An input output system (plant) follows this differential equation:

\[ \dot{y}(t) + c_1 y(t) = c_2 u(t), \]

with \( c_1, c_2 \in \mathbb{R} \).

1) Treating \( u(\cdot) \) as the input and \( y(\cdot) \) as the output, write the transfer function of the plant.

2) For what values of \( c_1, c_2 \) is the plant strictly stable?

3) What is the step response of the plant (i.e. the output resulting from input \( 1(t) \)) ?

Consider now a second plant with transfer function:

\[ H(s) = \frac{1}{\frac{c_2}{2} + s}. \]

5) The output signal of the first plant is fed into the second plant to form a combined plant. Assume that \( c_1, c_2 > 0 \). Describe the combined plant as a second order system:

5a) What are the parameters, \( \zeta \) and \( \omega_n \)?

5b) Does it have resonance in any frequencies?

6) Assume that the plant is now controlled in closed loop with a PD compensator with parameters \( K_P = \frac{1}{2} \) and \( K_D = 2 \). Consider now the controlled system:

6a) Is there a steady state error to the step response or is it zero? If there is a steady state error, explain briefly why? Further, suggest a modification to the controller that will remove the error.

6b) For values of \( c_1, c_2 = 1 \) is there a resonant peak? If so, explain briefly (1-3 lines) what this means.

6c) Assume that in the same controlled system, there is a pure delay of 3 time units on the feedback loop. I.e. the signal value that is subtracted from the reference at time \( t_0 \) is the output of the system at time \( t_0 - 3 \). Write the closed loop transfer function.

(Bonus) Return back to the single original plant. Set \( c_1, c_2 = 1 \). Let now \( y[k] \) be a discrete time sequence of samples at times \( k = 0, 1, 2, \ldots \), of the output resulting from the input \( u(t) = 1(t - 1) \) (and \( y(0) = y[0] = 0 \)). Write an expression for \( y[k] \).

Good Luck.