1) Consider the system,

\[ \dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \]

with state-feedback controller,

\[ u(t) = [\alpha \ 1] x(t). \]

Assume we want to show that the closed loop system (with 0 input is stable) using the Lyapounov function \( V(x) = x'x \). For which values of \( \alpha \) is this possible?

2) Consider discrete time linear systems,

\[ x(k + 1) = Ax(k), \]

with \( A \in \mathbb{R}^{n \times n} \). Assume that we want to find a Lyapounov function of the form,

\[ V(x) = Qx + x'Px. \]

Show that in that case the matrixes \( Q \) and \( P \) need to satisfy,

\[ (QA - Q)x + x'(APA - P)x < 0, \quad \forall x \neq 0. \]

3) Consider the system:

\[ \begin{align*}
\dot{x}_1 &= x_2(1 - x_1) \\
\dot{x}_2 &= -x_1(1 - x_2)
\end{align*} \]

a) Show that 0 is an equilibrium point.

b) Show that the system is stable. If you wish, try using the Lyapounov function,

\[ V(x_1, x_2) = -x_1 - \log(1 - x_1) - x_2 - \log(1 - x_2). \]

Good Luck.