MATH4406 (Control Theory)
Quiz 3 (Unit 5) - September 20, 2012.

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Quiz duration: 40 minutes.

1) Consider the system,

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t),$$

with state-feedback controller,

$$u(t) = [\alpha \quad 1]x(t).$$

Assume we want to show that the closed loop system (with 0 input is stable) using the Lyapounov function $V(x) = x'x$. For which values of α is this possible?

2) Consider discrete time linear systems,

$$x(k+1) = Ax(k),$$

with $A \in \mathbb{R}^{n \times n}$. Assume that we want to find a Lyapounov function of the form,

$$V(x) = Qx + x'Px.$$

Show that in that case the matrixes Q and P need to satisfy,

$$(QA - Q)x + x'(A'PA - P)x < 0, \qquad \forall x \neq 0.$$

3) Consider the system:

$$\begin{aligned} \dot{x_1} &= x_2(1 - x_1) \\ \dot{x_2} &= -x_1(1 - x_2) \end{aligned}$$

a) Show that 0 is an equilibrium point.
b) Show that the system is stable. If you wish, try using the Lyapounov function,

$$V(x_1, x_2) = -x_1 - \log(1 - x_1) - x_2 - \log(1 - x_2).$$

Good Luck.