9 Nov 2012

1. Consider the system $\dot{x}(t) = -10x(t) + u(t)$. Using the Hamilton-Jacobi-Bellman equation, find the control that minimizes the cost

$$J = \frac{1}{2}x^2(T) + \int_0^T \left(\frac{1}{4}x^2(t) + \frac{1}{2}u^2(t)\right) dt.$$

2. Consider the system $\dot{x}_1(t) = x_2(t)$, $\dot{x}_2(t) = -x_1(t) - 2x_2(t) + u(t)$. Define the cost as

$$J = 10x_1^2(T) + \frac{1}{2}\int_0^T (x_1^2(t) + 2x_2^2(t) + u^2(t))dt.$$

The control that minimizes J can be found in the form

$$u^*(t) = L(t)K(t)x(t),$$

where the matrix $L(t)$ can be found explicitly for every t, and the matrix $K(t)$ satisfies an ODE. Find $L(t)$ and write out the ODE for $K(t)$.

3. Write down the Euler equation for the functional

$$J(x) = \int_0^2 (x^2(t) + 2\dot{x}(t)x(t) + \dot{x}^2(t))\, dt$$

in the Banach space $C^2([0,2] \to \mathbb{R})$. Solve this equation under the boundary conditions $x(0) = 1$, $x(2) = -3$.

4. Determine the necessary conditions that must be satisfied by the extremals of the functional

$$\int_{t_0}^{t_f} (1 + w_3^2(t))\, dt$$

with the constraints $\dot{w}_1(t) = w_2(t)$, $\dot{w}_2(t) = w_3(t)$.

5. The system $\dot{x}_1(t) = x_2(t)$, $\dot{x}_2(t) = -x_1(t) + (1 - x_1^2(t))x_2(t) + u(t)$ is to be controlled to minimize the cost

$$\frac{1}{2}\int_0^1 (2x_1^2(t) + x_2^2(t) + u^2(t))\, dt.$$

Assume $x(0) = a$ and $x(1) = b$ for some $a, b \in \mathbb{R}^2$. Write down the corresponding costate equations. Find the control(s) which make(s) the partial derivative of the Hamiltonian in u equal to 0.