MATH4406 (Control Theory)
Practice for Quiz 1 (Units 2 and 3)

Prepared by Yoni Nazarathy, Last Updated: August 15, 2012

This note contains examples of questions dealing with PID controllers and second order systems. This is the subject of Quiz 1.

Consider a feedback system with controller $G_1(s)$ and plant $H(s)$ as depicted in class (see the file photos_2_3.pdf). Assume that the plant has transfer function

$$H(s) = \frac{2}{s(s+3)}.$$

- Find a controller $G_1(s) = K_1$ such that the overshoot is $\leq 10\%$ and that the gain of H_c at $s = 0$ is 1.
- Find the settling time (to $\pm 2\%$) of the controller.
- Are there any resonance points for the controller that you found? If so, what are they?
- What is the steady state error of $H_c(s)$ to a constant reference signal? What is the steady state error for a ramp ($r(t) = t$) input signal?

Consider now a controller of the form $K(s) = K_p + \frac{K_I}{s}$ (PI controller).

- Design K_P and K_I such that in closed loop the following demands are met: (1) $H_c(0) = 1$. (2) Overshoot $\leq 10\%$. (3) Settling time to 5% is less than 2 time units.
- What is the system type of the controlled system now?

Consider now a pure D controller, is this a good idea? What is the problem with this controller?

Another question: Consider the full PID controller (with parameters K_P, K_I, K_D) and a second order system with parameters ζ, ω_n and the constant gain (DC gain), A. Investigate $H_c(s)$ for the closed loop system. Write the $\tilde{\zeta}$, $\tilde{\omega}_n$ and $\tilde{A}$ for the closed loop system in terms of the parameters of the plant and the controller.