MATH4406 (Control Theory)
Quiz 1 (Units 2 and 3) - August 23, 2012.

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Quiz duration: 40 minutes.

An input output system (plant) follows this differential equation:

$$\dot{y}(t) + c_1 y(t) = c_2 u(t),$$

with $c_1, c_2 \in \mathbb{R}$.

1) Treating $u(\cdot)$ as the input and $y(\cdot)$ as the output, write the transfer function of the plant.

2) For what values of c_1, c_2 is the plant strictly stable?

3) What is the step response of the plant (i.e. the output resulting from input $\mathbf{1}(t)$) ?

Consider now a second plant with transfer function:

$$H(s) = \frac{1}{\frac{c_2}{2} + s}.$$

5) The output signal of the first plant is fed into the second plant to form a *combined plant*. Assume that $c_1, c_2 > 0$. Describe the combined plant as a second order system:

5a) What are the parameters, ζ and ω_n?

5b) Does it have resonance in any frequencies?

6) Assume that the plant is now controlled in closed loop with a PD compensator with parameters $K_P = \frac{1}{2}$ and $K_D = 2$. Consider now the controlled system:

6a) Is there a steady state error to the step response or is it zero? If there is a steady state error, explain briefly why? Further, suggest a modification to the controller that will remove the error.

6b) For values of $c_1, c_2 = 1$ is there a resonant peak? If so, explain briefly (1-3 lines) what this means.

6c) Assume that in the same controlled system, there is a pure delay of 3 time units on the feedback loop. I.e. the signal value that is subtracted from the reference at time t_0 is the output of the system at time $t_0 - 3$. Write the closed loop transfer function.

(Bonus) Return back to the single original plant. Set $c_1, c_2 = 1$. Let now $y[k]$ be a discrete time sequence of samples at times $k = 0, 1, 2, \ldots$, of the output resulting from the input $u(t) = \mathbf{1}(t-1)$ (and $y(0) = y[0] = 0$). Write an expression for $y[k]$.

Good Luck.