

MATH4406 (Control Theory)
Quiz 2 (Unit 4) - September 6, 2012.
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Quiz duration: 40 minutes.

Consider the system,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t),\end{aligned}$$

with,

$$A = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

1) Write the impulse response matrix of the system, i.e. the output $y(t)$ resulting from input, $u(t) = (\delta(t), \delta(t))'$ with $x(0) = (0, 0)$.

2) For what values of α is the system controllable?

3) For what values of α is the system observable?

4) Set now $\alpha = 0$. Determine a feedback control law,

$$u(t) = Fx(t) + r(t),$$

so that the eigenvalues of the resulting system are -1 and -2 and the resulting system is not observable.

Note: If you are not able to satisfy both of the above criteria, partial points will be given for satisfying one of the above. Further, if you can formulate the problem of satisfying both of the above criteria clearly, but are not able to solve it, you will also get partial points.

Good Luck.